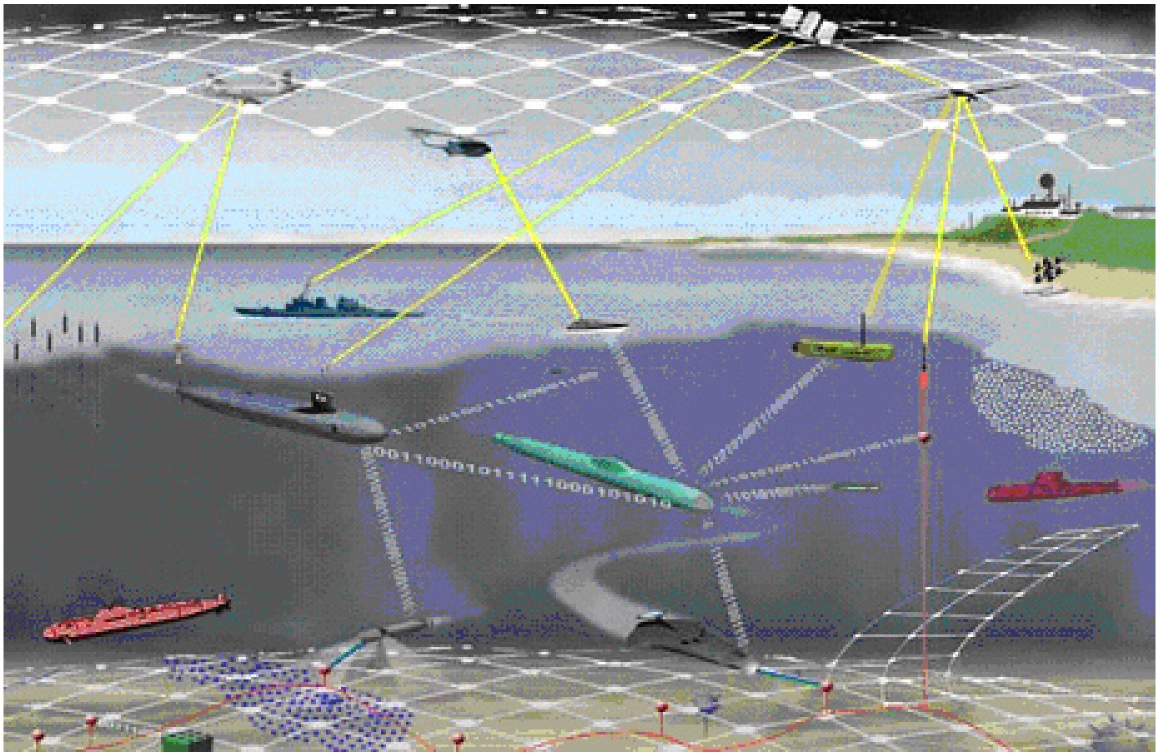
Passive SONAR Equation Intro

There are many forms of equations that comprise the passive SONAR equation but what is common amongst all forms of the equation is that they try to quantify all the affects on passive detection of sound from a contact or target. The form of the equation that will be presented this semester most often is:

$$L_{S/N} = SL-TL - (NL-DI)$$

In the below schematic of the undersea battle space, the sound emitted by one of the submarine platforms is represented by Source Level, SL. Losses sustained during sound propagation are represented by the Transmission Loss, TL. Back ground noise in the battle space are represented by the Noise Level, NL, and receiver characteristic, Directivity Index, DI.



Why are we presenting this now?

For the rest of the semester, we will take a closer look at each of the components of the passive SONAR equation. The goal is for each of you to enhance your understanding about each of the components and understand what actions can minimize or maximize the factors as appropriate to your tactical undersea warfare situation. Undersea warfare is a complex process where those with the best understanding of their craft, survive to fight again.

Parts of the Passive SONAR Equation

Signal to Noise Ratio

The intensity level on the left is the ratio of the signal received by a sonar receiver to the noise. Anyone who has ever tuned a radio station manually has experienced the station's signal in the static noise of the receiver. Signal to noise ratio is an important concept because it represents the degree to which an amplifier can be successfully employed to improve this situation. If signal to noise ratio (S/N or SNR) is two low, the noise is nearly equal to the signal. In this case, amplification will also increase the noise and provide no substantial improvement. For high signal to noise ratios, amplification will improve the magnitude of the signal relative to the noise.

