Oscillatory Motion

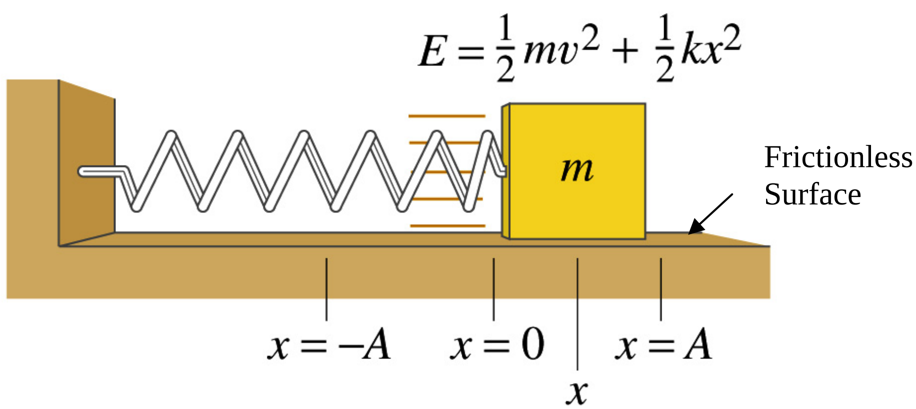
Our goal this semester is to understand how sound waves travel through the water so that we may exploit them to prosecute a target. We will start with simple models and increase complexity as we go. This course is meant to be directly applicable for the war-fighter.

To begin with lets go back to our childhood days looking out over the calm waters of the nearby pond. When you throw a rock in the water, you create a wave on the surface. If you closely watch a leaf on the surface, you will see it go up and down as the wave passes by, yet the leaf returns to its original position after the wave passes. This is a simple yet extremely important point regarding wave motion. The medium carrying the wave does not move with the wave, generally returning to its original position after the wave has gone past. The medium carrying the wave simply oscillates around an equilibrium position. To begin our study of underwater sound, we will look at the periodic nature of this motion. It is the basis of all mechanical wave motion.

Mass-Spring System

Hooke's Law and the Simple Harmonic Oscillator

An illustrative model to begin understanding acoustics is the problem of a simple mass-spring oscillating system. Begin with a mass attached to a perfect massless spring. The spring is attached to a firm wall and the mass sits on a frictionless surface. If the spring is displaced from the rest position of the system where x=0, the mass will move back and forth with a periodic motion centered about the x=0 position. This periodic motion can be described by a simple time varying equation, which should give us insight in to periodic wave motion.



From Hooke's Law, the restoring force of the spring is equal to:

$$F_{spring} = -kx$$

There is a minus sign in front of the spring constant because the force of the spring is in the opposite direction of the displacement of the mass. The displacement, x, is the distance the spring is stretched or compressed (and is equal to the displacement of the mass) from the x=0 or rest position of the spring.

We can now write an equation to relate the forces on the mass in the x-direction to the acceleration of the mass in the x-direction: (Or in other words apply Newton's second law for the motion only in the x-dimension.)

$$\sum \vec{F}_{block} = m_{block}\vec{a}$$

Since the only force on the block is due to the spring and all motion is along the x-axis, we can write the scalar equation,

$$F_{spring} = m_{block}a_x$$

$$-kx = m_{block}\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m_{block}}x = 0$$

This is a simple, second order differential equation that describes the motion of the mass. One solution for the position of the mass, x, as a function of time that satisfies the differential equation is:

$$x(t) = A\cos(\omega t + \phi)$$

where the angular frequency squared, $\omega^2 = \dfrac{k}{m_{block}}$ and A and ϕ, are unknown constants. Appendix A checks this solution and verifies the value of the angular frequency. We refer to quantity, $\omega t + \phi$, as the "phase" of the block's motion. The phase is generally expressed in radians and the motion repeats once the phase has changed by 2π. The amplitude of the oscillation, A and the initial phase of the oscillations ϕ, can **only** be solved for by knowing two initial conditions of the system.

Another solution to the second order differential equation is $x(t) = A\sin(\omega t + \phi)$. Another uses complex exponentials, $x(t) = Ae^{i(\omega t + \phi)}$ and is shorthand to signify only the real part of this expression is the solution to the second order differential equation. It is a worthwhile exercise for the student to show that both these solutions also satisfy the second order differential equation.

