Spectrum Level and Band Level

Intensity, Intensity Level, and Intensity Spectrum Level

As a review, earlier we talked about the intensity of a sound wave. We related the **intensity** of a sound wave to the acoustic pressure where:

$$\langle I \rangle = \frac{p_{a\,max}^2}{2\rho c} \quad \text{or,}$$

$$\langle I \rangle = \frac{\langle p_a^2 \rangle}{\rho c} = \frac{p_{a\,rms}^2}{\rho c}$$

Next we defined the **Intensity Level**, L, as the decibel quantity defined to be consistent with the fact that our ears registered intensity on a logarithmic vice linear scale.

$$L \equiv 10 \log \frac{\langle I \rangle}{I_{ref}}$$

In this definition, I_{ref} was determined by a standard convention. Intensity Levels in water were most usually referenced to 1 μPa pressure which is equivalent to an intensity of 6.67 x 10^{-19} W/m^2. In air the intensity reference is most usually 1 x 10^{-12} W/m^2 which is equivalent to a reference pressure of 20 μPa.

We will now define a new quantity, the spectrum level or intensity spectrum level (ISL).

The intensity spectrum level (ISL) is the intensity level of the sound wave within a 1 Hz band.

This is accomplished by comparing the intensity in a 1 Hz band to the reference level in a 1 Hz band. The equation for the ISL is:

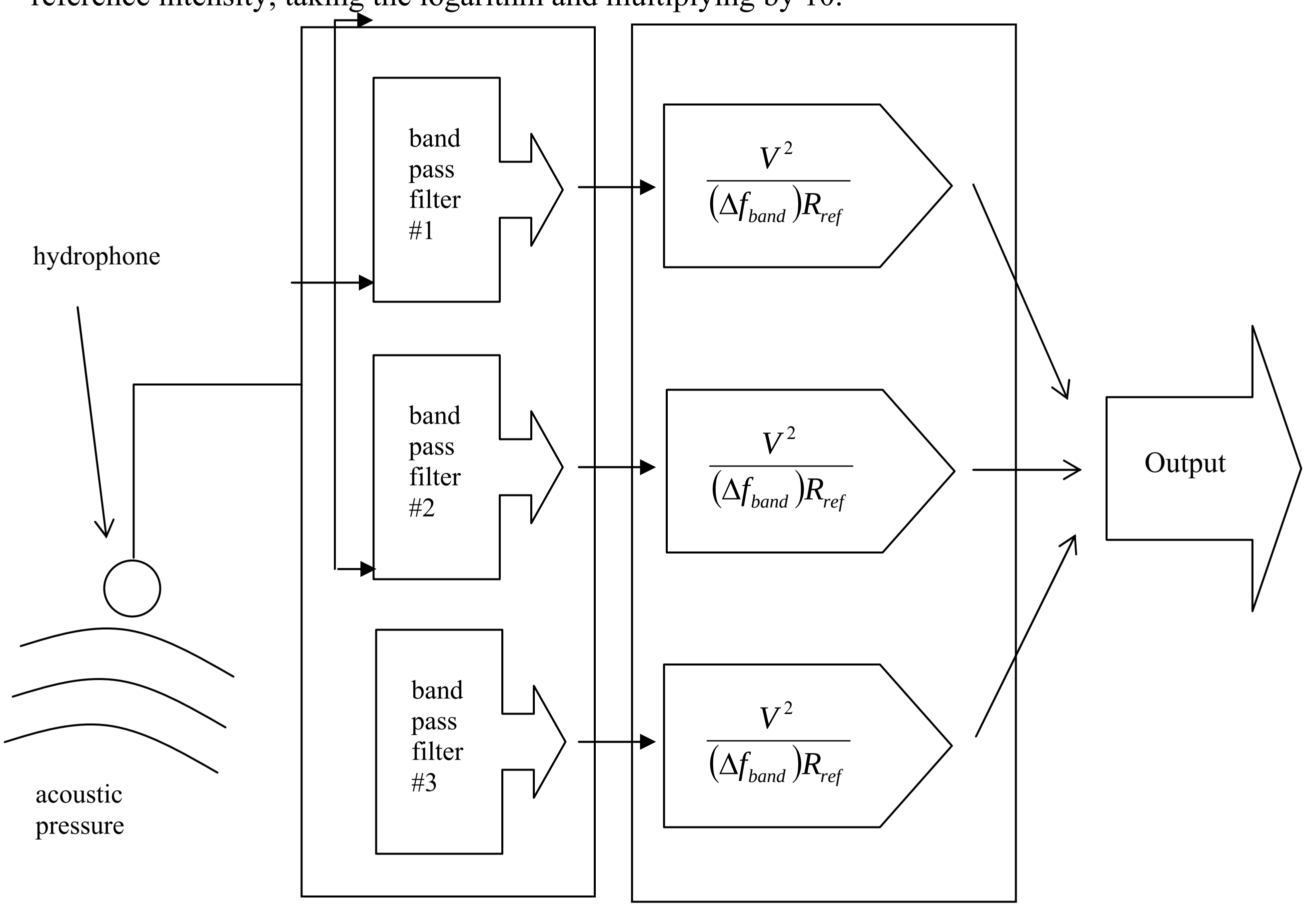
$$ISL = 10 \log \frac{I(\text{in 1 Hz band})}{I_{ref}(\text{in 1 Hz band})} = 10 \log \frac{I(\text{in 1 Hz band}) * 1\text{Hz}}{I_{ref}}$$

While this might seem a needless distinction, we can easily show in the lab that a pure tone with an intensity of 0.01 W/m2 is painfully loud. On the other hand, the same intensity spread as noise over the entire audible frequency bandwidth (20 Hz to 20 kHz) is nowhere near as loud. The ISL gives us the intensity in the 1 Hz band compared to the reference level (normally 1 μPa corresponding to 6.67 x 10^{-19} W/m^2 in water). This allows us to truly compare apples to apples. We saw in our brief study of Fourier analysis that most sound waves are made up of the combination of many different frequencies of sound waves. To compare one level to another, we must compare both levels within the <u>same</u> 1 Hz band. But what if the bandwidth of our equipment is different than 1 Hz?