A very good question to ask is, how large a signal to noise ratio is necessary? For consumer electronic audio, listeners demand a very high SNR. If all that is necessary is the identification of information, low SNRs might be tolerated. In fact, some systems adopt the convention that the minimum SNR required is 1.0. Regardless of the exact nature of the detection criteria used, we refer to the criteria as Detection Threshold (DT). Any actual signal above the Detection Threshold is referred to as Signal Excess. Sometimes we set the minimum signal to noise ratio such that a trained sonar operator will be able to pick a target out of noise 50% of the time. We refer to this signal to noise ratio for 50% detection as the "Recognition Differential."

Remember that the passive sonar equation compares "levels" (in dB) vice the actual intensities. As such, $L_{S/N}$ is defined

$$L_{S/N} = 10\log\left(\frac{\text{Signal}_{\text{required}}}{\text{Noise}}\right)$$

Detection Threshold and Recognition Differential are also a decibel quantities.

Signal Level Received

The signal level received at the detector is the difference of the first two quantities on the right side of the SONAR equation above. The origin of these two terms is the intensity of the signal that is transmitted to the water from the target. This is called the Source Level (SL).

$$SL = 10\log\frac{I_S}{I_0}$$

$$I_S \equiv \text{ Signal Intensity}$$

$$I_0 \equiv \text{ Reference Intensity}$$

As the signal travels through the water, some of the signal is lost through various mechanisms. The totality of this loss is quantified as the Transmission Loss (TL).

$$TL = 10\log\frac{I_S}{I_R} \qquad \text{(For a plane wave)}$$

$$I_R \equiv \text{ Received signal intensity}$$

The source level minus the transmission loss determines how much signal is received at the detector.

$$L_s = (SL - TL) = \text{Signal Level}$$

As a general rule, Transmission Loss is a dependent on the distance between the source and the receiver. Since this distance is often the tactically significant quantity in an undersea engagement, we often rearrange the passive sonar equation to solve for the Transmission Loss. The loss that can be tolerated and still meet the detection criterion is termed Figure of Merit. This quantity provides a means to estimate the distance at which detection can be achieved.

$$TL_{allowable} = \text{Figure of Merit} = SL - L_{S/N\ Threshold} - (NL\text{-}DI)$$

There are several conventions we will adopt in refining these basic definitions. Specific items to pay attention to are the location of the source level. Additionally, the frequency bands that contain the signal and noise must always be considered.

Noise Level Present

The Noise Level (L_n or NL) is the sum of the total effect of background and self-noise hindering our ability to detect the target signal. Background noise much be estimated from a variety of sources including wind and weather, shipping, biologic activity, and industrial activity

$$NL = 10\log\frac{I_n}{I_0}$$

$$I_n \equiv \text{ Noise intensity}$$

The Directivity Index (DI) is a ratio of the noise level detected by the detector, to the total noise level over 360°.

$$DI = 10\log\frac{N_{ND}}{N_D}$$

$$N_{ND} \equiv \text{ Noise power from non-directional receiver}$$

$$N_D \equiv \text{ Noise power generated by actual receiver}$$

When a detector is omni-directional, the power ratio is one, corresponding to 0 dB. If a sonar receiver is an array of elements, beams (directions) are formed where the system is more responsive due the interference of coherent sound. In this case, all isotropic noise does not reach the receiver. Since only the noise in the correct beam reaches the receiver, it is effectively lowered compared to the omni-directional case.

The Noise Level (L_n) is the sum of the total effect of background and self-noise hindering our ability to detect the target signal.

$$L_n = (NL - DI)$$

Nomenclature conventions and the Passive SONAR Equation

There is little standardization in the symbols used in various references. In writing the passive sonar equation above, the nomenclature adopted by various academic sources (Urick) was used. In the Naval Warfare Publications (NWPs), the Navy has a slightly different set of symbols.

An "L" quantity is an absolute level (dB referenced to a standard). An "N" quantity uses level subtraction to compare two intensities or pressures. These two acoustic parameters can be measured at different platforms or different times. The below chart compares some of the terms used in the passive (and active) sonar equation.

