Logarithms and Levels

Logarithms are used to compare two quantities to one another quickly with an easy frame of reference. It is particularly useful if there is a large difference in orders of magnitude between quantities as in acoustic pressure or acoustic energy calculations. We will see how useful logarithms can be in our next lesson. For now, let's concentrate on review of some of the basic principles leading up to our use of logarithms.

Unless otherwise stated, we will be working solely with logarithms that are in base 10 (Briggsian) . Some useful relationships to remember when working with logarithms are:

1. $y = 10^x$ then $\log_{10}(y) = x$

2. $\log(xy) = \log(x) + \log(y)$

3. $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$

4. $10\log(x^n) = n10\log(x)$

Intensity Level

In the last lesson, we defined the time average intensity in relation to the time average or rms pressure as well as the maximum acoustic pressure.

$$\langle I \rangle = \frac{\langle p^2 \rangle}{\rho c} = \frac{p_{max}^2}{2\rho c}$$

The intensity is a useful quantity because it quantifies the power in an acoustic wave, but because of the large variation in magnitudes of Intensity, it is more useful to use logarithms to compare intensities. The below table demonstrates the wide variation in Intensity for typical sounds in air.

We will start by defining a new quantity, **L**, the **intensity level**, which has units of dB.

$$L \equiv 10\log\frac{\langle I \rangle}{I_0}$$

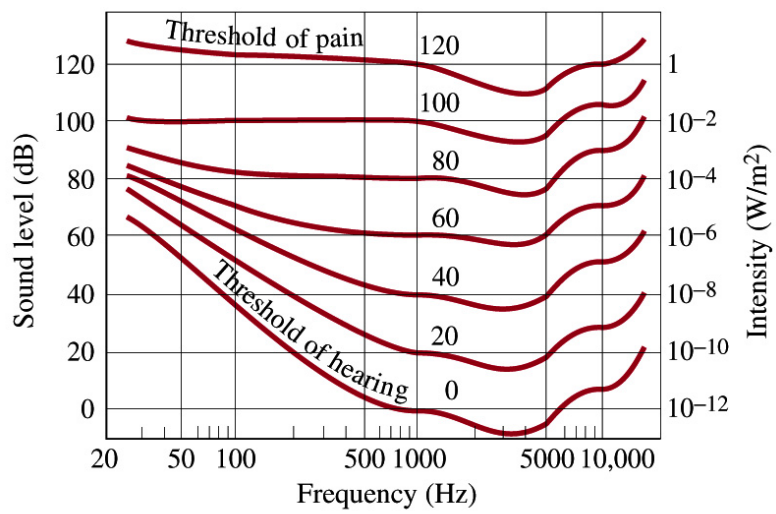
where:

<I> is the time average intensity of the sound wave.

I_0 is the reference level used for comparison purposes.

Source	Intensity (W/m^2)	Intensity Level (dB)
Jet Plane	100	140
Pain Threshold	1	120
Siren	$1x10^{-2}$	100
Busy Traffic	$1x10^{-5}$	70
Conversation	$3x10^{-6}$	65
Whisper	$1x10^{-10}$	20
Rustle of leaves	$1x10^{-11}$	10
Hearing Threshold	$1x10^{-12}$	1

The reference intensity in air is typically 1×10^{-12} W/m^2. Using this simple definition you see that intensities spanning 14 orders of magnitude become **intensity levels** between 1 and 140. This is an appealing scale because our ears seem to judge loudness on a logarithmic vice linear scale. Additionally, if you tried to graph various intensities, say as a function of frequency, your scale would likely only display the loudest noise with all others jammed along the abscissa. When intensity levels are plotted, the graph becomes much more useful.



The units of decibels were constructed for intensity level definition. A “bel” was named after Alexander Graham Bell and defined:

$$\text{"bel"} \equiv \log \frac{I}{I_0}$$

A “decibel” adopts the standard metric prefix and is $1/10^{th}$ of a bel.

Reference Intensity

We have already noted above that the reference intensity when calculating intensity level for sounds in air is conventionally 1 x 10^{-12} W/m^2, the hearing threshold. This is not always the case. In fact for water, it is conventional to use a standard reference pressure, p_o. The most common reference pressure for water is 1μPa. This should not alarm you since the two can be converted using the specific acoustic impedance and assuming a plane wave.

$$I_0 = \frac{p_0^2}{\rho c}$$

Thus for water with a nominal density, ρ=1000 kg/m^3 and the nominal speed of sound, c = 1500 m/s, the reference intensity would be:

$$I_0 = \frac{(1\,\mu\text{Pa})^2}{(1000\text{ kg/m}^3)(1500\text{ m/s})} = 6.67\text{x}10^{-19}\ \text{W/m}^2$$

Similarly, one can work backwards from the reference intensity in air and determine that the reference pressure is about 20 μPa (ρ = 1.21 kg/m^3, c = 343 m/s).

