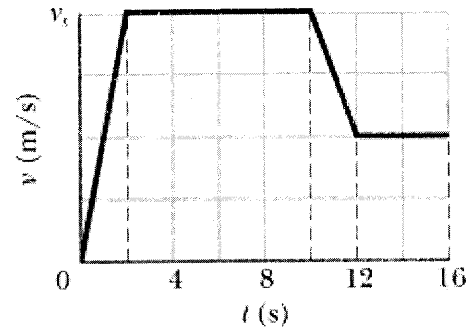
Name Solutions Section: 4341/6541 EXAM-1, SP211 Fall 2012

1. The figure's vertical scaling is set by $v_s = 8.0$ m/s



$v = v_0 + at$ [t: 0-2 sec]

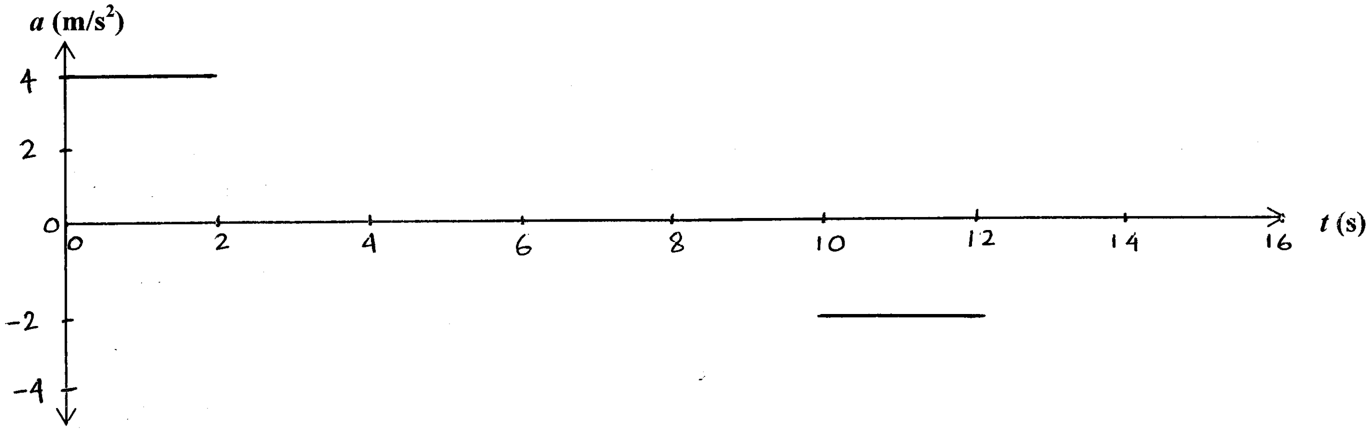
$\Rightarrow 8 = 0 + a(2)$

$\Rightarrow a = 4\ m/s^2$

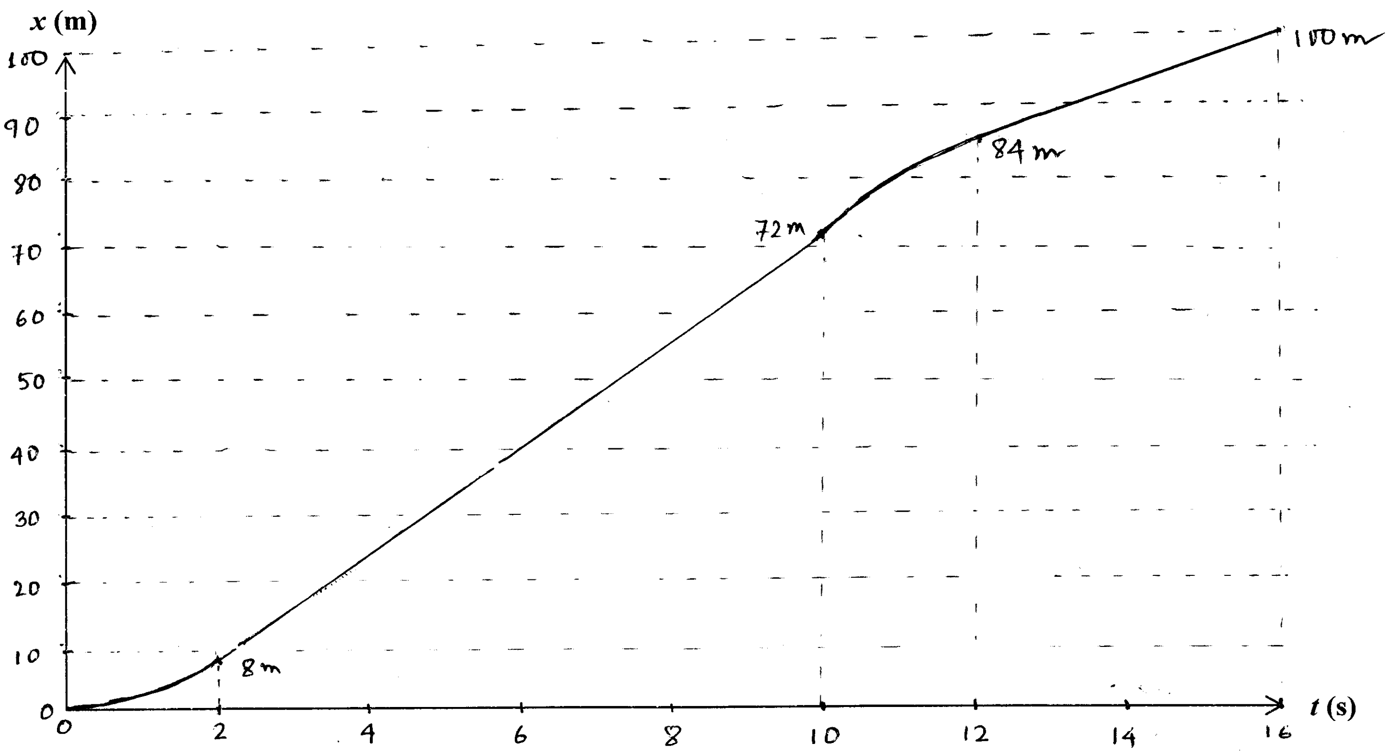
$v = v_0 + at$ (t: 10 - 12 sec)

$\Rightarrow 4 = 8 + a.2 \Rightarrow a = -2\ m/s^2$

a) [10 points] Draw the ***a*** *vs.* ***t*** graph. Be sure to label and scale *a* (m/s²) in the graph.



b) [15 points] Draw the ***x*** *vs.* ***t*** graph. Be sure to label and scale *x* (m) in the graph. (Assume at $t = 0$, $x = 0$)



Points earned on this page______