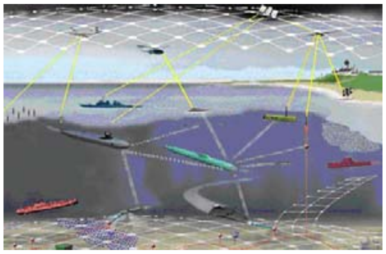
Urick	Description	Navy
SL	Source Level	L_S
TL	Transmission Loss	N_W
NL	Noise Level	L_N
DI	Directivity Index	N_{DI}
DT	Detection Threshold	N_{RD}
$L_{S/N}$ SNR	Signal to Noise Ratio	N_{SN}
RL	Reverberation Level	L_R
TS	Target Strength	N_{TS}

Problems

1. A submarine is conducting a passive barrier patrol against a transiting enemy submarine. The friendly sub has a sonar with a directivity index of 15 dB and a detection threshold of 8 dB. The enemy sub has a source level of 140 dB. Environmental conditions are such that the transmission loss is 60 dB and the equivalent isotropic noise level is 65 dB.
   a) What is the received signal level?
   b) What is the received signal-to-noise ratio in dB?
   c) What is the figure of merit?
   d) Can the enemy sub be detected? Why?

2. A submarine is attempting to detect an aircraft carrier transiting the Straits of Malacca. The aircraft carrier has a source level of 90 dB. The submarine's passive sonar has a directivity index of 20 dB and a detection threshold of 15 dB. Biological noise in the Straits is 54 dB. The submarine's self noise is 50 dB. Given that TL = 10 log r, where r is the range in yards (we will show you where this comes from soon), at what range can the carrier just be detected?

The Passive Sonar Equation



Will the sensor detect the red submarine?

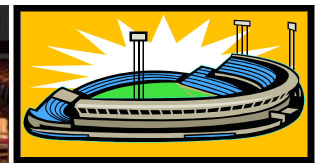
Signal to Noise Ratio

Signal

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\text{Signal Intensity}}{\text{Noise Intensity}}$$

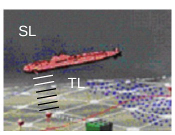
The higher the SNR, the more likely you are to hear (detect) the signal.

Noise (quiet)



Noise (Loud

Source Level and Transmission Loss



$$SL = 10\log\left(\frac{I_S}{I_0}\right)$$

$$TL = 10\log\left(\frac{I_S}{I_R}\right)$$

$$L_R = SL - TL$$

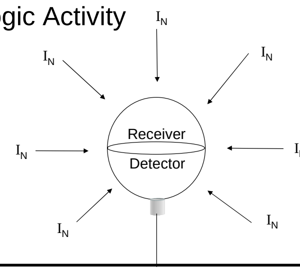
$$L_{S/N} = 10\log\left(\frac{\text{Signal}}{\text{Noise}}\right) = L_R - 10\log\left(\frac{\text{Noise Intensity}}{I_0}\right) = SL - TL - 10\log\left(\frac{\text{Noise Intensity}}{I_0}\right)$$

Sources of Noise

- Shipping Noise
- Wind and Weather
- Marine Life – Biologic Activity
- Self Noise
  - Flow of Water
  - Machinery

Omni directional Noise

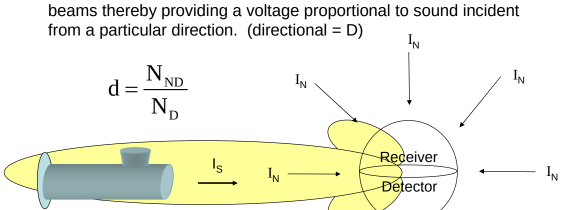
Isotropic Noise



Directivity Factor

- Some detectors are only able to provide a voltage proportional to all incident sound from all directions. (non-directional = ND)
- Other detectors use more sophisticated signal processing and form beams thereby providing a voltage proportional to sound incident from a particular direction. (directional = D)