Unfortunately, you must be very observant when using decibels to understand the reference level used in the calculation of an intensity level. While the numbers stated here for water and air are the most common today, up until the early 1970's, the standard reference pressure level for sound in water was the microbar (μbar). To remove any ambiguity, intensity levels are generally stated with the reference included as follows:

$$L = 40\ \text{dB}_{\text{re } 1\ \mu\text{Pa}}$$

Of course, this puts additional burden on you when submitting answers on homework, tests and quizzes.

Sound Pressure Level (SPL)

If the reference is provided as a pressure, and we know the about the pressure of the sound wave, we do not actually need to convert both to intensities because we can relate the pressure of a sound wave directly to the reference pressure using our basic rules for logarithms.

$$L = 10\log\frac{\langle I\rangle}{I_0} = 10\log\frac{\dfrac{\langle p^2\rangle}{\rho c}}{\dfrac{p_0^2}{\rho c}} = 10\log\frac{\langle p^2\rangle}{p_0^2} = 10\log\left(\frac{\sqrt{\langle p^2\rangle}}{p_0}\right)^2$$

A better equation for the intensity level is then:

$$L = 20\log\left(\frac{\sqrt{\langle p^2\rangle}}{p_0}\right) = 20\log\frac{p_{rms}}{p_o} \qquad \text{where,}\quad p_{rms} = \sqrt{\langle p^2\rangle} = \frac{p_{max}^2}{2}$$

In this form, the intensity level is often called the "sound pressure level." The sound pressure level and the intensity level must be equal provided the reference values correspond ($I_0 = \frac{p_0^2}{\rho c}$).

Note that the form $10\log\left(\frac{X_1}{X_2}\right)$ is used for energy quantities (power, intensity). These are sometimes called "mean squared" quantities. The form $20\log\left(\frac{x_1}{x_2}\right)$ is used for acoustic pressure and other "root mean squared" quantities such as voltage.

As a quick example, a sound wave in water with an rms pressure of 100 μPa would have an intensity level or sound pressure level (in dB):

$$L = 20\log\frac{100\ \mu\text{Pa}}{1\ \mu\text{Pa}}$$

$$L = 40\ \text{dB}_{\text{re}\,1\,\mu\text{Pa}}$$

As stated above, the reference pressure is given in this answer so that we know the intensity level is a comparison of the intensity to the reference pressure. In the future, all intensity levels for sound in water can be **assumed** to be referenced to 1 μPa unless otherwise stated. For sound in air, the standard reference pressure is 20 μPa.

About the Decibel (dB)

A couple of things to note about this new unit, dB:

1) Remember that decibels are often used to deal with values that differ over many orders of magnitude thus allowing for much smaller differences in dB. For instance, a hydrophone with a source level of 120 dB emits a sound wave with a rms pressure of 1,000,000 Pa. A hydrophone with a source level of 100 dB emits a sound wave with an rms pressure of 100,000 Pa. Thus in this instance, a difference of 20 dB equals a **difference of 900,000 Pa**.

2) Every time you see the units of dB, you should think of a **ratio**. By definition, a level in dB is related to the **ratio** of rms pressure to a reference pressure (in water p_{ref} = 1μPa). When expressing a sound pressure level referenced to 1 μPa, the units are noted as **dB/1 μPa** or **dB re 1 μPa**. Sound levels in air use 20 μPa as the reference level, the average human hearing threshold at for a 1 kHz signal. Acoustic signals in water were originally referenced to 1 μbar. You can show that sound levels referenced to the new 1 μPa reference level are therefore 100 dB higher than those referenced to 1 μbar.

$$20\log\left(\frac{10^5\ \mu\text{Pa}}{1\ \mu\text{Pa}}\right) = 100\ \text{dB}$$

Later we will see how a difference of levels of two sources, in dB, is related to the **ratio** of the pressure (or intensity) of the two sources.

3) Since $L = 10\log\frac{\langle I\rangle}{I_0}$, a 10 dB intensity level means that $\langle I\rangle$ is 10 times greater than I_0 and a 3 dB intensity level increase corresponds to a doubling of the energy level.

4) Intensity levels and sound pressure levels both use the symbol, L. As we move through the course, we will discuss source levels, noise levels, and reverberation levels. A common procedure in the Navy is to assign a subscript such as L_S for a source level. Several standard textbook have adopted the convention of putting the subscript before the "L." In this case, SL would mean source level.