We must be able to find the velocity and acceleration of the mass as a function of time to use the initial conditions of the system. To calculate these quantities, we must just take the derivative as shown below.

$$v_x(t) = \frac{dx(t)}{dt} = A\frac{d\left[\cos(\omega t + \phi)\right]}{dt} = -\omega A \sin(\omega t + \phi)$$
$$a_x(t) = \frac{d^2x(t)}{dt^2} = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$$

Looking at the above equations , we can obtain the maximum values of the velocity and acceleration. These maximum values are:

$$v_{max} = \omega A$$
$$a_{max} = \omega^2 A$$

An important characteristic of the system is the angular frequency. Using the above equations, and knowing a couple of the parameters of the system as a function of time, we can solve for the more easily understandable quantities, the frequency and period of the system. These can be calculated from the following equations:

$$f = \frac{\omega}{2\pi}$$
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Example Problem

Let's look at an example: A mass of 200 grams is connected to a light spring that has a spring constant (k) of 5.0 N/m and is free to oscillate on a horizontal, frictionless surface. If the mass is displaced 5.0 cm from the rest position and released from rest find: a) the period of its motion, b) the maximum speed and c) the maximum acceleration of the mass.

Using the relationships given above, the following can be calculated:

a) $\omega = \sqrt{k/m} == \sqrt{\frac{5.0\ \text{N/m}}{200\text{x}10^{-3}\ \text{kg}}} = 5.0\ \text{rad/sec}$

$T = \frac{2\pi}{\omega} = 1.26\ \text{seconds}$

b) Using the initial conditions that the mass was displaced 5.0 cm and let go from rest at $t = 0$ seconds

$x(0\ \text{sec}) = 5.0\ \text{cm} = A\,cos([(5.0\ \text{rad/s})(0\ \text{seconds})] + \phi)$

and $v(0\ \text{sec}) = 0\ \text{cm/s} = -(5.0\ \text{rad/s})A\,sin([(5.0\ \text{rad/s})(0\ \text{sec})] + \phi)$

therefore soving for A and ϕ: $A = 5.0$ cm and $\phi = 0.0$ rad

$v_{max} = \omega A = (5.0\ \text{rad/s})(5.0\text{x}10^{-2}\ \text{m}) = 0.25\ \text{m/s}$

c) $a_{max} = \omega^2 A = 1.25\ \text{m/s}^2$

Energy in the Mass Spring System

The energy of the mass spring system can be found at any time by summing the kinetic energy of the mass with the potential energy of the spring.

$$E = K + U = \frac{1}{2}m_{block}v^2 + \frac{1}{2}kx^2$$

When the displacement of the mass from the equilibrium position is at the maximum displacement, x=A, the velocity of the spring is instantaneously zero. As there are no non-conservative forces such as friction, energy is conserved and the total energy at any time is simply

$$E_{max} = \frac{1}{2}kA^2$$

This is very powerful because it allows us to calculate the total energy of an oscillating mass very simply and then calculate the velocity when the position is known or vice versa. Conceptually, we view the continuous motion of a mass spring oscillator as the perpetual transfer of energy back and forth between kinetic and potential forms. Without any energy loss (due, for example, to friction) this transfer will continue indefinitely.

The average energy in a simple harmonic oscillator is calculated using the following definition for the average of a periodic function:

$$\left\langle f(t)\right\rangle \equiv \frac{1}{T}\int_0^T f(t)\,dt$$

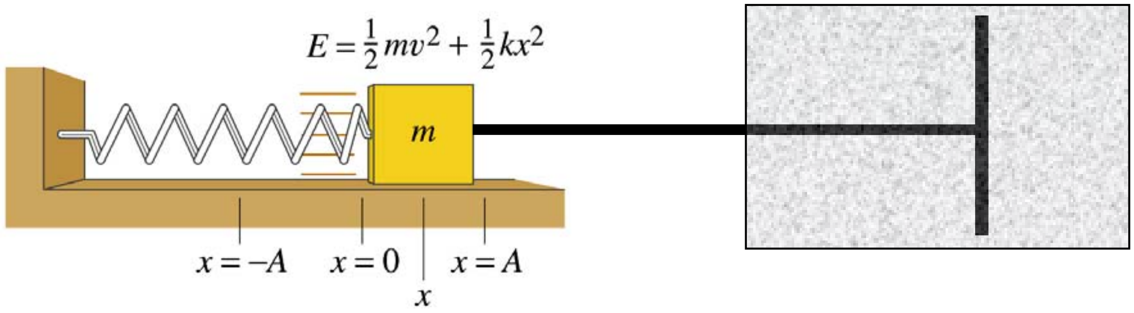
For kinetic and potential energy we find that since the time average of the square of the sine and cosine is one half, i.e. $\left\langle \sin^2\theta(t)\right\rangle = \left\langle \cos^2\theta(t)\right\rangle = \frac{1}{2}$, then