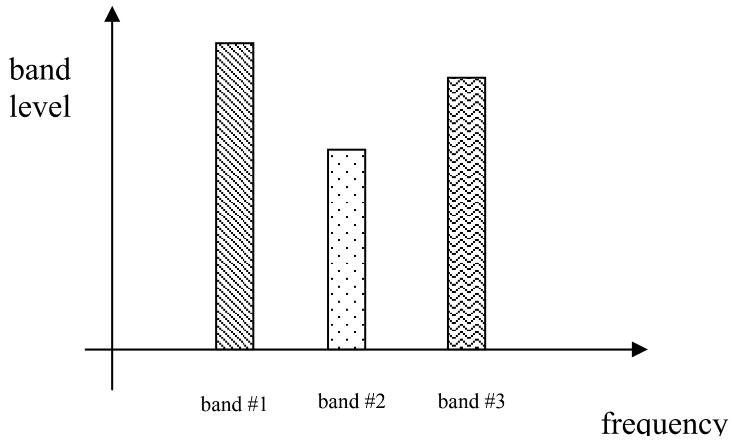
Band Level

Let's describe the processing within a typical sonar system. In this system, a sound wave is incident upon a transducer or hydrophone, which converts the varying acoustic pressure to a voltage level using a piezoelectric material such a naturally occurring quartz crystals or man made ceramic materials like PZT. Piezoelectric materials respond to external applied stresses by building up charge on their surfaces. This charge redistribution is sensed as a voltage by electrodes attached to the surfaces of the piezoelectric material.

This voltage is then passed through a set of parallel band pass filters to separate the voltage signal into the different frequency bands. After passing through the band pass filters, all frequencies outside the particular band are eliminated. The voltage representing pressure in the band is then converted to an rms power level by squaring the voltage and taking the average over a period of time called the integration time. An intensity level is created by dividing by the reference intensity, taking the logarithm and multiplying by 10.



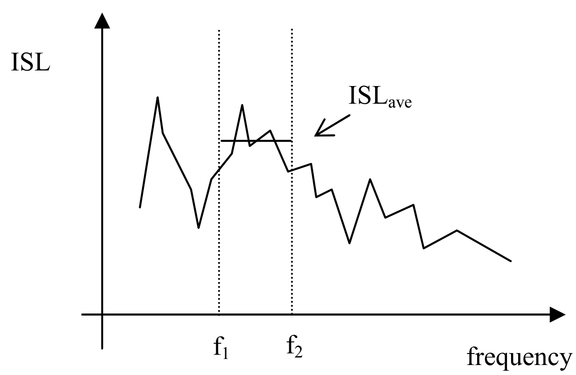
The result is then an average intensity level in each of the bands that we have split the signal into, as shown on the following graph. This is called a "Band Level" and given the symbol, BL. In this simple example system, there are only 3 bands. Real systems have many more bands. After the display is created as a plot of average intensity level in a band versus the frequency of the band, we see we have essentially created the Fourier Transform of the time domain signal. Devices capable of measuring and displaying frequency components of a signal in this manner are called spectrum analyzers.



The Band Level is the intensity level over a band other than 1 Hz.

Below is a plot representing the spectrum level of environmental noise within an imaginary environment. To determine the band level in the frequency band shown on the plot, we can use the following equation:

$$BL = 10\log\frac{I_{tot}}{I_{ref}} = 10\log\frac{I(\text{in a 1 Hz band})\Delta f}{I_{ref}}$$



The Intensity in a 1 Hz band is often called the Intensity Spectral Density. Using the multiplication rule for logarithms,

$$BL = 10\log\frac{I(\text{in a 1 Hz band})\text{x}1\text{Hz}}{I_{ref}} + 10\log\frac{\Delta f}{1\text{Hz}}$$

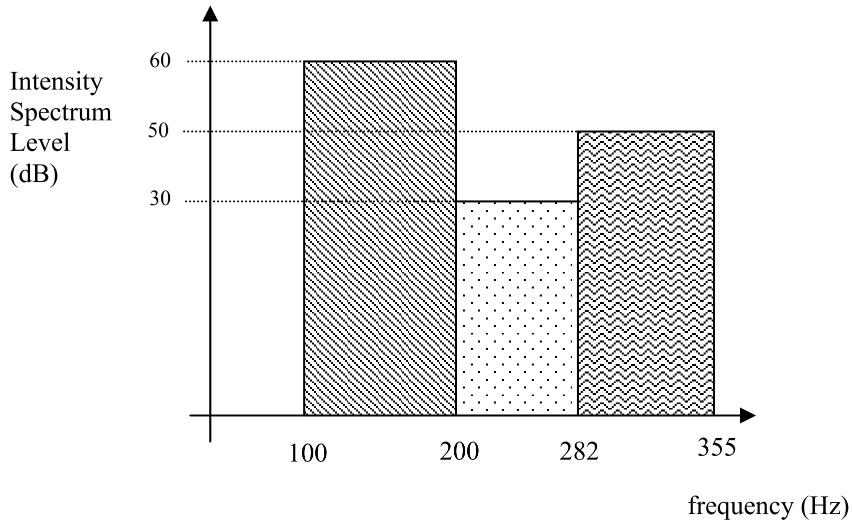
$$BL = ISL_{ave} + 10\log(\Delta f)$$

$$\text{where } \Delta f = f_2 - f_1$$

This equation can be used to compare the energy in a band other than a 1 Hz band. It might appear in the second term that we are taking the logarithm of quantity that has units. This is only because it is conventional to drop the 1 Hz in the denominator when writing the equation.