1 (b) {calculation]

t: [0 - 2 s] $x = v_0 t + \frac{1}{2} a t^2$

$= 0 + \frac{1}{2} 4 \cdot 4 = 8 m$

t: [2 - 10 s] $x = x_0 + v_0 t + \cancel{\frac{1}{2} a t^2}^{0}$

$= 8m + 8(8) m = 72 m$

t: [10 - 12 s] $x = x_0 + v_0 t + \frac{1}{2} a t^2$

$= 72 + 8(2) + \frac{1}{2}(-2) 4$

$= 72 + 16 - 4$

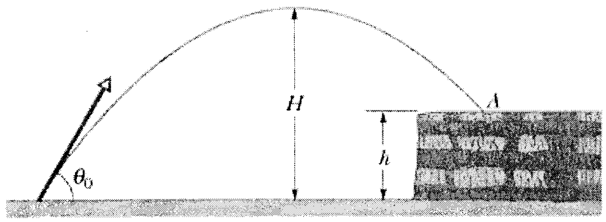
$= 84 m$

t: [12 - 16 s] $x = x_0 + v_0 t + \cancel{\frac{1}{2} a t^2}^{0}$

$= 84 + 4 \times 4$

$= 84 + 16 = 100 m$

2. In the figure, a stone is projected at a cliff of height $h = 52.0$ m with an initial speed of 42.0 m/s directed at angle θ_o above the horizontal. The stone strikes at A, 5.50 s after launching.