$$d = \frac{N_{ND}}{N_D}$$



N_D = Electrical power generated by actual directional receiver

N_{ND} = Electrical power generated by equivalent non-directional receiver

The Passive Sonar Equation

$$L_{S/N} = 10\log\left(\frac{\text{Signal}}{\text{Noise}}\right) = L_R - 10\log\left(\frac{\text{Noise Intensity}}{I_0}\right) = SL - TL - 10\log\left(\frac{\text{Noise Intensity}}{I_0}\right)$$

$$L_{S/N} = 10\log\left(\frac{\text{Signal}}{\text{Noise}}\right) = L_R - L_{N\ \text{Received}} = SL - TL - (NL - DI)$$

$$L_{S/N} = SL - TL - (NL - DI)$$

$$SL = 10\log\left(\frac{I_S}{I_0}\right) \qquad NL = 10\log\left(\frac{I_N}{I_0}\right)$$

$$TL = 10\log\left(\frac{I_S}{I_R}\right) \qquad DI = 10\log(d)$$

Figure of Merit

- Often a **detection threshold** is established such that a trained operator should be able to detect targets with that $L_{S/N}$ half of the time he hears them. Called "**Recognition Differential.**" **(RD)**
- Passive sonar equation is then solved for TL allowable at that threshold. Called "**Figure of Merit.**" **(FOM)**

$TL_{allowable}$ = Figure of Merit = SL- $L_{S/N\ Threshold}$ - (NL-DI)

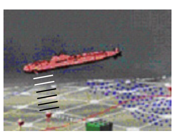
- Since TL logically depends on range, this could provide an estimate of range at which a target is likely to be detected. Called "**Range of the Day.**" **(ROD)**
- Any $L_{S/N}$ above the Recognition Differential is termed "**Signal Excess**." **(SE)** Signal Excess allows detection of targets beyond the Range of the Day.

Example

- A hostile submarine with a Source Level, SL = 130 dB re 1 μPa is near a friendly submarine in a part of the ocean where the Noise Level from all sources, NL = 70 dB re 1 μPa. The directivity factor is 3000 for the friendly submarine's sonar. If the Recognition Differential for the friendly submarine is 20 dB, what is the Figure of Merit?
- If the actual Transmission Loss is 50 dB, what is the Signal Excess.

Signal to Noise Level

$$L_{S/N} = 10\log\left(\frac{\text{Signal Intensity}}{\text{Noise Intensity}}\right) = 10\log\left(\frac{\text{Signal Intensity}}{I_0}\right) - 10\log\left(\frac{\text{Noise Intensity}}{I_0}\right)$$



But we will be measuring the signal intensity level at the receiver/detector, I_R (in the frequency band of the detector)

This is different from the signal intensity level leaving the target, I_S (in the frequency band of the detector)

$$\frac{I_R}{I_0} = \frac{I_S}{I_0}\frac{I_R}{I_S} = \frac{\frac{I_S}{I_0}}{\frac{I_S}{I_R}} \quad \text{OR} \quad 10\log\left(\frac{I_R}{I_0}\right) = 10\log\left(\frac{I_S}{I_0}\right) - 10\log\left(\frac{I_S}{I_R}\right)$$

$$L_R = SL - TL$$

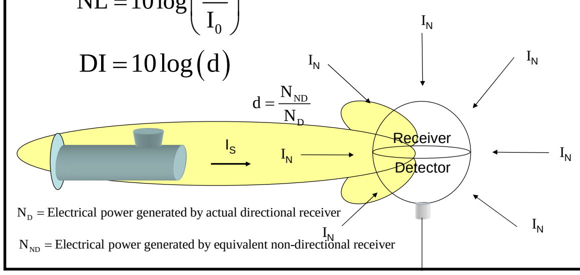
Fraction of emitted intensity reaching receiver

Noise Level and Directivity Index

$$\frac{I_{N\ Received}}{I_0} = \frac{I_N}{I_0}\frac{1}{d} \quad \text{OR} \quad L_{N\ Received} = 10\log\left(\frac{I_{N\ Received}}{I_0}\right) = 10\log\left(\frac{I_N}{I_0}\right) - 10\log(d)$$

$$NL = 10\log\left(\frac{I_N}{I_0}\right) \qquad L_{N\ Received} = NL - DI$$

$$DI = 10\log(d)$$



$$d = \frac{N_{ND}}{N_D}$$

N_D = Electrical power generated by actual directional receiver

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Adding Decibels