Example

Using the plot of ISL as a function of frequency shown below, calculate a) the band level of every band and b) the total band level.



a) To calculate the band levels:

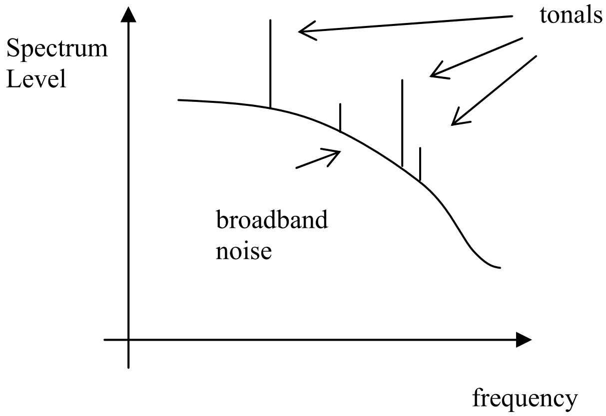
$$BL_1 = ISL_{ave} + 10\log(\Delta f)$$
$$BL_1 = 60 + 10\log(100)$$
$$BL_1 = 80 \text{ dB}$$
$$BL_2 = 49.1 \text{ dB}$$
$$BL_3 = 68.6 \text{ dB}$$

b) To calculate the total band level:

$$BL_{tot} = 10\log\left(10^{BL_1/10} + 10^{BL_2/10} + 10^{BL_3/10}\right)$$
$$BL_{tot} = 80.3 \text{ dB}$$

Types of Spectrums

A broadband spectrum is one where the sound pressure levels are spread continuously across a spectrum. A tonal spectrum is one where there is a discrete non-continuous spectrum with different frequency components. Sound pressure level measurements may contain components of broadband sounds as well as tonals as shown below.

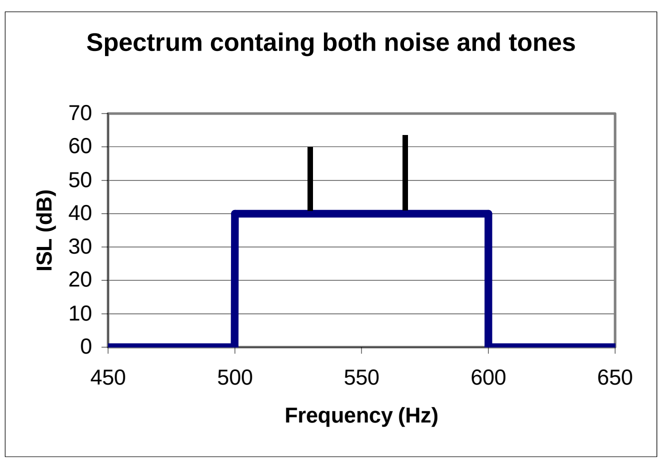


To calculate the total band level of broadband and tonal spectrums, we must add in the band level (or ISL if they have a 1 Hz bandwidth) of each tonal separately with the band level of the broadband noise. An equation would be:

$$BL_{tot} = \left[ISL_{ave} + 10\log\left(\Delta f_{BB}\right)\right] \oplus L_{tonal\ \#1} \oplus L_{tonal\ \#2} \oplus \ldots$$

Example

In the following figure, the background noise is constant (40 dB) over the band width from 500 to 600 Hz. There are two tones with levels of 60 dB and 63 dB respectively. What is the total Band Level?



$$BL_{tot} = \left[ISL_{ave} + 10\log\left(\Delta f_{BB}\right)\right] \oplus L_{tonal\ \#1} \oplus L_{tonal\ \#2}$$

$$BL_{tot} = 40\text{dB} + 10\log\left(100\text{Hz}\right) \oplus 60\text{dB} \oplus 63\text{dB} = 66\text{dB}$$

Bandwidth and Common Bandwidths

Using the example above, let's describe some features of the frequency bands. First, it is often easier to describe a frequency band by stating the center frequency and the bandwidth versus stating the bottom and top frequency of the band. The center frequency or "average" frequency (though it is not a true average) of a frequency band can be found by using the following definition:

$$f_c = \sqrt{f_1 f_2}$$

Mathematically this frequency is the geometric mean of the upper and lower frequencies. The bandwidth is simply:

$$\Delta f = f_2 - f_1$$

Constant Bandwidth

Where all bands are the same number bandwidth, i.e. all bands are 10 Hz wide.

Proportional Bandwidth

Where the ratio of the upper frequency to the lower frequency are constant.