$$\left\langle K\right\rangle = \frac{1}{2}m_{block}\left\langle v^2\right\rangle = \frac{1}{2}m_{block}\omega^2A^2\left\langle \sin^2(\omega t+\phi)\right\rangle = \frac{1}{4}m_{block}\omega^2A^2 = \frac{1}{4}kA^2$$

$$\left\langle U\right\rangle = \frac{1}{2}k\left\langle x^2\right\rangle = \frac{1}{2}kA^2\left\langle \cos^2(\omega t+\phi)\right\rangle = \frac{1}{4}kA^2$$

This shows that on average, the kinetic energy of a simple harmonic oscillator and the potential energy of a simple harmonic oscillator are the same, each being exactly one half the total energy of the harmonic oscillator.

Damped Mass-Spring System



Hooke's Law Revisited

The approach used above for the simple harmonic oscillator will work for a damped oscillator with a small modification. Some device such as a “dashpot” provides a mechanism by which energy is removed from the system. A dashpot is like a shock absorber with a piston moving through a viscous fluid. We model the dashpot such that it provides a resistive force to the system that is proportional to the speed of the mass.

$$F_{damping} = -bv$$

The constant of proportionality, b, depends on such factors as fluid viscosity, size, shape and roughness of the piston, and the space between the piston and the fluid chamber walls. Because of this new force, our x component equation from Newton’s second law gains an additional term.

$$-kx - bv_x = m_{block}a_x$$

The new equation of motion then becomes:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

A solution to the equation of motion is:

$$x = Ae^{-\alpha t}\cos(\omega' t + \phi)$$

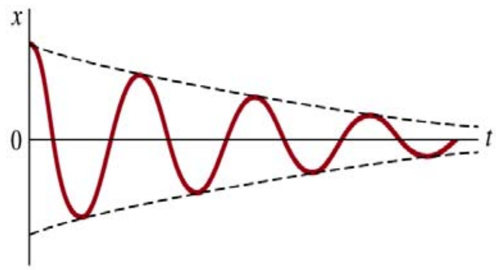
Again the initial amplitude of oscillation, A, and the initial phase, ϕ, are arbitrary constants of the second order differential equation. The angular frequency is slightly different from the undamped case:

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

The amplitude decays exponentially with time with a decay constant, α:

$$\alpha = \frac{b}{2m}$$

Appendix B shows that our solution satisfies the equation of motion and that the angular frequency and damping constants are correct. When plotted for typical values of k, m, and b, the motion of the mass looks like the graph below. As the amplitude decreases we can see that energy is leaving the system, mostly as heat generated from friction as the piston moves through the viscous fluid in the dashpot. Later in the course we will discuss losses of energy due to various mechanisms in the ocean draining energy from an acoustic wave. Although greatly simplified, the damped oscillator provides a satisfactory model of what the medium must be experiencing as the wave passes.

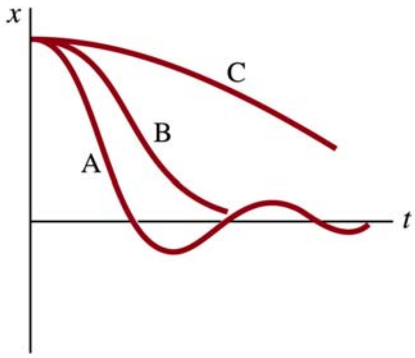


Overdamped and critically damped motion

One interesting result of the expression for the angular frequency is that if the damping constant is large enough, ω can become zero or even an imaginary number. This occurs whenever the damping constant is sufficiently large compared to the mass and the spring constant.

$$b^2 \geq 4mk$$

When this happens we say that the system is "over damped" and the motion resembles that of curve C below. Note that it can take significant time for the mass to relax to its equilibrium position in this case. When the angular frequency is exactly zero, the system is said to be "critically damped" as shown by curve B. In this case, the mass returns to the equilibrium position faster and without overshoot.



