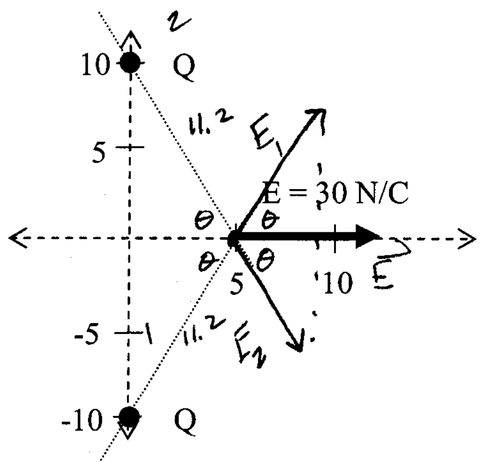
Name KEY Exam #1 SP212R Summer 2008

Total points ____ \ 85

1. The electric field on the x-axis at 5.0 cm has a magnitude of 30 N/C and is pointed to the right as shown. Two identical positive charges are located on the y-axis at +10 cm and -10 cm. (15 points)



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E_x = E_{1x} + E_{2x}$$

$$E_x = E_1 \cos\theta + E_2 \cos\theta$$

a. What is the charge, Q, causing this field? (5)

$$E_x = \frac{2kQ}{r^2}\cos\theta$$

$$\Rightarrow Q = \frac{r^2 E_x}{2k\cos\theta} = \frac{(11.2\times10^{-2}\,m)^2 (30\,N/C)}{2(8.988\times10^9 \frac{Nm^2}{C^2})\left(\frac{5}{11.2}\right)}$$

$$\underline{Q = 4.69\times10^{-11}\,C}$$

b. What is the electric potential at the same point as the field (5.0 cm, 0)? (5)

$$V = V_1 + V_2 = \frac{2kQ}{r} = \frac{2(8.988\times10^9 \frac{Nm^2}{C^2})(4.69\times10^{-11}\,C)}{(11.2\times10^{-2}\,m)}$$

$$\boxed{V = 7.53\,V}$$

c. At a different location in space the electric potential is given as $V(x,y,z) = 5y - 3x^2y + 2yz^2$. Find an expression for the Electric Field. Remember, the electric field is a vector and has three components. (5)

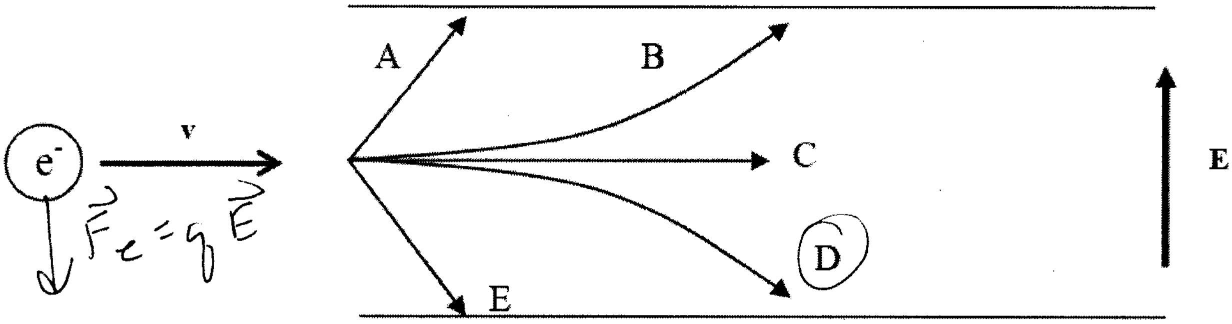
$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$E_z = -\frac{\partial V}{\partial z} = -4yz$$

$$E_x = -\frac{\partial V}{\partial x} = +6x$$

$$E_y = -\frac{\partial V}{\partial y} = -5 + 3x^2 - 2z^2$$

2. An electron with a velocity of $\mathbf{v} = 5.0 \times 10^5\,\mathbf{i}$ m/s enters a tunnel that contains a uniform electric field $\mathbf{E} = 20.0$ N/C $\mathbf{j}$. Which curve best describes the path of the electron after it enters the tunnel? (10 pts)



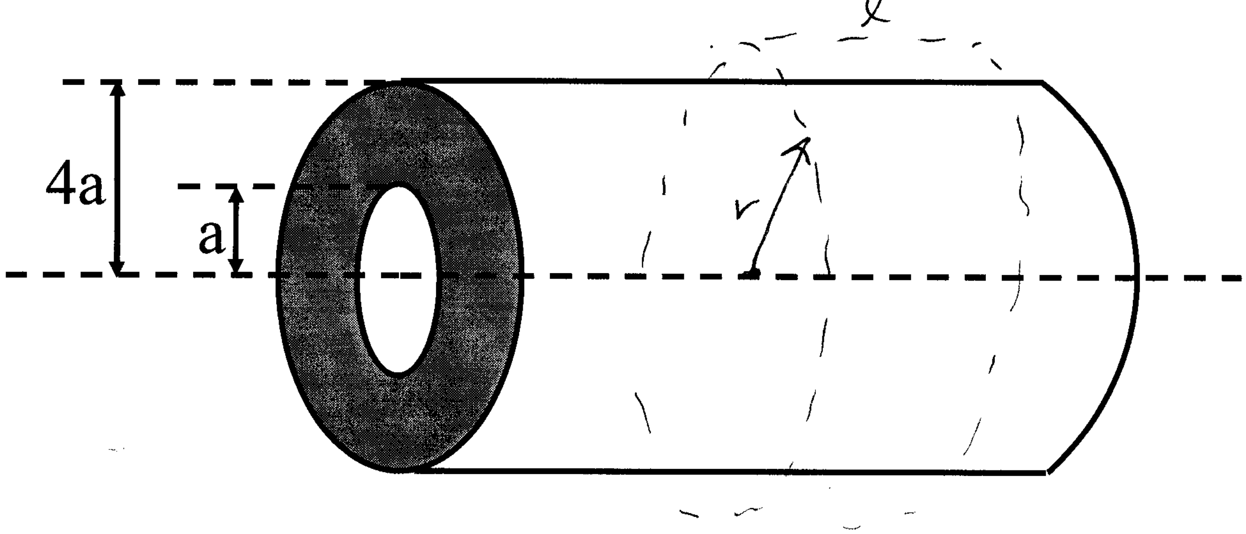
a) Why? (5 pts)