One-octave bandwidth

The first band from 100 Hz to 200 Hz, used in the previous example is an example of a one-octave bandwidth. A one-octave bandwidth is where:

$$f_2 = 2^1 f_1$$

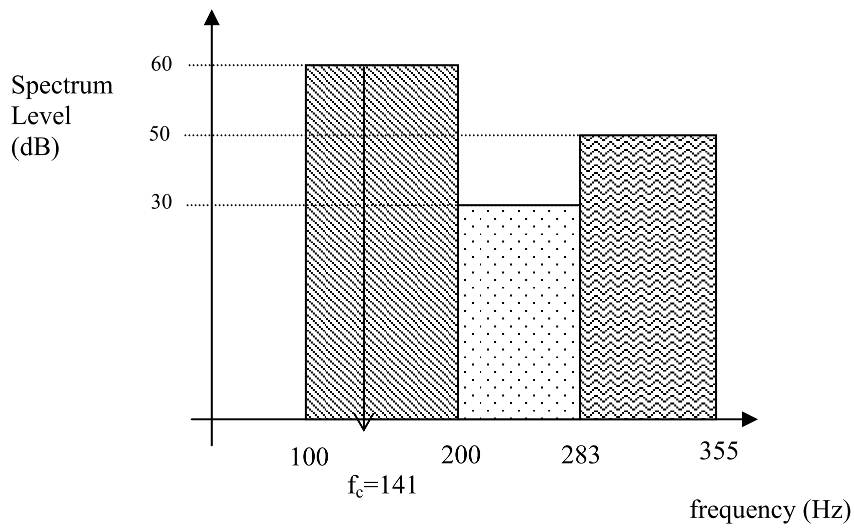
Also, using the definition to calculate the center frequency we find it is 141 Hz.

Half-octave bandwidth

The second band is an example of a half-octave bandwidth. A half-octave bandwidth is where:

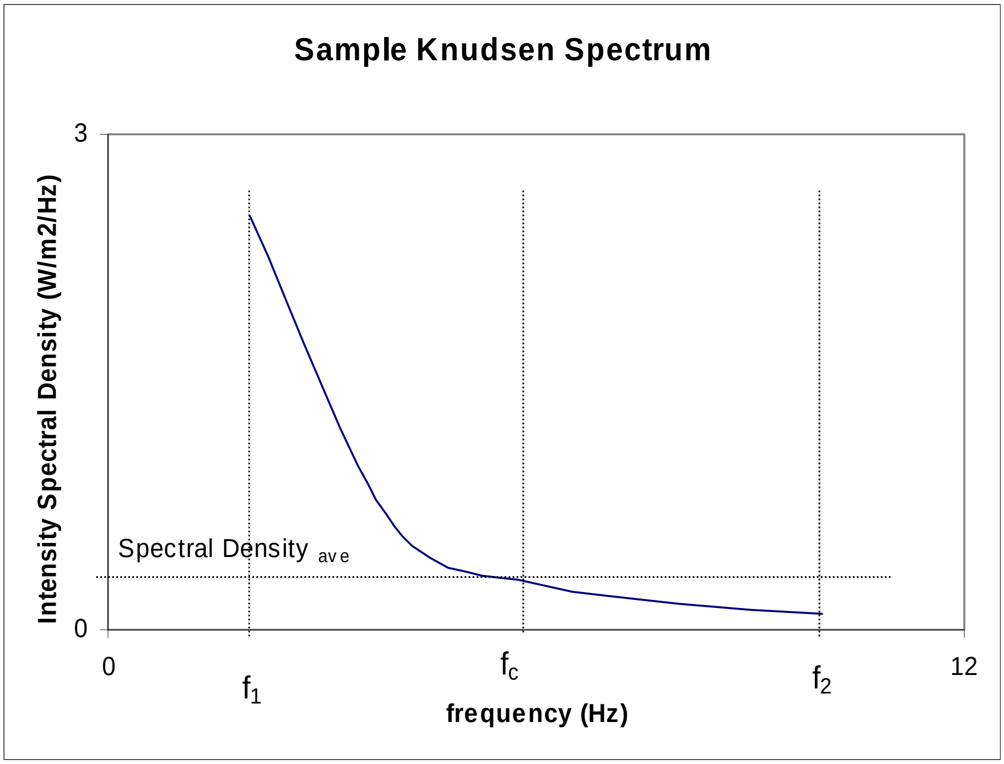
$$f_2 = 2^{1/2} f_1$$

Note also that the center frequency of the one-octave band above is where the octave is split into two half-octave bands. An octave band with a 200 Hz lower frequency has an upper frequency of 400 Hz. The center frequency is $\sqrt{200\text{Hz} * 400\text{Hz}} = 283\text{Hz}$, exactly the same number calculated from the definition of the half octave band. It may have seemed odd that the center frequency was not the simple average of the upper and lower frequencies. Hopefully this observation explains the use of the geometric mean for calculating the center frequency.



Appendix A – Why is center frequency a geometric mean vice a simple arithmetic average?

It has been observed that noise in the sea from the wind driven surface is not flat across all frequency bands. Instead we see that noise decreases with increasing frequency. The exact shape of non constant noise is called a Knudsen spectrum and Intensity Spectral Density is proportional to $1/f^2$.



