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Name Key SP211 Exam 3

1. a. The radius of the earth is 6.37×10^6 m. The mass of the earth is 5.97×10^{24} kg. The universal gravitation constant is 6.67×10^{-11} N-m^2/kg^2. Find the acceleration due to gravity at a distance of two earth radii from the center of the earth (i.e. $r = 2\ R_{earth}$).
(5 points)

$$mg = \frac{G M_e m}{r^2}$$

$$= \frac{(6.67\times10^{-11}\ \frac{N\cdot m^2}{kg^2})(5.97\times10^{24}\ kg)}{(2\times6.37\times10^6\ m)^2}$$

$$= \boxed{2.45\ m/s^2}$$

b. What is the speed required for a satellite to move in a circular orbit at this distance ($r = 2\ R_{earth}$) from the center of the earth? Hint: What is the centripetal acceleration? (5 points)

$$m\frac{v^2}{r} = \frac{G M_e m}{r^2} = g$$

$$v^2 = gr \Rightarrow v = \sqrt{gr}$$

$$v = \sqrt{(2.45\ m/s^2)(2\times6.37\times10^6\ m)} = \boxed{5.60\times10^3\ m/s}$$

c. What is the orbital period of this satellite? (5 points)

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(2\times6.37\times10^6\ m)}{5.60\times10^3\ m/s}$$

$$= 1.43\times10^4\ s$$

$$= \boxed{3.97\ Hr}$$

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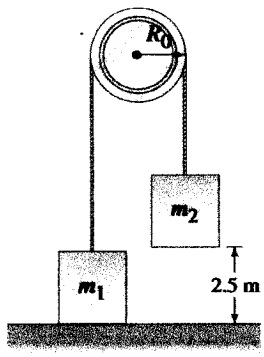
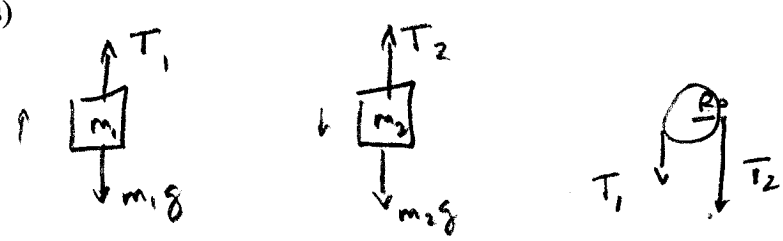
2. Two masses, $m_1 = 35.0$ kg and $m_2 = 38.0$ kg are connected by massless rope and a pulley which is a uniform cylinder of radius $R_o = 0.30$ m and mass, $m = 4.8$ kg ($I=mR^2/2$). With m_1 initially on the ground and m_2 2.5 m above the ground, the system is released from rest.

a. What is the moment of inertia for the pulley wheel? (2 points)

$$I = \frac{1}{2} mR^2 = \frac{1}{2}(4.8\,kg)(.3\,m)^2 = .216 \ kg\,m^2$$

b. Solve for the acceleration of m_2. Assume the pulley to be frictionless. Note the tension on the side of the rope attached to m_1 cannot be the same as that on the side attached to m_2 or the pulley will not turn. (6 points)





$$T_1 - m_1 g = m_1 a \Rightarrow T_1 - 343N = 35kg\, a$$

$$m_2 g - T_2 = m_2 a \Rightarrow 372N - T_2 = 38kg\, a$$

$$R_0 T_2 - R_0 T_1 = I\alpha = I\frac{a}{R_0}$$

$$T_2 - T_1 = I\frac{a}{R_0^2} = \frac{1}{2} M \cancel{R_0^2} \frac{a}{\cancel{R_0^2}}$$

$$T_2 - T_1 = (2.4kg)\, a$$

$$\boxed{a = .385\,m/s^2}$$

$a = .384\,m/s^2$ $T_1 = 356.5\,N$ $T_2 = 357.4$

b. What is the velocity of m_2 just before it strikes the ground. (2 points)

$$v_f^2 = \cancel{v_0^2} + 2a\Delta x = 2(.385\,m/s^2)(2.5\,m)$$

$$v_f = 1.39\,m/s$$

For full credit, you must draw free body diagrams for m_1, m_2 and the pulley wheel and show equations associated with each.