Appendix A - Checking the solution for simple harmonic motion

$$x(t) = A\cos(\omega t + \phi)$$
$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$
$$a = \frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$$

Substituting into the equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m_{block}} x = -A\omega^2 \cos(\omega t + \phi) + \frac{k}{m_{block}} A\cos(\omega t + \phi) = 0$$
$$-\omega^2 \cos(\omega t + \phi) + \frac{k}{m_{block}} \cos(\omega t + \phi) = \left(-\omega^2 + \frac{k}{m_{block}}\right)\cos(\omega t + \phi) = 0$$

So this solution works so long as $\omega^2 = \frac{k}{m_{block}}$

You should be able to repeat this process for other solutions.

Appendix B - Checking the solution for damped harmonic motion

$$x = Ae^{-\alpha t}\cos(\omega' t + \phi)$$

$$v = \frac{dx}{dt} = -Ae^{-\alpha t}\omega'\sin(\omega' t + \phi) + A(-\alpha)e^{-\alpha t}\cos(\omega' t + \phi)$$

$$= -Ae^{-\alpha t}\left[\omega'\sin(\omega' t + \phi) + \alpha\cos(\omega' t + \phi)\right]$$

$$a = \frac{d^2x}{dt^2} = -Ae^{-\alpha t}\omega'^2\cos(\omega' t + \phi) + A\alpha e^{-\alpha t}\omega'\sin(\omega' t + \phi) + A\alpha e^{-\alpha t}\omega'\sin(\omega' t + \phi) + A\alpha^2 e^{-\alpha t}\cos(\omega' t + \phi)$$

$$= Ae^{-\alpha t}\left\{2\alpha\omega'\sin(\omega' t + \phi) + \left[\alpha^2 - \omega'^2\right]\cos(\omega' t + \phi)\right\}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$Ae^{-\alpha t}\left\{2\alpha\omega'\sin(\omega' t + \phi) + \left[\alpha^2 - \omega'^2\right]\cos(\omega' t + \phi)\right\} - \frac{b}{m}Ae^{-\alpha t}\left[\omega'\sin(\omega' t + \phi) + \alpha\cos(\omega' t + \phi)\right] + \frac{k}{m}Ae^{-\alpha t}\cos(\omega' t + \phi) = 0$$

$$Ae^{-\frac{b}{2m}t}\left\{\left[2\alpha\omega' - \frac{b}{m}\omega'\right]\sin(\omega' t + \phi) + \left[\alpha^2 - \omega'^2 - \frac{b}{m}\alpha + \frac{k}{m}\right]\cos(\omega' t + \phi)\right\} = 0$$

$$\alpha = \frac{b}{2m} \qquad \frac{k}{m} - \left(\frac{b}{2m}\right)^2 - \omega'^2 = 0$$

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

Problems

1. A particle oscillates with simple harmonic motion so that its displacement varies according to the expression $x=(5.0\text{ cm})\cos(2t+\pi/6)$, where x is in centimeters and t is in seconds. At t=0, find
   a) the displacement of the particle,
   b) its velocity, and
   c) its acceleration.
   d) Find the period and amplitude of the motion.

2. A piston in an automobile engine is in simple harmonic motion. If its amplitude of oscillation from the centerline is ±5.0 cm and its mass is 2.0 kg, find the maximum velocity and acceleration of the piston when the auto engine is running at the rate of 3600 rev/min.

3. A 20.0 g particle moves in simple harmonic motion with a frequency of 3.0 oscillations/sec and amplitude of 5.0 cm.
   a) Through what total distance does the particle move during one cycle of its motion?
   b) What is its maximum speed? Where does this occur?
   c) Find the maximum acceleration of the particle. Where in the motion does the maximum acceleration occur?