The exact mathematical description of a Knudsen spectrum is:

$$dI = \frac{A}{f^2} df$$

where A is a constant. The intensity in a band from f_1 to f_2 is then:

$$I_{Band} = \int_{f_1}^{f_2} \frac{A}{f^2} df = A\left(\frac{1}{f_1} - \frac{1}{f_2}\right) = \frac{A}{f_1 f_2}\left(f_2 - f_1\right) = \frac{A}{f_1 f_2}\Delta f$$

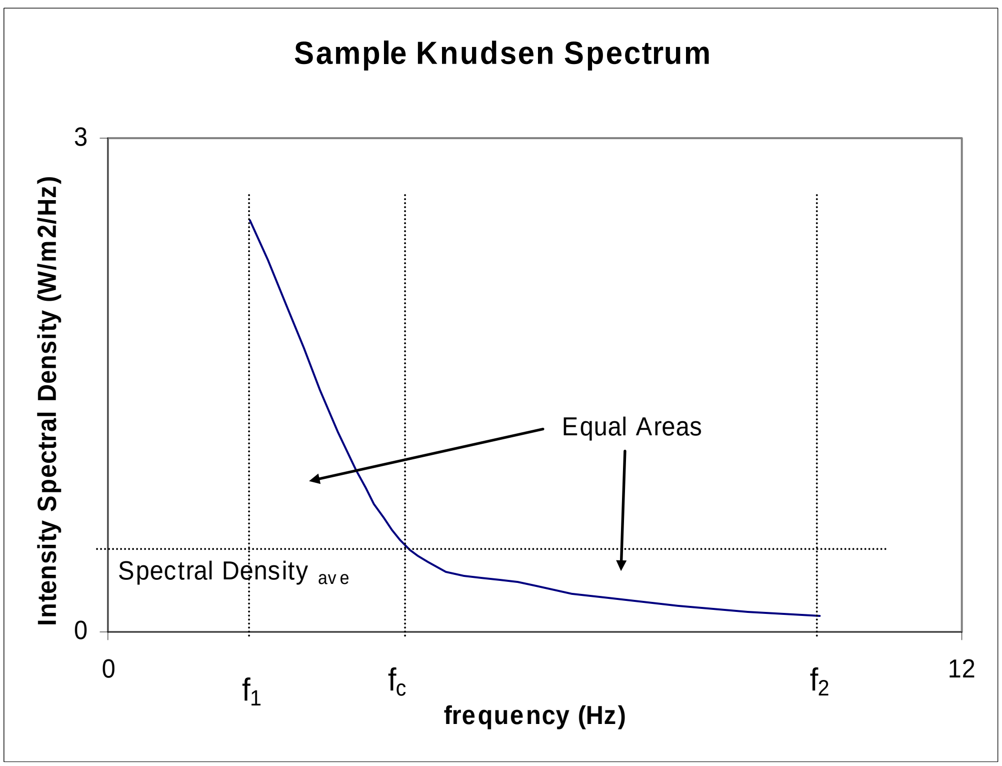
If the noise spectrum where constant, the total intensity in the band is just the Intensity Spectral Density x bandwidth. In the case of the non-constant spectral density, the term $\frac{A}{f_1 f_2}$ represents the best average value of the Intensity spectral density. It is evaluated at the center frequency.

$$\frac{A}{f_1 f_2} = \text{Spectral Density}_{ave} = \left.\frac{dI}{df}\right|_{ave} = \frac{A}{f_c^2}$$

As such, we see that the center frequency must be:

$$f_c = \sqrt{f_1 f_2}$$

Simply looking at the Knudsen spectrum above shows us the problem with using the arithmetic average for the center frequency. Our knowledge of approximate integration tells us the area above and below the average Intensity Spectral Density must be approximately equal. This is clearly not the case. Instead, a lower average value must be to balance the equal areas.



Problems

1. Given the following FFT for pressure p(t) {where T = 1 sec, p_o = 1 μPa, ρ = 1000 Kg/m^3, and c = 1500 m/s}:

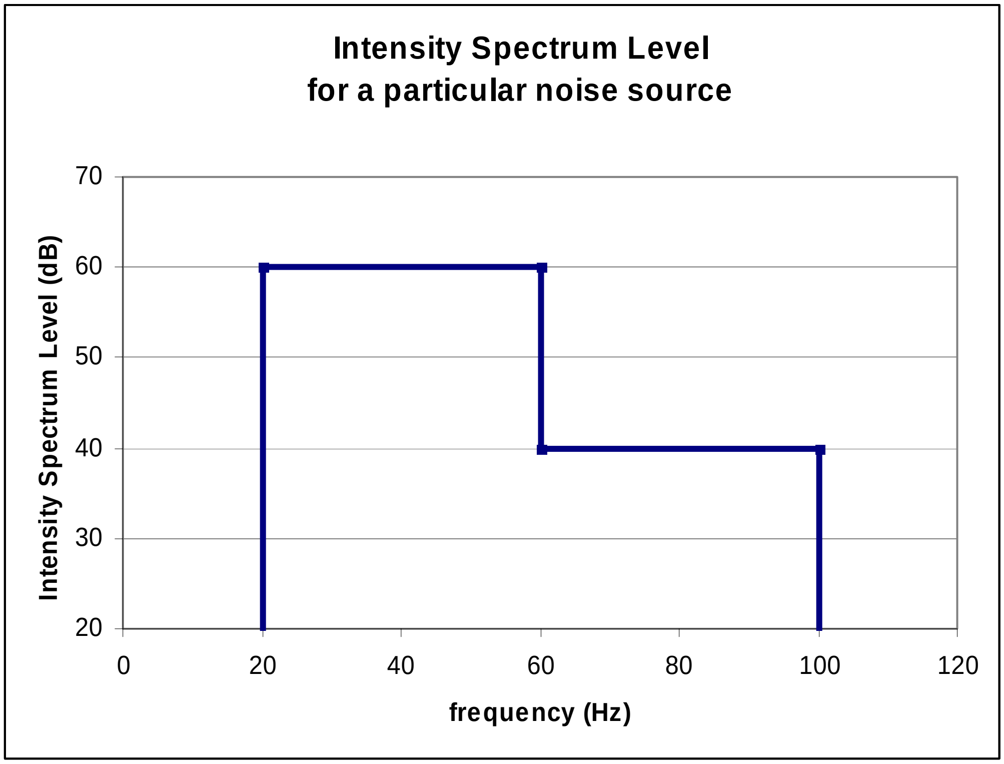
$$p(t) = [1 + 4\cos\left(2\pi 1t/T\right) - 3\cos(2\pi 2t/T) + 6\cos(2\pi 5t/T) +$$
$$2\sin(2\pi 1t/T) + 5\sin(2\pi 3t/T) + 8\sin(2\pi 5t/T)]\ Pa$$