Alternatively, you might use energy considerations to solve part b first then work back to part a. If you use this method, part b is worth 6 points and b becomes worth 2 points.

$$E_i = E_f$$

$$m_2 g y_2 = m_1 g y_2 + \frac{1}{2} m_1 v_2^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I \omega^2$$

$$m_2 g y_2 - m_1 g y_2 = \frac{1}{2} m_1 v_2^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I \frac{v^2}{R_0^2}$$

$$(m_2 - m_1) g y_2 = \left(\frac{1}{2} m_1 + \frac{1}{2} m_2 + \frac{1}{4} M\right) v^2$$

$$(38\,kg - 35\,kg)(9.8\,m/s^2)(2.5\,m) = \left(\frac{1}{2}\,38\,kg + \frac{1}{2}\,35\,kg + \frac{1}{4}\,9.8\,kg\right) v^2$$

$$73.5\,J = (37.7\,kg)\, v^2$$

$$1.9496\,\frac{m^2}{s^2} = v^2$$

$$v = 1.396\,m/s = \boxed{1.4\,m/s}$$

$$v^2 = \cancel{v_0^2} + 2a\,\Delta x$$

$$(1.4\,m/s)^2 = 2a(2.5\,m)$$

$$a = .389 \approx \boxed{.39\,m/s^2}$$

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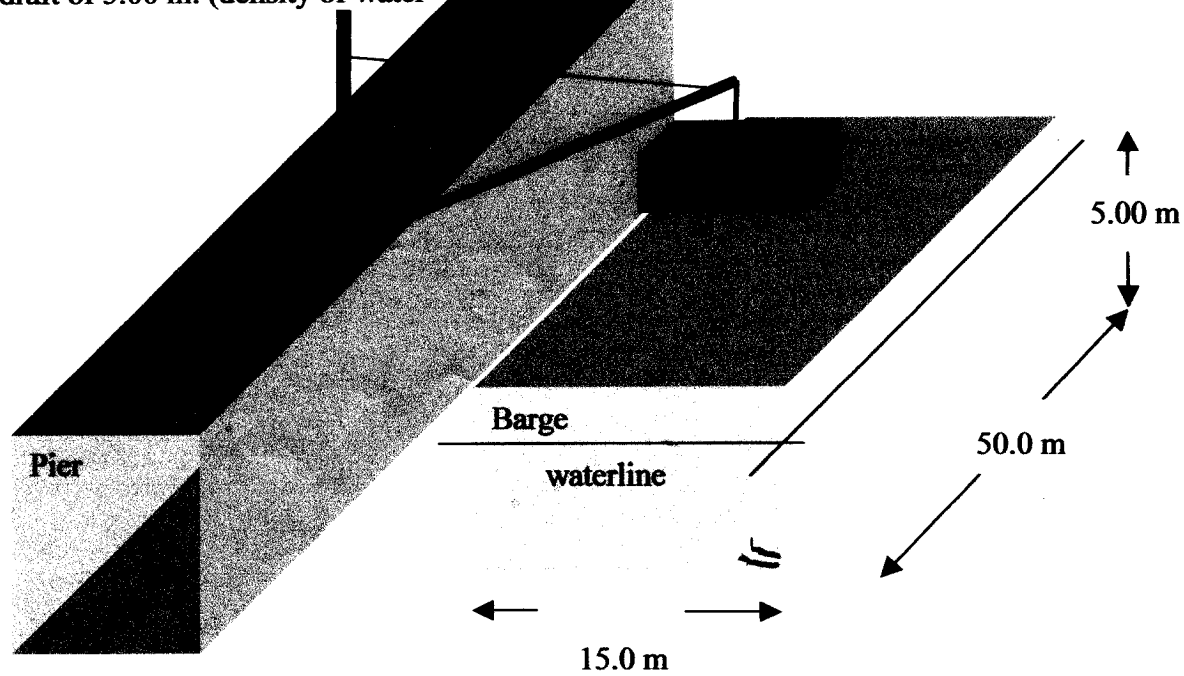
3. A rectangular barge with length 50.0 m and a beam of 15.0 m is floating alongside a pier with a draft of 5.00 m. (density of water =