Working with intensity levels

For this course, we will need to work with intensity levels in many ways. Some examples of using intensity levels are given below:

Subtracting Intensity Levels

Finding the difference between two intensity levels is a little bit different. **The difference in the two intensity levels represents the ratio of the intensities or pressure**:

$$\mathrm{L}_2 - \mathrm{L}_1 = 10\,log\frac{\langle \mathrm{I}_2\rangle}{\mathrm{I}_0} - 10\,log\frac{\langle \mathrm{I}_1\rangle}{\mathrm{I}_0}$$

$$\mathrm{L}_2 - \mathrm{L}_1 = 10\,log\langle \mathrm{I}_2\rangle - 10\,log\,\mathrm{I}_0 - \left[10\,log\langle \mathrm{I}_1\rangle - 10\,log\,\mathrm{I}_0\right]$$

$$\mathrm{L}_2 - \mathrm{L}_1 = 10\,log\langle \mathrm{I}_2\rangle - 10\,log\langle \mathrm{I}_1\rangle$$

$$\mathrm{L}_2 - \mathrm{L}_1 = 10\,log\frac{\langle \mathrm{I}_2\rangle}{\langle \mathrm{I}_1\rangle}$$

or substituting in the definition for the intensity:

$$\mathrm{L}_2 - \mathrm{L}_1 = 10\,log\frac{\dfrac{\langle \mathrm{p}_2^2\rangle}{\rho c}}{\dfrac{\langle \mathrm{p}_1^2\rangle}{\rho c}}$$

$$\mathrm{L}_2 - \mathrm{L}_1 = 10\,log\frac{\langle \mathrm{p}_2^2\rangle}{\langle \mathrm{p}_1^2\rangle}$$

$$\mathrm{L}_2 - \mathrm{L}_1 = 20\,log\frac{\langle \mathrm{p}_2\rangle}{\langle \mathrm{p}_1\rangle}$$

So if a noisy sub was emitting a source level of 140 dB and a quiet sub was emitting a source level of 80 dB, the difference between the two intensity levels would be:

$$L_2 - L_1 = 140\text{ dB} - 80\text{ dB} = 60\text{ dB}$$

For perspective, this represents a **ratio** of the intensity of both submarines To find the actual ratio (not in dB):

$$L_2 - L_1 = 10\,log\frac{\langle I_2\rangle}{\langle I_1\rangle} \text{ or}$$

$$\frac{\langle I_2\rangle}{\langle I_1\rangle} = 10^{\frac{L_2 - L_1}{10}} \text{ or}$$

$$I_2 = 10^6 * I_1$$

Or ratio of the acoustic pressures emitted:

$$L_2 - L_1 = 20\log\frac{\langle p_2\rangle}{\langle p_1\rangle} \text{ or}$$

$$\frac{\langle p_2\rangle}{\langle p_1\rangle} = 10^{\frac{L_2 - L_1}{20}} \text{ or}$$

$$p_2 = 10^3 * p_1$$

In other words, the acoustic pressure of the sound wave from the louder sub is 3 orders of magnitude or a thousand times greater than that of the quiet sub.

This example illustrates why it is so much more efficient to reference all intensities or pressures to intensity levels to provide an easier comparison between numbers that can be so many orders of magnitude different.

Adding Incoherent Intensity Levels

Noise in the ocean is the combination of noise from many different sources. How can we add two intensity levels together? We want to add two intensity levels, L_1 and L_2, where:

$$L_1 = 10\log\frac{\langle I_1\rangle}{I_0} \quad \text{and} \quad L_2 = 10\log\frac{\langle I_2\rangle}{I_0}$$

$$"L_1 + L_2" = 10\log\frac{I_{tot}}{I_0}$$

$$\text{but } I_{tot} = \langle I_1\rangle + \langle I_2\rangle$$

First, let's rewrite equation, $L_1 = 10\log\frac{\langle I_1\rangle}{I_0}$ to solve for $\langle I_1\rangle$

$$\frac{L_1}{10} = \log\frac{\langle I_1\rangle}{I_0}$$

$$10^{\frac{L_1}{10}} = \frac{\langle I_1\rangle}{I_0}$$

$$\text{similarly, } 10^{\frac{L_2}{10}} = \frac{\langle I_2\rangle}{I_0}$$

$$\text{therefore}\langle I_1\rangle = I_0 10^{\frac{L_1}{10}} \text{ and } \langle I_2\rangle = I_0 10^{\frac{L_2}{10}}$$

$$\text{so } I_{tot} = I_0\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}}\right)$$

$$L_{tot} = L_1 \oplus L_2 = 10\log\left(\frac{I_{tot}}{I_0}\right)$$

$$L_{tot} = 10\log\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}}\right)$$

We used the notation with a circle around the plus sign to represent the power sum of two decibel quantities.

$$L_{tot} = L_1 \oplus L_2$$

To add intensity levels, there are two shortcuts that can be used for some problems to make it easier than using the above equation:

1. if $L_1 = L_2$ then $L_{tot} = 10\log\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}}\right) = L_1 + 3\text{ dB}$ this is because:

$$L_{tot} = 10\log\left(10^{\frac{L_1}{10}} + 10^{\frac{L_1}{10}}\right) = 10\log\left[2\left(10^{\frac{L_1}{10}}\right)\right] = 10\log\left[10^{\frac{L_1}{10}}\right] + 10\log[2] = L_1 + 3\text{ dB}$$

The rules of logarithms, specifically the second rule above, tells us the only time it is appropriate to actually add dB would be when intensity was multiplied by some quantity as in the case of the gain provided by an amplifier. In this example, if an amplifier had doubled the intensity as if there were two source intensities, we say the amplifier provided a 3 dB increase and we simply add the 3 dB to the initial intensity level.