a) Complete the table below for $0 \le n \le 6$:

n	a_n (Pa)	b_n (Pa)	T_n (sec)	f_n (Hz)	P^2_{rms} (Pa)	$\langle I_n \rangle$W/ m^2	BL_n (dB)
0							
1							
2							
3							
4							
5							
6							

b) Plot the Cosine Amplitude spectrum ***a_n* (Pa) vs f (Hz)**. Use your own graph paper
c) Plot the Sine Amplitude spectrum ***b_n* (Pa) vs f (Hz)**. Use your own graph paper.
d) Plot the time averaged Intensity spectrum, ***$\langle I_n \rangle$* (W/m^2) vs f (Hz)**. Use your own graph paper.
e) Plot the discrete Band Level spectrum, ***BL_n* (dB) vs f (Hz)**. Use your own graph paper.
f) Determine the total Band Level, ***BL_{TOT}*** for $0Hz \le f \le 6Hz$ and the average Intensity Spectrum Level, ***ISL_{AVE}*** using the information above.

2. The intensity Spectrum Level (ISL) is given below for a source of noise:



a) Over what frequency interval is the Intensity Spectrum Level a constant 60 dB?
b) What is the Intensity Spectrum Level in the range 60 Hz $<f<$ 100 Hz?
c) What is the Band Level (BL) for the noise in the frequency range 20 Hz $<f<$ 60 Hz?
d) What is the Band Level (BL) for the noise in the frequency range 60 Hz $<f<$ 100 Hz?
e) What is the Band Level (BL) for the noise in the frequency range 20 Hz $<f<$ 100 Hz?

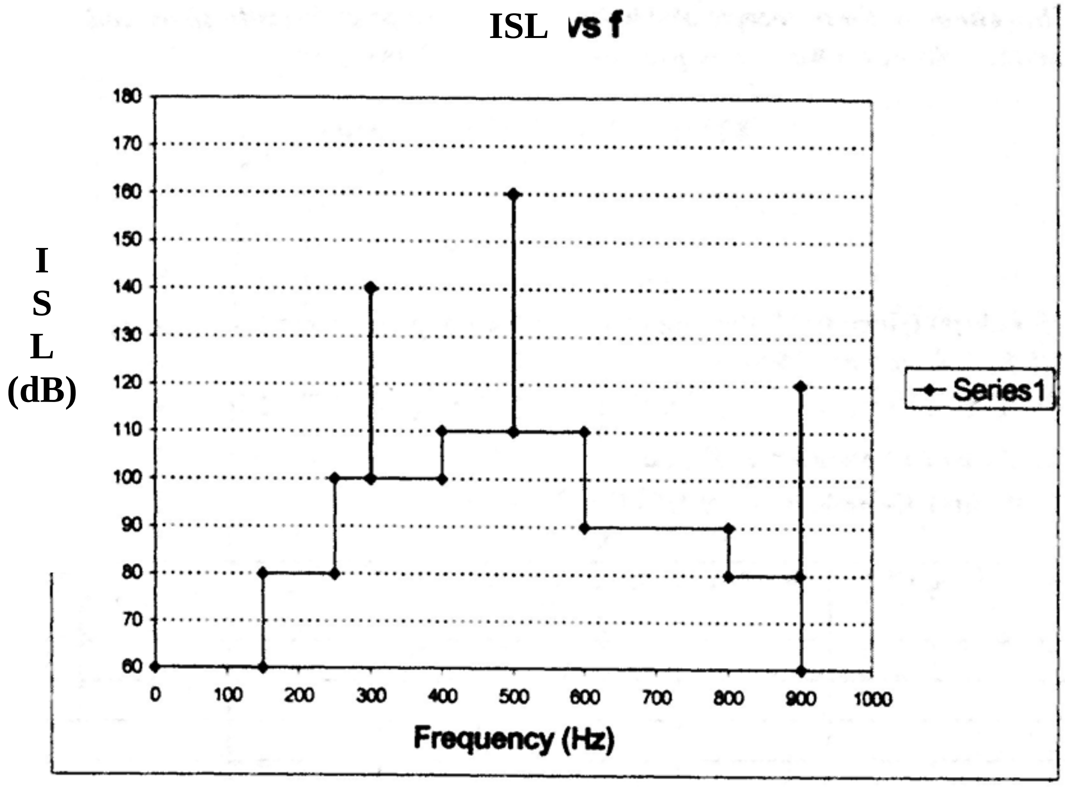
3. If the Band Level is 100 dB in a white noise bandwidth of 50 Hz, what is the (average) Intensity Spectrum Level?

4. For aone-third octave band centered on a frequency of 1000 Hz, calculate the lower and upper frequencies and the bandwidth.