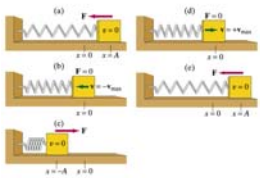
4. A 1.0 kg mass attached to a spring of force constant 25.0 N/m oscillates on a horizontal, frictionless track. At t=0, the mass is released from rest at x = -3.0 cm. (That is, the spring is compressed by 3.0 cm) Find
   a) the period of its motion,
   b) the maximum values of its speed and acceleration, and
   c) the displacement, velocity, and acceleration as functions of time.

5. A 5.0 kg mass attached to a spring of force constant 8.0 N/m vibrates in simple harmonic motion with amplitude of 10.0 cm. Calculate
   a) the maximum value of its speed and acceleration,
   b) the speed and acceleration when the mass is 6.0 cm from the equilibrium position, and
   c) the time it takes the mass to move from x = 0 to x = 8.0 cm.
   d) the total energy of the system
   e) the speed of the 5.0 kg mass when x = 5.0 cm

6. A block of unknown mass is attached to a spring of force constant 6.5 N/m and undergoes simple harmonic motion with an amplitude of 10.0 cm. When the mass is halfway between its equilibrium position and endpoint, its speed is measured to be +30 cm/s. Calculate
   a) the mass of the block,
   b) the period of the motion, and
   c) the maximum acceleration of the block.

Lesson 1 - Oscillations

- Harmonic Motion
  Circular Motion
- Simple Harmonic Oscillators
  - Linear - Horizontal/Vertical Mass-Spring Systems
- Energy of Simple Harmonic Motion



Math Prereqs

$$\frac{d}{d\theta}\sin\theta = \qquad \cos\theta$$

$$\frac{d}{d\theta}\cos\theta = \qquad -\sin\theta$$

$$\int_0^{2\pi}\cos\theta d\theta = \int_0^{2\pi}\sin\theta d\theta = \qquad 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta d\theta = \frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta d\theta = \qquad \frac{1}{2}$$

Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\cos\theta + \cos\phi = 2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$

Math Prereqs

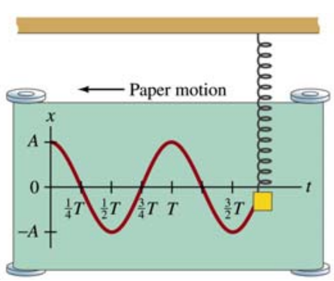
$$\langle\ \rangle = \text{"Time Average"}$$

$$\langle f(t)\rangle \equiv \frac{1}{T}\int_0^T f(t)\,dt$$

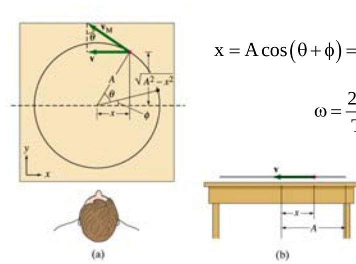
Example:

$$\left\langle \cos^2\left(\frac{2\pi}{T}t\right)\right\rangle = \frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi}{T}t\right)dt = \frac{1}{T}\int_0^T\left[\frac{1}{2} + \frac{1}{2}\cos\left(2\frac{2\pi}{T}t\right)\right]dt = \frac{1}{2}$$

Harmonic



Relation to circular motion



$$x = A\cos(\theta + \phi) = A\cos(\omega t + \phi)$$

$$\omega = \frac{2\pi}{T}$$

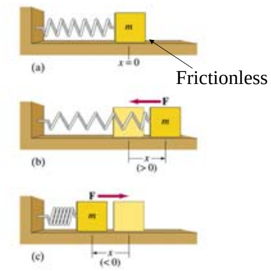
Horizontal mass-spring

$$\sum F = ma$$

Hooke's Law: $F_s = -kx$

$$-kx = m_{block}\frac{d^2x}{dt^2}$$

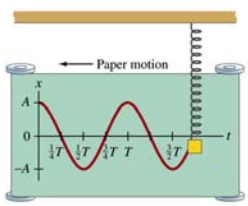
$$\frac{d^2x}{dt^2} + \frac{k}{m_{block}}x = 0$$



Solutions to differential equations

- Guess a solution
- Plug the guess into the differential equation
  - You will have to take a derivative or two
- Check to see if your solution works.
- Determine if there are any restrictions (required conditions).
- If the guess works, your guess is a solution, but it might not be the only one.
- Look at your constants and evaluate them using initial conditions or boundary conditions.