2. if $L_1 >> L_2$ (or vice versa), then $L_{tot} \approx L_1$ (or vice versa). Here, “much more than” is defined as 10 dB or $I_1 > 10 * I_2$,

Problems

1. Using the rules for logs:

a) Simplify the following relationship for this empirical sound level of a signal in water ($dB_{re\ 1\mu Pa}$): $L = 20\log_{10}\left(\frac{x^a y^b}{z^c}\right)\ dB_{re\,1\mu Pa}$.

b) If x=5, y=8, z=10, a=10, b=5, and c=11, what is the resulting level (in $dB_{re\ 1\mu Pa}$).

2. Cavitation may take place at the face of a sonar transducer when the sound peak pressure amplitude being produced exceeds the hydrostatic pressure in water.

a) For a hydrostatic pressure of 600,000 Pa, what is the highest intensity that may be radiated without producing cavitation?

b) What is the intensity level in dB re 1 μPa?

c) How much acoustic power is radiated if the transducer face has an area of 1/3 m^2?

3. If $P_{2\ rms}$ = 100 μPa and $P_{1\ rms}$ = 25 μPa, what is:

a) $\frac{\langle I_2 \rangle}{\langle I_1 \rangle} = ?$

b) $L_2 - L_1 = ?$

4. If $L_1 = L_2 = 60\ dB_{re\ 1\ \mu Pa}$, $L_3 = 57\ dB_{re\ 1\ \mu Pa}$, $L_4 = 50\ dB_{re\ 1\ \mu Pa}$, and $L_5 = 65\ dB_{re\ 1\ \mu Pa}$, what is L_{tot}, the some of all the levels.

5. What is the intensity of a 0 $dB_{(re\ 1\ \mu Pa)}$ acoustic wave in water?

6. If $L_1 = L_2$, then prove that $L_{tot} = L_1$+3 dB (the 3 dB rule).

7. If $P_{1,rms}$ = 200 μPa and $P_{2,rms}$ = 10μPa, determine (assume P_0 = 1μPa):

a) L_1,

b) L_2,

c) $L_1 \oplus L_2$,

d) L_1-L_2,

e) What does the previous result tell us?

8. If L_1=30 dB re 1μPa and L_2=65 dB re 1μPa, what is P_2/P_1?

9. Show that a plane wave having an effective acoustic pressure of 1 μbar in air has an intensity level of 74 dB re 0.0002 μbar.

10. Find the intensity (W/m^2) produced by an acoustic plane wave in water of 120 dB sound pressure level relative to 1 μbar.

11. What is the ratio of the sound pressure in water for a plane wave to that of a similar wave in air of equal intensity? c_{air} = 343 m/s, ρ_{air} = 1.21 kg/m^3, c_{water} = 1500 m/s, ρ_{water} = 1000 kg/m^3.

12. If the intensity level in seawater is 160 dB re 1 μPa, what is the rms acoustic pressure in μPa?

a) What is the rms acoustic pressure in μPa if the intensity level is 160 dB re 1 μbar?

b) What is the rms acoustic pressure in μPa if the intensity level is 160 dB re 0.0002 μbar?

13. Over a certain band of frequencies in the deep ocean basis, the noise level due to surface water turbulence (due to wind) is 62 dB and the noise level due to distant shipping is 65 dB. What is the total noise level?

14. The rms pressure from a low frequency sound source is 200 μPa. What is the combined rms pressure for both sources? What is the combined source level in dB re 1 μPa?

Also published in: *Canadian Acoustics / Acoustique Canadienne 26(2) 29-31 (1998)* **Technical note / Note technique**

THE ELUSIVE DECIBEL: THOUGHTS ON SONARS AND MARINE MAMMALS

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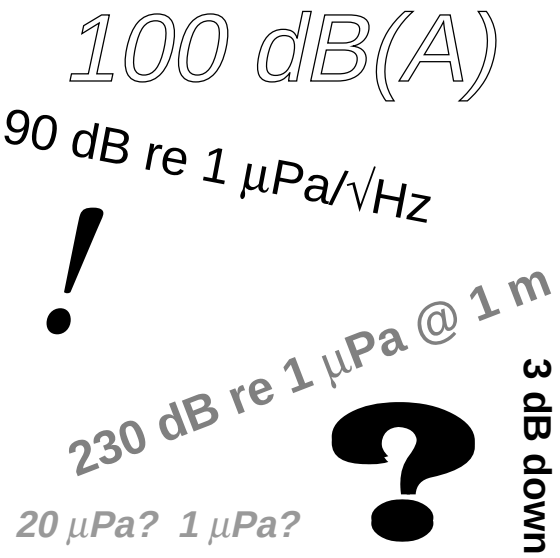
INTRODUCTION