5. The lower frequency of a one-third octave band is 200 Hz. Find the upper frequency, the center frequency, and the bandwidth.

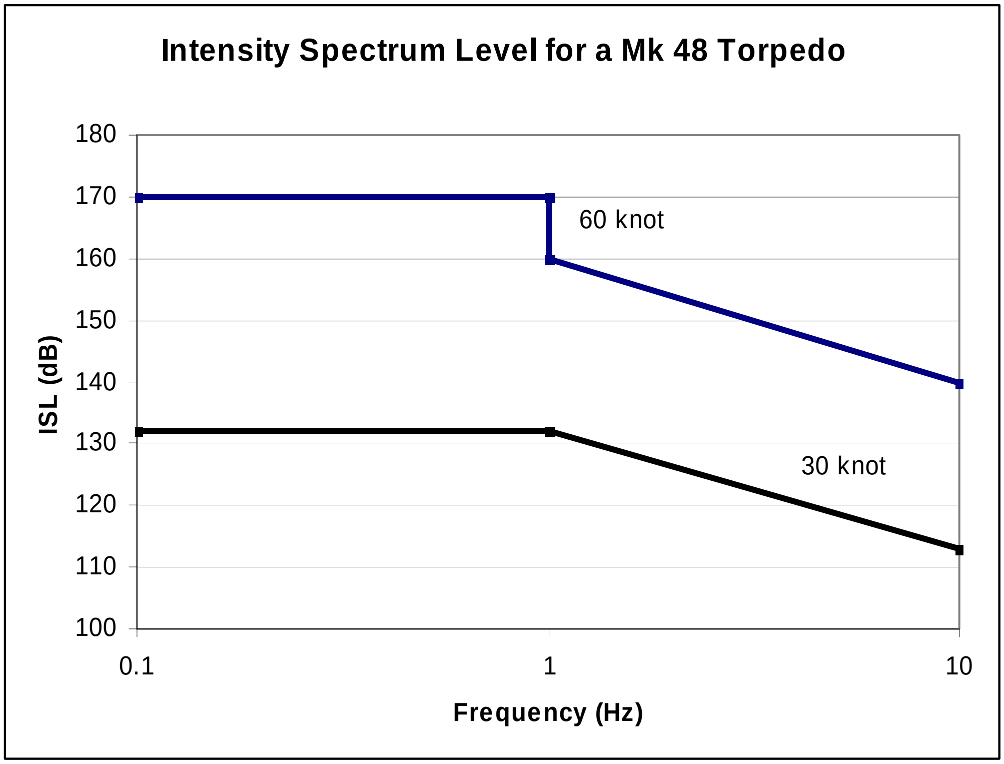
6. a) For a one-octave bandwidth, show that the bandwidth is about 71% of the center frequency.
b) For a half-octave bandwidth, show that the bandwidth is about 35% of the center frequency.
c) For a third-octave bandwidth, show that the bandwidth is about 23% of the center frequency.

7. Given the following graph of “white noise” and tonals:



a) Compute the total band level BL_{tot}, for a receiver having a one octave bandwidth centered around 637.4 Hz. Assume each tonal has a Δf=1 Hz.
b) Compute the average “white noise” intensity spectrum level, ISL_{ave}.

8. Using the figure below, estimate the noise level at 1 m from a U.S. Mark 48 torpedo traveling at 30 knots in the frequency band ranging 200 Hz to 10 kHz.



a) Repeat for a torpedo traveling at 30 knots.
b) Repeat for a torpedo traveling at 30 knots but for a sonar receiver with a band ranging from 100 Hz to 10 kHz.

Intensity, Intensity **Level**, and Intensity **Spectrum Level**

$$\langle I\rangle = \frac{p_{a\,max}^2}{2\rho c} \quad \text{or,}$$

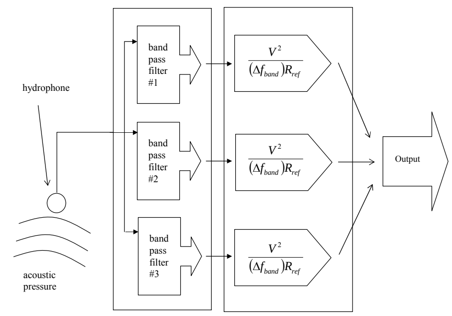
$$\langle I\rangle = \frac{\langle p_a^2\rangle}{\rho c} = \frac{p_{a\,rms}^2}{\rho c}$$

$$L \equiv 10\log\frac{\langle I\rangle}{I_{ref}}$$

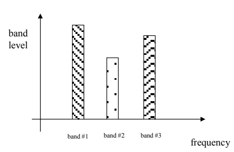
The spectrum level is the intensity level of the sound wave **within a 1 Hz band**.

$$ISL = 10\log\frac{I(\text{in 1 Hz band})}{I_{ref}(\text{in 1 Hz band})} = 10\log\frac{I(\text{in 1 Hz band})\bullet 1\text{Hz}}{I_{ref}}$$

A Sonar System



System Output

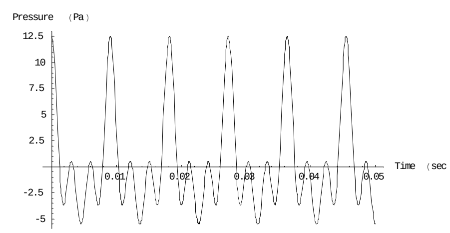


- Fourier Transform of the Time Domain Signal
- Frequency Analyzer

Example

$$p(t) = \left\{\frac{1}{2} + 5\cos\left[2\pi(110\text{Hz})t\right] + 3\cos\left[(2)2\pi(110\text{Hz})t\right] + 4\cos\left[(3)2\pi(110\text{Hz})t\right]\right\}\text{Pa}$$