(a) [6 points] Find the angle θ_o.

$$v_{0x} = v_0 \cos\theta_0 \; ; \; v_{0y} = v_0 \sin\theta_0$$

$$y = v_{0y} t - \frac{1}{2} g t^2 = (v_0 \sin\theta_0) t - \frac{1}{2} g t^2$$

$$\Rightarrow \sin\theta_0 = \frac{y + \frac{1}{2} g t^2}{v_0 t} = \frac{(52\text{ m}) + \frac{1}{2}(9.8\text{ m/s}^2)(5.5\text{ s})^2}{(42\text{ m/s})(5.5\text{ s})} = 0.8667$$

$$\Rightarrow \theta_0 = \sin^{-1}(0.8667) = 60.1^\circ$$

$$\boxed{\theta_o = 60.1^\circ}$$

(b) [6 points] Find the <u>*velocity vector*</u> $\vec{v}$ of the stone just before impact at A.

$$v_x = v_{0x} = v_0 \cos\theta_0 = (42\text{ m/s}) \cos 60.1^\circ = 20.9\text{ m/s}$$

$$v_y = v_{0y} - gt = v_0 \sin\theta_0 - gt = (42\text{ m/s}) \sin 60.1^\circ - (9.8\text{ m/s}^2)(5.5\text{ s})$$
$$= -17.5\text{ m/s}$$

$$\boxed{\vec{v} = 20.9\text{ m/s}\,\hat{i} - 17.5\text{ m/s}\,\hat{j}}$$

(c) [5 points] Find the <u>*magnitude*</u> of $\vec{v}$ and the <u>*angle*</u> it makes with the positive x axis.

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} \; ; \; \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\boxed{|\vec{v}| = 27.3\text{ m/s} \; ; \; \theta = -39.9}$$

(d) [8 points] Find the maximum height H reached above the ground. (Show your complete work.)

$v_y = 0$, at $y = y_{max} = H$

$$v_y = 0 = v_0 \sin\theta_0 - (9.8\text{ m/s}^2) t_H$$

$$\Rightarrow t_H = \frac{(42\text{ m/s}) \sin 60.1}{9.8\text{ m/s}^2} = 3.7\text{ s}.$$

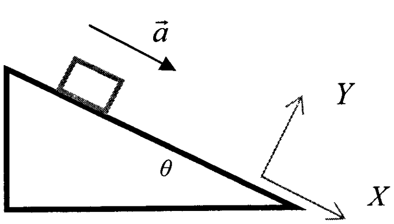
$$H = v_{0y} t_H - \frac{1}{2} g t_H^2$$
$$= (42\text{ m/s})(\sin 60.1^\circ)(3.7\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(3.7\text{ s})^2$$
$$= 67.6\text{ m}$$

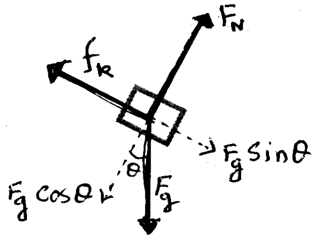
$$\boxed{H = 67.6\text{ m}}$$

Name *Solutions* Section: 4341/6541 EXAM-1, SP211 Fall 2012

3. A block of mass m accelerates down a slope of angle θ. The coefficient of kinetic friction is μ_k.

a. [4 + 4 points] Draw the free body diagram of the block and clearly label the forces acting on the block. Write Newton's second law for X and Y.





Forces on block

X: $F_g \sin\theta - f_k = ma$

Y: $F_N - F_g \cos\theta = 0$

$F_N = F_g \cos\theta = mg\cos\theta$

b. [4 points] Find the acceleration $\vec{a}$ in terms of m, g, θ, and μ_k.

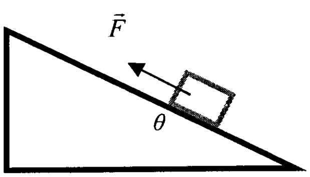
$$f_k = \mu_k F_N = \mu_k mg\cos\theta$$

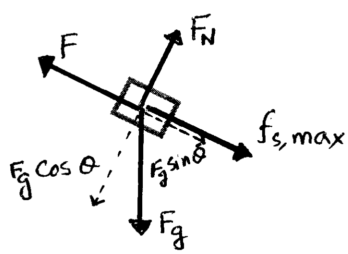
$$ma = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = (g\sin\theta - \mu_k \cos\theta)$$

$$\boxed{\vec{a} = (g\sin\theta - \mu_k \cos\theta)\,\hat{i}}$$

c. A force $\vec{F}$ slows the acceleration $\vec{a}$ to zero, and now the block is <u>on the verge of sliding up</u> the slope. [4 points] Draw the free body diagram of the block. [4 points] Write Newton's Second Law for X and Y (use the same coordinate system as part **a**). [5 points] Find the coefficient of static friction μ_s between the block and the slope. Express your answer in terms of m, g, θ, and F.