a. What is the weight of the barge (in Newtons)? (5 points)

$$F_B = F_w = \rho V g = (1025\ kg/m^3)(15\ m)(50\ m)(5\ m)(9.8\ m/s^2)$$
$$= \boxed{3.77 \times 10^7\ N}$$

b. A crane lowers a palate containing several small cars with a total mass of 20000 kg onto the barge. What is the change in the draft? (5 points)

$$\Delta F_B = \rho \Delta V g = \rho A \Delta z g$$
$$(20000\ kg)\cancel{g} = (1025\ kg/m^3)(50\ m)(15\ m)\ \Delta z\ \cancel{g}$$
$$\Delta z = \boxed{26\ cm}$$

c. A hole develops near the keel at a depth of 5.00 m. What is the water pressure (gauge pressure) just outside the hole? (5 points)

$$P_G = \rho g z = (1025\ kg/m^3)(9.8\ m/s^2)(5\ m)$$
$$= 5.02 \times 10^4\ N/m^2$$

d. Use Bernoulli's Equation to find the velocity of the water entering the barge through the hole near the keel. The pressure inside the barge is 1 Atmosphere. (5 points)

$$\cancel{P_{IN}}^{0} + \frac{1}{2}\rho v_{IN}^2 + \cancel{\rho g z_{IN}} = P_{OUT} + \frac{1}{2}\cancel{\rho v_{OUT}^2} + \cancel{\rho g z_{OUT}}$$

$$\frac{1}{2}\cancel{\rho} v_{IN}^2 = \cancel{\rho} g z_{OUT}$$

$$v_{IN} = \sqrt{2 g z_{OUT}} = \sqrt{2(9.8\ m/s^2)(5\ m)} = 9.90\ m/s$$

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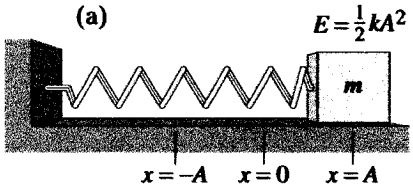
4. For the mass oscillating in the figure below, the x coordinate of the position of the mass is given by the following equation

$$x = 8\cos\left(4\pi t + \frac{\pi}{2}\right)$$

where x is in meters and t is in seconds.



a. **What is the amplitude? (1 point)**

$A = 8\,m$

b. What is the angular frequency, ω? (1 point)

$\omega = 4\pi\ rad/s$

c. What is the frequency, f? (2 points)

$f = \frac{\omega}{2\pi} = \frac{4\pi\ rad/s}{2\pi} = 2\ Hz$

d. What is the period? (2 points)

$T = \frac{1}{f} = \frac{1}{2}\ sec$

e. What is the phase t = 0 sec? (1 point)

$\frac{\pi}{2}\ rad$

f. What is the velocity at 2 sec? (2 points)

$v = -8m(4\pi)\frac{r}{s}\sin\left(4\pi t + \pi/2\right) = -32\pi(1)\ m/s = -32\pi\ m/s$
$= -100.5\ m/s$

g. What is the total energy at any time? (2 points)

$E = \frac{1}{2}kA^2 = \frac{1}{2}32\pi^2(8m)^2 = 1024\pi^2\ J = 1.01\times10^4\ J$

h. What is the kinetic Energy at 2 sec? (1 point)

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(2kg)(-32\pi\ m/s)^2 = 1024\pi^2\ J$

i. What is the potential energy at 2 sec? (2 point)

$x = 8\cos\left(4\pi(2) + \pi/2\right) = 0 \Rightarrow E_p = 0\ J$

j. If the mass is 2 kg, what is the value of the spring constant? (2 points)