A few years ago, there was considerable controversy over the effects of a proposed global acoustic experiment designed to measure the temperature of the world's oceans[1]. The focus of concern was the possible effect of the acoustic signals on whales and other marine life. There is continued interest in the effects of underwater sound on marine animals, according to a recent news item in *The Economist*[2] based on related scientific correspondence in *Nature*[3]. The thesis is that loud signals from experimental sonars harm marine mammals, or at least harass them enough to unacceptably alter their behaviour patterns. In the various discussions of this important issue that can be found in the press and on the internet, one often sees questionable comparisons being made, such as the acoustic output of a naval sonar being compared with the noise from a jet aircraft. Some misunderstandings between professionals in different fields can be traced to the multiple uses of the term "decibel". Acoustical terms can be confusing, even for experts. It is not at all surprising that well-intentioned articles sometimes fail to present situations clearly. By definition, the decibel is a relative unit, not an absolute unit with a physical dimension; unless the standard of comparison is cited, the term "decibel" is to all intents and purposes useless. The confusion is not helped by the use of the decibel to specify distinctly different physical quantities, or the same physical quantity with different reference levels. Some reporters—and even some scientists—are getting their "apple" decibels mixed up with their "orange" decibels, as it were.

The decibel (abbreviated dB) is simply a numerical scale used to compare the values of like quantities, usually power or intensity. Acousticians introduced the decibel to devise a compressed scale to represent the large dynamic range of sounds experienced by people from day to day, and also to acknowledge that humans—and presumably other animals—perceive loudness increases in a logarithmic, not linear, fashion. An intensity ratio of 10 translates into a level difference of 10 decibels[4]; a ratio of 100 translates into a level difference of 20 dB; 1000 into 30 dB; and so on. (The term "level" usually implies a decibel scale.) In a uniform acoustic medium, the magnitude of the acoustic intensity is proportional to the square of the pressure for a freely-propagating sound wave. Accordingly, the level difference in



decibels associated with two sound pressure values (measured in the same medium) is determined by calculating the ratio of the pressures, squaring this number, taking the logarithm (base 10), and multiplying by 10.[5] If one chooses a standard reference pressure value, then sound pressure levels can be specified in decibels relative to that reference, but this should be stated along with the number, for clarity[6].

The following is a typical erroneous statement found in the press, on radio, on television, and on internet discussion groups. Referring to an experimental sonar source that produces very loud low-frequency sound, *The Economist* wrote: "It has a maximum output of 230 decibels, compared with 100 decibels for a jumbo jet." Regardless of the author's intention, the implication is that a whale would experience an auditory effect from the sonar that would be substantially greater than that of a person exposed to the jet aircraft. However, this type of comparison is misleading for at least three reasons: (1) the reference sound pressures used in underwater acoustics and in-air acoustics are not the same; (2) it compares a source level with a received level; and (3) there is no obvious connection between an annoying or harmful sound level for a human in air and an annoying or harmful sound level for a marine animal in water. In the remainder of this note, we will expand on these topics somewhat, attempt to correct the mistaken impression, and

try to direct attention to the real issue at the heart of the controversy.

1. STANDARD REFERENCE SOUND PRESSURES IN AIR AND IN WATER

The standard reference pressures used in underwater acoustics and in-air acoustics are not the same. In water, acousticians use a standard reference sound pressure of 1 micropascal (i.e. 10^{-6} newtons per square metre), abbreviated μPa. In air, acousticians use a higher standard reference sound pressure of 20 μPa. The in-air standard was chosen so that the threshold of hearing for a person with normal hearing would correspond to 0 dB at a frequency of 1000 Hz. Adopting different standards for air and water inevitably leads to a confusing consequence: the same sound pressure that acousticians label 0 decibels in air would be labelled 26 decibels in water. Presumably, both factions of acousticians had equally good reasons for proposing their respective standards, and this dichotomy is now entrenched in an ANSI standard[6], which is unlikely to change. Accordingly, the following dictum should always be observed, especially when dealing with cross-disciplinary issues: *It is essential that sound levels stated in decibels include the reference pressure.*

2. SOURCE LEVEL AND RECEIVED LEVEL

The erroneous statement compares a *source* level with a *received* level. In underwater acoustics, a source level usually represents the sound level at a distance of one metre from the source, while a received level is the sound level at the listener's actual position, which could be considerably more distant with a correspondingly reduced sound level. In an unbounded uniform medium, loudness decreases rapidly with increasing source-receiver distance, 6 dB less per doubling of distance. For example, *The Economist* (and even *Nature*), in referring to the 230 dB sonar source level, neglected to mention the reference distance of 1 metre. In contrast, the 100 dB number that *The Economist* associated with a jumbo jet is not a source level at all, but is typical of a received noise level measured during jet airplane take-off, averaged over several microphones situated several hundred to some thousands of metres from the runway[7]. *It is incorrect to compare a source level at 1 metre with a received noise level at an unspecified (and probably much larger) distance.*