Forces on block

X: $F_g \sin\theta + f_{s,max} - F = 0$

Y: $F_N - F_g \cos\theta = 0$

$F_N = F_g \cos\theta = mg\cos\theta$

$$f_{s,max} = \mu_s F_N = \mu_s mg\cos\theta$$

From X: N2L

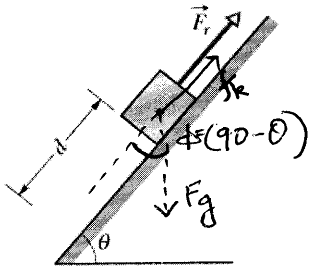
$$mg\sin\theta + \mu_s mg\cos\theta - F = 0$$

$$\Rightarrow \mu_s mg\cos\theta = F - mg\sin\theta$$

$$\Rightarrow \mu_s = \frac{F - mg\sin\theta}{mg\cos\theta}$$

$$\boxed{\mu_s = \frac{F - mg\sin\theta}{mg\cos\theta}}$$

4. In the figure below, a block of mass m slides down a ramp at angle θ while a worker pulls on the block (via a rope) with a force $\vec{F}_r$. The coefficient of kinetic friction between the block and the ramp is μ_k. As the block slides down a distance d along the ramp, find **(a)** [4 points] the work done by the gravitational force $\vec{F}_g$, **(b)** [4 points] the work done by the force $\vec{F}_r$, and **(c)** [4 points] the work done by the friction force. **(d)** [4 points] What is the total work done? **(e)** [5 points] If the initial speed of the block is v_o, find the final speed of the block after it slides the distance d down the ramp? [Hint: $\Delta K = W_{tot}$]
[Note: Express your symbolic answers in terms of m, g, F_r, μ_k, d, θ, and v_o]



$$W_g = \vec{F}_g \cdot \vec{d} = F_g d \cos(90-\theta) = mgd\sin\theta$$

$$W_{F_r} = \vec{F}_r \cdot \vec{d} = F_r d \cos(180) = -F_r d$$

$$W_{friction} = \vec{f}_k \cdot \vec{d} = f_k d \cos(180)$$

$$= -f_k d = \mu_k F_N d = -\mu_k mg\cos\theta\, d$$

$$\Delta K = W_{tot}$$

$$\Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mgd\sin\theta - F_r d - \mu_k mgd\cos\theta$$

$$\Rightarrow v_f^2 = v_o^2 + 2gd\sin\theta - \frac{2F_r}{m}d - 2\mu_k gd\cos\theta$$

(a)	$W_g = mgd\sin\theta$
(b)	$W_{Fr} = -F_r d$
(c)	$W_{friction} = -\mu_k mgd\cos\theta$
(d)	$W_{tot} = mgd\sin\theta - F_r d - \mu_k mgd\cos\theta$
(e)	$v_f = \left[v_o^2 + 2gd\sin\theta - \frac{2F_r}{m}d - 2\mu_k gd\cos\theta\right]^{1/2}$

Points earned on this page______

4 (f) [4 points] A block of mass *m* is held at rest against a compressed spring (spring constant *k*) on a frictionless horizontal surface. The spring is compressed a distance *d*. As soon as the spring is released, it shoots the block along the surface. What is the speed *v* of the block just after the spring is released? (Express your answer in terms of *m*, *k*, and *d*.)

$$E_{\text{before release}} = E_{\text{after release}}$$

$$\Rightarrow \cancel{K_i}^{\,0} + U_i = K_f + \cancel{U_f}^{\,0}$$

$$\Rightarrow \frac{1}{2} k d^2 = \frac{1}{2} m v^2$$

$$\Rightarrow v = \sqrt{k/m}\, d$$

$$\boxed{v = \sqrt{k/m}\; d}$$

Extra points:

[5 points] Name two conservative and two non-conservative forces.

Conservative

1. Force of gravity
2. Spring force

non-Conservative

1. Friction
2. Tension

Your score

Q#	Points
1	
2	
3	
4	
Total	