$\omega = \sqrt{\frac{k}{m}} \Rightarrow m\omega^2 = k = (2kg)(4\pi\ rad/s)^2$
$= 32\pi^2\ N/m = 315.8\ N/m$
$\approx 316\ N/m$

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5. An ice skater is initially rotating 100 rpm. The body, head, etc. of the ice skater are modeled by a *constant* moment of inertia of 0.10 kg-m^2. The hands, etc. are modeled by 0.5 kg point masses at the ends of massless rods. The point masses are initially 0.8 m from the center of rotation. If the point masses are pulled in to 0.4 m from the center of rotation, the new angular speed of the ice skater is closest to

A. 200 rpm.
B. 250 rpm.
C. 285 rpm.
D. 355 rpm.
E. 445 rpm.

$I_1 \omega_1 = I_2 \omega_2$

$(.1\,kg\,m^2 + 2(.5\,kg)(.8\,m)^2)\,100\,rpm = \omega_f(.1\,kg\,m^2 + 2(.5\,kg)(.4\,m)^2)$

$.74\ 100\,rpm = .26\ \omega_f$

$\omega_f = 285\,rpm$

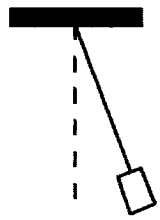
6. A simple pendulum takes 3.0 seconds for a complete oscillation. (The period of oscillation is 3.0 s.) The length of the pendulum is closest to

A. 3.0 m.
B. 2.2 m.
C. 6.1 m.
D. 3.5 m.
E. 5.2 m.

$\omega = \sqrt{\frac{g}{L}}$

$f = \frac{\omega}{2\pi}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L}{9.8\,m/s^2}} = 3\,s$



$\frac{L}{9.8\,m/s^2} = \frac{9\,s^2}{4\pi^2}$

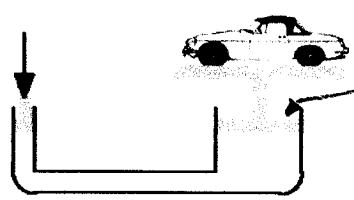
$L = \frac{9\,s^2(9.8\,m/s^2)}{4\pi^2} = \boxed{2.23\,m}$

7. A car sits on a platform (hydraulic lift) connected to a piston with an area of 1.0 m^2. The total weight of the car, platform and piston is 2.0x10^4 N. The massless piston on the left has an area of 0.010 m^2. The force on the piston on the left is closest to

A. 200 N.
B. 400 N.
C. 800 N.
D. 1600 N.
E. 2000 N.

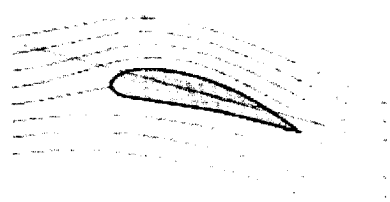
$\frac{F}{.01\,m^2} = \frac{2\times10^4\,N}{1\,m^2}$

$F = 2\times10^2\,N$



7. The top of an airplane wing has an area of 90 m^2. Assume that the area of the bottom of the wing is also 90 m^2. Air of density 1.29 kg/m^3 passes over the top with a speed of 350 m/s and over the bottom with a speed of 270 m/s. The dynamic lift on the wing (due to Bernoulli's principle) is closest to

A. 1.7x10^6 N.
B. 2.2x10^6 N.
C. 2.9x10^6 N.
D. 3.5x10^6 N.
E. 4.1x10^6 N.



$P_T + \frac{1}{2}\rho v_T^2 = P_B + \frac{1}{2}\rho v_B^2$

$P_B - P_T = \frac{1}{2}\rho(v_T^2 - v_B^2) = \frac{1}{2}(1.29\,kg/m^3)(350\,m/s^2 - 270\,m/s^2)$

$F = 90\,m^2(\frac{1}{2})(1.29\,kg/m^3)(\quad) = 2.88\times10^6$

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