Combining these two remarks, the output of the sonar source should have been written as 230 dB re 1 μPa at 1 m, while the jumbo jet noise level should have been written as 100 dB re 20 μPa. The inclusion of the reference values shows that these are not like quantities, and that the numbers are not directly comparable. *The Encyclopedia of Acoustics*[8] offers 120 dB re 20 μPa as a typical noise level associated with jet aircraft take-off measured at 500 m distance (although there is sure to be a wide variation about this number, depending on the type of aircraft, etc.). With the assumption of spherical spreading, referencing this level back to 1 metre distance adds 54 dB. Switching to the 1 μPa standard reference adds another 26 dB. Accordingly, the source level of a large jet looks more like 120 + 54 + 26 = 200 dB re 1 μPa at 1 m, compared with 230 dB re 1 μPa at 1 m for the sonar. Both of these are loud sources, but now at least the comparison is sensible. The ratio of sound pressures is around 32, rather than over 3 million, as some commenters would have you believe!

There are other minor issues that could be discussed. The signal from the sonar source is narrowband, and the concentration of all the signal at one frequency may be particularly troublesome for an animal who has a cavity that resonates at that frequency. On the other hand, the jet noise is broadband, and the acoustic signal was probably passed through a filter that approximately matches the sensitivity of the human ear before the measurement was made, so this measurement would be meaningless for an animal with a different hearing sensitivity curve. Much more could be said about these issues, but the principal reason for raising them is to underscore the message that the sonar / jet plane comparison has little validity.

3. WHAT HURTS?

There is no clear connection between a harmful sound level for a human in air and that for an animal in water. All creatures have evolved and adapted to their respective environments and there is no reason why human hearing characteristics should apply to any other animal, including whales. If a given sound pressure hurts a human, would the same sound pressure level in water hurt a whale (or a fish, or a shrimp)? Is the threshold of pain higher? Is it lower? Particularly when comparing acoustic effects in media of widely different impedance, is acoustic pressure the relevant acoustic quantity, or is it acoustic intensity?[9] In the end, it is the answers to these and related questions that really matter, not juggling decibels. To properly answer these questions and to determine the "community" noise standards for marine animals, scientific research is necessary—just as it was for humans. Some of this work has already been done, and an excellent review[10] of the state of knowledge up to 1995 is a good starting point for acousticians and biologists interested in deepening their understanding. A single example cannot represent the whole range of species under consideration, but is typical: The response threshold (determined through behavioural studies) of a Beluga at 1000 Hz is just over 100 dB re 1 μPa. Of course, this says nothing about the Beluga's threshold of pain, and says

nothing about what sound level would unacceptably alter its behaviour. *It is unwise to assume that the auditory experience of any animal would be the same as that of a human exposed to the same sound level.*

CONCLUSION

As sonar engineers, marine biologists, and environmentally conscious citizens continue to discuss these important issues, we should at least agree to use the same acoustical units to convey our points of view, to avoid confusion and misrepresentation. Some sensible acousticians have advocated abandoning the use of the decibel—which is partly to blame for our woes—in favour of good old SI (i.e., metric) units for sound pressure, acoustic intensity, power, etc. Until that happy day dawns, let us include reference values with our decibels, so we don't end up with fruit salad dBs. Ultimately, what is important is to determine what underwater sound levels are harmful to marine life. We must develop mitigation measures to allow underwater acoustic systems to be operated while ensuring the protection of the marine environment with due diligence.

ACKNOWLEDGEMENT

The authors thank Harold M. Merklinger for his helpful comments on the manuscript.

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[2] "Quiet, please. Whales navigating", *The Economist*, 1998 March 7, page 85.

[3] R. Frantzis, "Does acoustic testing strand whales?", *Nature* **392**, 1998 March 5, page 29.

[4] In fact, this defines 1 bel, named after Alexander Graham Bell. The bel turned out to be too large for practical purposes and the decibel—which is 1/10 of a bel—is the preferred unit. Also, one decibel is about the smallest incremental change of sound pressure level a person can sense.

[5] Mathematically, this is equivalent to taking the logarithm of the pressure ratio and multiplying by 20, but knowing when to multiply by 10 or 20 in such calculations is an endless source of confusion to the neophyte, so we advocate the definition in the main text.

[6] *American National Standard Preferred Reference Quantities for Acoustical Levels*, ANSI S1.8-1969, page 8.

[7] Malcolm J. Crocker, editor, *The Encyclopedia of Acoustics* (John Wiley and Sons, Inc., New York, 1997), page 1095.