Our guess



$$x = A\cos(\omega t + \phi)$$

Definitions

$$x = A\cos(\omega t + \phi)$$

- Amplitude - (A) Maximum value of the displacement (radius of circular motion). Determined by initial displacement and velocity.
- Angular Frequency (Velocity) - (ω) Time rate of change of the phase.
- Period - (T) Time for a particle/system to complete one cycle.
- Frequency - (f) The number of cycles or oscillations completed in a period of time
- Phase - ($\omega t + \phi$) Time varying argument of the trigonometric function.
- Phase Constant - (ϕ) Initial value of the phase. Determined by initial displacement and velocity.

The restriction on the solution

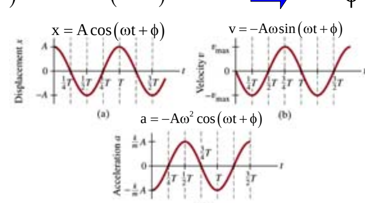
$$\omega^2 = \frac{k}{m_{block}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m_{block}}}$$

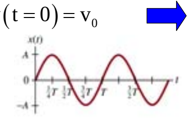
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_{block}}{k}}$$

The constant – phase angle

$x(t=0) = A$ $\quad v(t=0) = 0$ $\quad \Rightarrow \quad \phi = 0$

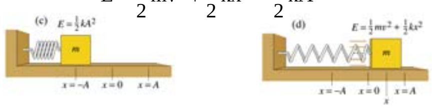


$x(t=0) = 0$ $\quad v(t=0) = v_0$ $\quad \Rightarrow \quad \phi = \frac{\pi}{2}$



Energy in the SHO

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$



$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

Average Energy in the SHO

$$x = A\cos\left(\omega t + \phi\right)$$

$$\left\langle U\right\rangle = \frac{1}{2}k\left\langle x^2\right\rangle = \frac{1}{2}kA^2\left\langle \cos^2\left(\omega t+\phi\right)\right\rangle = \frac{1}{4}kA^2$$

$$v = \frac{dx}{dt} = -A\omega\sin\left(\omega t+\phi\right)$$

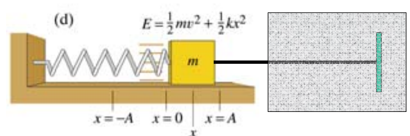
$$\left\langle K\right\rangle = \frac{1}{2}m\left\langle v^2\right\rangle = \frac{1}{2}m\omega^2A^2\left\langle \sin^2\left(\omega t+\phi\right)\right\rangle = \frac{1}{4}m\omega^2A^2 = \frac{1}{4}kA^2$$

$$\left\langle K\right\rangle = \left\langle U\right\rangle$$

Example

- A mass of 200 grams is connected to a light spring that has a spring constant (k) of 5.0 N/m and is free to oscillate on a horizontal, frictionless surface. If the mass is displaced 5.0 cm from the rest position and released from rest find:
- a) the period of its motion,
- b) the maximum speed and
- c) the maximum acceleration of the mass.
- d) the total energy
- e) the average kinetic energy
- f) the average potential energy

Damped Oscillations



"Dashpot"

$$F_{damping} = -bv$$

$$-kx - b\frac{dx}{dt} = ma$$

Equation of Motion $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

Solution $$x = Ae^{-\alpha t}\cos\left(\omega' t + \phi\right)$$

$$x = Ae^{-\alpha t}\cos\left(\omega' t+\phi\right)$$

$$v = \frac{dx}{dt} = -Ae^{-\alpha t}\omega'\sin\left(\omega' t+\phi\right) + A\left(-\alpha\right)e^{-\alpha t}\cos\left(\omega' t+\phi\right)$$

$$= -Ae^{-\alpha t}\left[\omega'\sin\left(\omega' t+\phi\right) + \alpha\cos\left(\omega' t+\phi\right)\right]$$

$$a = \frac{d^2x}{dt^2} = -Ae^{-\alpha t}\omega'^2\cos\left(\omega' t+\phi\right) + A\alpha e^{-\alpha t}\omega'\sin\left(\omega' t+\phi\right) + A\alpha e^{-\alpha t}\omega'\sin\left(\omega' t+\phi\right) + A\alpha^2 e^{-\alpha t}\cos\left(\omega' t+\phi\right)$$