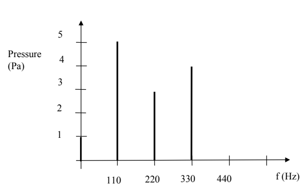
$$f(t) = \frac{1}{2}a_0 + a_1\cos(\omega t) + a_2\cos(2\omega t) + \cdots$$
$$+ b_1\sin(\omega t) + b_2\sin 2(\omega t) + \cdots$$



Fourier Coefficients

n	f_n (Hz)	P_{max} (Pa)	P^2_{rms} (Pa)	I (W/m^2)	L (dB)
0	0	1	0.5	3.3×10^{-7}	117
1	110	5	12.5	8.3×10^{-6}	131
2	220	3			
3	330	4			

$$p(t) = \left\{\frac{1}{2} + 5\cos\left[2\pi(110\text{Hz})t\right] + 3\cos\left[(2)2\pi(110\text{Hz})t\right] + 4\cos\left[(3)2\pi(110\text{Hz})t\right]\right\}\text{Pa}$$



$$P_{max}^2 = a_n^2 + b_n^2$$

$$P_{rms}^2 = \frac{P_{max}^2}{2}$$

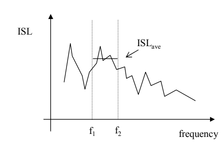
$$\langle I\rangle = \frac{P_{rms}^2}{\rho c} = \frac{P_{max}^2}{2\rho c}$$

Band Level

$$BL = 10\log\frac{I_{tot}}{I_{ref}} = 10\log\frac{I(\text{in a 1 Hz band})\Delta f}{I_{ref}}$$

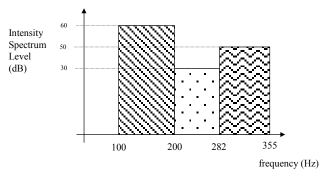
$$BL = 10\log\frac{I(\text{in a 1 Hz band})\square 1\text{Hz}}{I_{ref}} + 10\log\frac{\Delta f}{1\text{Hz}}$$

$$BL = ISL_{ave} + 10\log(\Delta f)$$
where $\Delta f = f_2 - f_1$



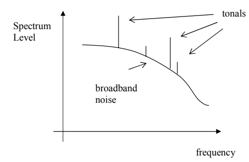
The Band Level is the intensity level over a band other than 1 Hz.

Example



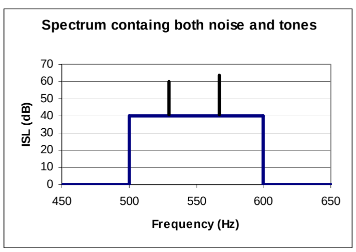
- Using the plot of ISL as a function of frequency shown, calculate
  - the band level of every band
  - the total band level
  - the total band level in a band from 150 Hz to 300 Hz

Types of Spectrums



$$BL_{tot} = \left[ISL_{ave} + 10\log\left(\Delta f_{BB}\right) \right] \oplus L_{tonal\ \#1} \oplus L_{tonal\ \#2} \oplus \ldots$$

Example



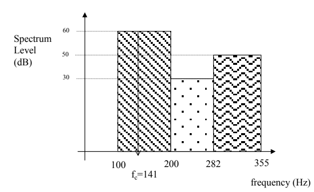
What is the total BL?

Common Bandwidths

- Constant Bandwidth

  $\Delta f = \text{constant}$
- Proportional Bandwidth
  - Octave Bandwidth
    - $f_2 = 2^1 f_1$
  - Half Octave Bandwidth
    - $f_2 = 2^{1/2} f_1$
- Center Frequency

  $f_c = \sqrt{f_1 f_2}$



Preferred Octave Bands

TABLE 4.2

Comparison of 1-octave and 1/3-octave bands

1 Octave			1/3 Octave		
Lower cutoff frequency (Hz)	*Center frequency (Hz)*	*Upper cutoff frequency (Hz)*	*Lower cutoff frequency (Hz)*	*Center frequency (Hz)*	*Upper cutoff frequency (Hz)*
11	16	22	14.1	16	17.8
			17.8	20	22.4
			22.4	25	28.2
22	31.5	44	28.2	31.5	35.5
			35.5	40	44.7
			44.7	50	56.2
44	63	88	56.2	63	70.8
			70.8	80	89.1
			89.1	100	112
88	125	177	112	125	141
			141	160	178
			178	200	224
177	250	355	224	250	282
			282	315	355
			355	400	447
355	500	710	447	500	562
			562	630	708
			708	800	891
710	1,000	1,420	891	1,000	1,122
			1,122	1,250	1,413
			1,413	1,600	1,778
1,420	2,000	2,840	1,778	2,000	2,239
			2,239	2,500	2,818
			2,818	3,150	3,548
2,840	4,000	5,680	3,548	4,000	4,467
			4,467	5,000	5,623
			5,623	6,300	7,079
5,680	8,000	11,360	7,079	8,000	8,913
			8,913	10,000	11,220
			11,220	12,220	14,130
11,360	16,000	22,720	14,130	16,000	17,780
			17,780	20,000	22,390

Why do we care?

Source Level (SL) and Noise Level (NL) are both examples of Band Levels (BL) where the frequency band(s) are defined by the frequencies of our Sonar System