[8] Malcolm J. Crocker, editor, *The Encyclopedia of Acoustics* (John Wiley and Sons, Inc., New York, 1997), page 11.

[9] The suggestion that acoustic intensity has more bearing than sound pressure in this context has been seriously proposed by some acousticians; however, the available evidence gives the nod to sound pressure, not intensity.

[10] W. John Richardson *et al*., *Marine Mammals and Noise* (Academic Press, New York, 1995).

Lesson 3 - Logs and Levels

Math Prereqs

$$y = 10^x \quad \text{then} \quad \log_{10}(y) = x$$

$$\log(xy) = \log(x) + \log(y)$$

$$\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

$$10\log(x^n) = n10\log(x)$$

Examples

- Without using your calculator, find the following: ($\log_{10}(2) = 0.30$)
- $\log_{10}(10^{-3}) =$
- $\log_{10}(1 \text{ x } 10^{12}) =$
- $\log_{10}(2 \text{ x } 10^{12}) =$
- $\log_{10}(200) =$
- $\log_{10}(200) - \log_{10}(10) =$
- $\log_{10}(2^{10}) =$

-3, 12, 12.3, 2.3, 1.3, 3

Pitch is frequency

Audible	20 Hz – 20000 Hz
Infrasonic	< 20 Hz
Ultrasonic	>20000 Hz

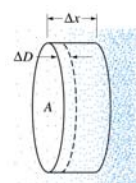
Middle C on the piano has a frequency of 262 Hz. What is the wavelength (in air)?

1.3 m

Intensity of sound

$$\langle I \rangle = \frac{\langle p^2 \rangle}{\rho c} = \frac{p_{max}^2}{2\rho c}$$

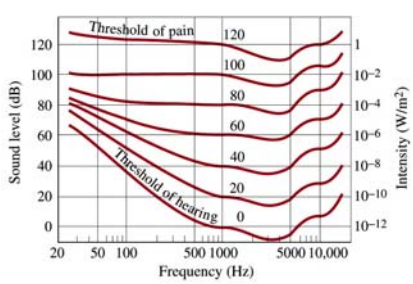
- Loudness – intensity of the wave. Energy transported by a wave per unit time across a unit area perpendicular to the energy flow.



Source	Intensity (W/m²)	Sound Level
Jet Plane	100	140
Pain Threshold	1	120
Siren	$1\text{x}10^{-2}$	100
Busy Traffic	$1\text{x}10^{-5}$	70
Conversation	$3\text{x}10^{-6}$	65
Whisper	$1\text{x}10^{-10}$	20
Rustle of leaves	$1\text{x}10^{-11}$	10
Hearing Threshold	$1\text{x}10^{-12}$	1

Sound Level - Decibel

$$L = 10\log\left(\frac{\langle I \rangle}{I_0}\right) \qquad I_0 = 1\text{x}10^{-12}\,\frac{W}{m^2}$$



Why the decibel?

- Ears judge loudness on a logarithmic vice linear scale
- Alexander Graham Bell $\quad \text{"bel"} \equiv \log\frac{I}{I_0}$
- deci = $\frac{1}{10}$
- 1 bel = 10 decibel

$$L(\text{in dB}) = 10\log\left(\frac{\langle I \rangle}{I_0}\right)$$

Reference Level Conventions

$$I_0 = \frac{p_0^2}{\rho c}$$

Location	Reference Intensity	Reference Pressure
Air	1×10^{-12} W/m^2	20 μPa
Water	6.67×10^{-19} W/m^2	1 uPa

Historical Reference

- 1 microbar
- 1 bar = 1×10^5 Pa
- 1 μbar = 1×10^5 μPa

$$20\log\left(\frac{10^5\ \mu\text{Pa}}{1\ \mu\text{Pa}}\right) = 100\ \text{dB}$$

- So to convert from intensity levels referenced to 1 μbar to intensity levels referenced to 1 μPa, simply add 100 dB

Sound Pressure Level

Mean Squared Quantities:
Power, Energy, **Intensity**

$$L = 10\log\left(\frac{\langle I\rangle}{I_0}\right)$$

"Intensity Level"

Root Mean Squared Quantities:
Voltage, Current, **Pressure**

$$L = 20\log\left(\frac{\sqrt{\langle p^2\rangle}}{p_0}\right) = 20\log\frac{p_{rms}}{p_o}$$

"Sound Pressure Level"

Subtracting Intensity Levels

$$L_2 - L_1 = 10\log\frac{\langle I_2\rangle}{I_0} - 10\log\frac{\langle I_1\rangle}{I_0}$$

$$L_2 - L_1 = 10\log\frac{\langle I_2\rangle}{\langle I_1\rangle}$$

$$L_2 - L_1 = 20\log\frac{\langle p_2\rangle}{\langle p_1\rangle}$$

Two Submarines

- If a noisy sub was emitting a source level of 140 dB and a quiet sub was emitting a source level of 80 dB,
- What is the difference in noise levels?
- what does this mean in terms of relative intensity and acoustic pressure?