$$= Ae^{-\alpha t}\left\{2\alpha\omega'\sin\left(\omega' t+\phi\right) + \left[\alpha^2 - \omega'^2\right]\cos\left(\omega' t+\phi\right)\right\}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$Ae^{-\alpha t}\left\{2\alpha\omega'\sin\left(\omega' t+\phi\right) + \left[\alpha^2-\omega'^2\right]\cos\left(\omega' t+\phi\right)\right\} - \frac{b}{m}Ae^{-\alpha t}\left[\omega'\sin\left(\omega' t+\phi\right) + \alpha\cos\left(\omega' t+\phi\right)\right] + \frac{k}{m}Ae^{-\alpha t}\cos\left(\omega' t+\phi\right) = 0$$

$$Ae^{-\frac{b}{2m}t}\left\{\left[2\alpha\omega' - \frac{b}{m}\omega'\right]\sin\left(\omega' t+\phi\right) + \left[\alpha^2 - \omega'^2 - \frac{b}{m}\alpha + \frac{k}{m}\right]\cos\left(\omega' t+\phi\right)\right\} = 0$$

$$\boxed{\alpha = \frac{b}{2m}}$$

$$\frac{k}{m} - \left(\frac{b}{2m}\right)^2 - \omega'^2 = 0 \Rightarrow \boxed{\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

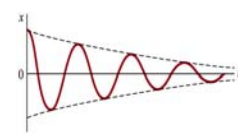
Damped frequency oscillation

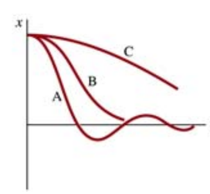
$$\alpha = \frac{b}{2m}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$b^2 \geq 4mk$$

B - Critical damping (=)
C - Over damped (>)

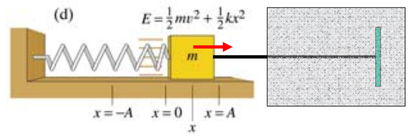




Giancoli 14-55

- A 750 g block oscillates on the end of a spring whose force constant is k = 56.0 N/m. The mass moves in a fluid which offers a resistive force F = -bv where b = 0.162 N-s/m.
  - What is the period of the motion? What if there had been no damping?
  - What is the fractional decrease in amplitude per cycle?
  - Write the displacement as a function of time if at t = 0, x = 0; and at t = 1.00 s, x = 0.120 m.

Forced vibrations



$$F_{ext} = F_0 \cos \omega t \qquad -kx - b\frac{dx}{dt} + F_0 \cos \omega t = ma$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos \omega t$$

$$x = A_0 \sin(\omega t + \phi_0)$$

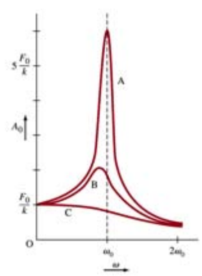
Resonance

$$x = A_0 \sin(\omega t + \phi_0)$$

Natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$

$$A_0 = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \frac{b^2\omega^2}{m^2}}}$$

$$\phi_0 = \tan^{-1}\left(\frac{m\left(\omega^2 - \omega_0^2\right)}{b\omega}\right)$$

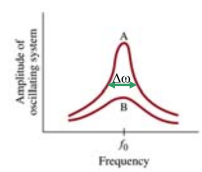


Quality (Q) value

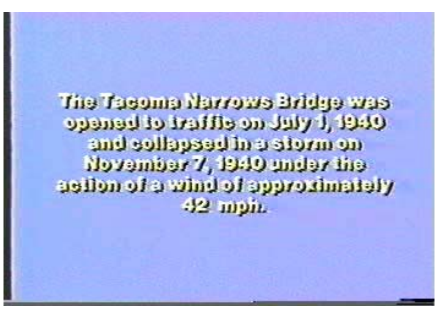
- Q describes the sharpness of the resonance peak
- Low damping give a large Q
- High damping gives a small Q
- Q is inversely related to the fraction width of the resonance peak at the half max amplitude point.

$$Q = \frac{m\omega_0}{b}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$



Tacoma Narrows Bridge



Tacoma Narrows Bridge (short clip)