Adding Levels

$$L_1 = 10\log\frac{\langle I_1\rangle}{I_0} \quad \text{and} \quad L_2 = 10\log\frac{\langle I_2\rangle}{I_0}$$

$$"L_1 + L_2" = 10\log\frac{I_{tot}}{I_0}$$

$$\text{but } I_{tot} = \langle I_1\rangle + \langle I_2\rangle$$

$$L_{tot} = L_1 \oplus L_2 = 10\log\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}}\right)$$

Total Noise

- On a particular day, noise from shipping is 53 dB and noise from rain and biologics is 50 dB. What is the total noise level from the two sources?

Adding Equal Noise Levels

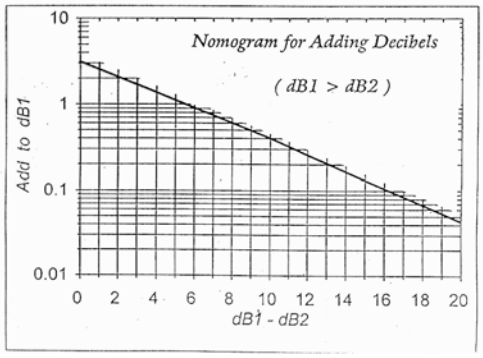
- Two transducers are both transmitting a source intensity level of 90 dB. What is the total source intensity level?

If $L_1 = L_2$ then $L_{Total} = L_1 \oplus L_2 = L_1 + 3\text{dB}$

Two Submarines

- If a noisy sub was emitting a source level of 140 dB and a quiet sub was emitting a source level of 80 dB, what is the total noise from the two submarines?

Adding Decibels



Backups

Reference Values

$$I_0 = \frac{p_0^2}{\rho c}$$

Air: $$I_0 = \frac{(20\ \mu\text{Pa})^2}{(1.2\ \text{kg/m}^3)(343\ \text{m/s})} = 1\text{x}10^{-12}\ \text{W/m}^2$$

Water: $$I_0 = \frac{(1\ \mu\text{Pa})^2}{(1000\ \text{kg/m}^3)(1500\ \text{m/s})} = 6.67\text{x}10^{-19}\ \text{W/m}^2$$

Sound Pressure Level

$$L = 10\log\frac{\langle I\rangle}{I_0} = 10\log\frac{\frac{\langle p^2\rangle}{\rho c}}{\frac{p_0^2}{\rho c}} = 10\log\frac{\langle p^2\rangle}{p_0^2} = 10\log\left(\frac{\sqrt{\langle p^2\rangle}}{p_0}\right)^2$$

$$L = 20\log\left(\frac{\sqrt{\langle p^2\rangle}}{p_0}\right) = 20\log\frac{p_{rms}}{p_o}$$

Subtracting Intensity Levels

$$L_2 - L_1 = 10\log\frac{\langle I_2\rangle}{I_0} - 10\log\frac{\langle I_1\rangle}{I_0}$$

$$L_2 - L_1 = 10\log\langle I_2\rangle - 10\log I_0 - \left[10\log\langle I_1\rangle - 10\log I_0\right]$$

$$L_2 - L_1 = 10\log\langle I_2\rangle - 10\log\langle I_1\rangle$$

$$L_2 - L_1 = 10\log\frac{\langle I_2\rangle}{\langle I_1\rangle}$$

$$L_2 - L_1 = 10\log\frac{\frac{\langle p_2^2\rangle}{\rho c}}{\frac{\langle p_1^2\rangle}{\rho c}}$$

$$L_2 - L_1 = 10\log\frac{\langle p_2^2\rangle}{\langle p_1^2\rangle}$$

$$L_2 - L_1 = 20\log\frac{\langle p_2\rangle}{\langle p_1\rangle}$$

Addition

$$"L_1 + L_2" = 10\log\frac{I_{tot}}{I_0}$$

but $I_{tot} = \langle I_1\rangle + \langle I_2\rangle$

$$\frac{L_1}{10} = \log\frac{\langle I_1\rangle}{I_0}$$

$$10^{\frac{L_1}{10}} = \frac{\langle I_1\rangle}{I_0}$$

similarly, $10^{\frac{L_2}{10}} = \frac{\langle I_2\rangle}{I_0}$

therefore $\langle I_1\rangle = I_0 10^{\frac{L_1}{10}}$ and $\langle I_2\rangle = I_0 10^{\frac{L_2}{10}}$

so $I_{tot} = I_0\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}}\right)$

$$L_{tot} = L_1 \oplus L_2 = 10\log\left(\frac{I_{tot}}{I_0}\right)$$

$$L_{tot} = 10\log\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}}\right)$$