Constant force in $\hat{j}$ direction $\Rightarrow$ constant acceleration $\Rightarrow$ parabolic path

b) Neglecting gravity, calculate the net force acting on the electron while in the field. What direction is this force in? (5 pts)

$$\vec{F} = q\vec{E} = (-1.60\times10^{-19}\,C)(20.0\,N/C)\,\hat{j}$$
$$= \boxed{-3.2\times10^{-18}\,N\,\hat{j} \quad \downarrow\ -\hat{j}}$$

3. A fairly long non-conducting cylindrical shell has inner radius "a" and outer radius "4a" as shown. The cylinder has uniform volume charge density, ρ. (20 points)



a. Sketch a Gaussian surface that could be used to find the electric field at a distance r from the longitudinal axis when r is greater than 4a on the above figure. Be sure to stay away from the ends of the cylindrical shell. **Label the dimensions of this surface.** (3)

b. Why did you choose this surface? Include a description of the expected magnitude and direction of the electric field. (1)

$\vec{E}$ is $\perp$ to curved surface + constant

$\vec{E}$ is $\parallel$ to ends $\Rightarrow$ flux $= 0$

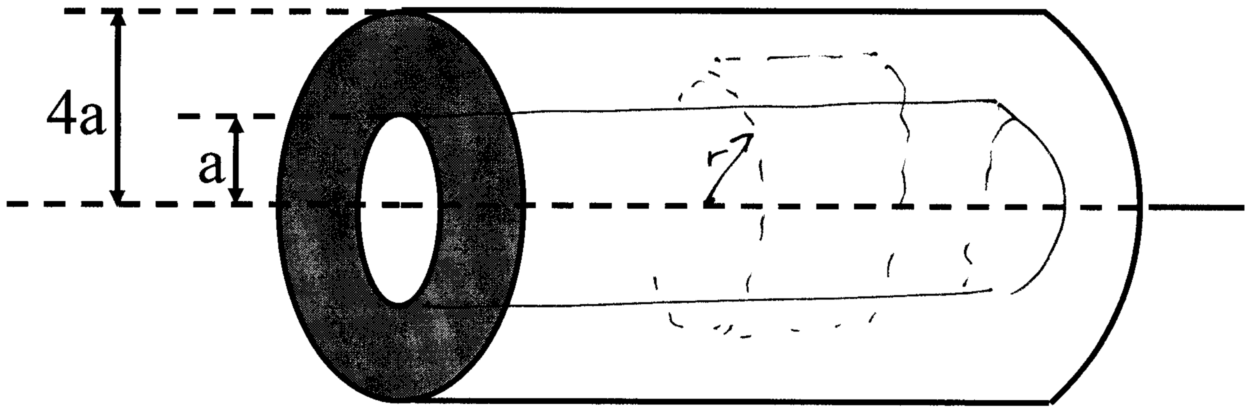
c. In terms of the charge density, ρ, the dimensions of your Gaussian surface and/or the cylindrical shell, and any appropriate physical constants, what is the expression for the flux passing through your surface? Hint – what is the charge inside your surface? (2)

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{\rho\left[\pi(4a)^2\ell - \pi a^2\ell\right]}{\varepsilon_0} = \frac{15\rho\pi a^2\ell}{\varepsilon_0}$$

d. Use Gauss's Law to solve for an expression for the magnitude of the electric field at a distance r from the longitudinal axis when r > 4a. (3)

$$\oint \vec{E}\cdot d\vec{A} = E\int_{curved} dA = E(2\pi r\ell) = \frac{q_{encl}}{\varepsilon_0} = \frac{15\rho\pi a^2\ell}{\varepsilon_0}$$

$$\boxed{E = \frac{15\rho a^2}{2\varepsilon_0 r}}$$



e. Use Gauss's Law to solve for an expression for the magnitude of the electric field at a distance r from the longitudinal axis when a < r < 4a. (5) Sketch the Gaussian surface you are using for this region on the above figure. (3) Again, label the dimensions of this surface.

$$\oint \vec{E} \cdot d\vec{A} = E \int_{curved} dA = \frac{q_{encl}}{\varepsilon_0} = \frac{\rho\left[\pi r^2 l - \pi a^2 l\right]}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{\rho \pi l (r^2 - a^2)}{\varepsilon_0}$$

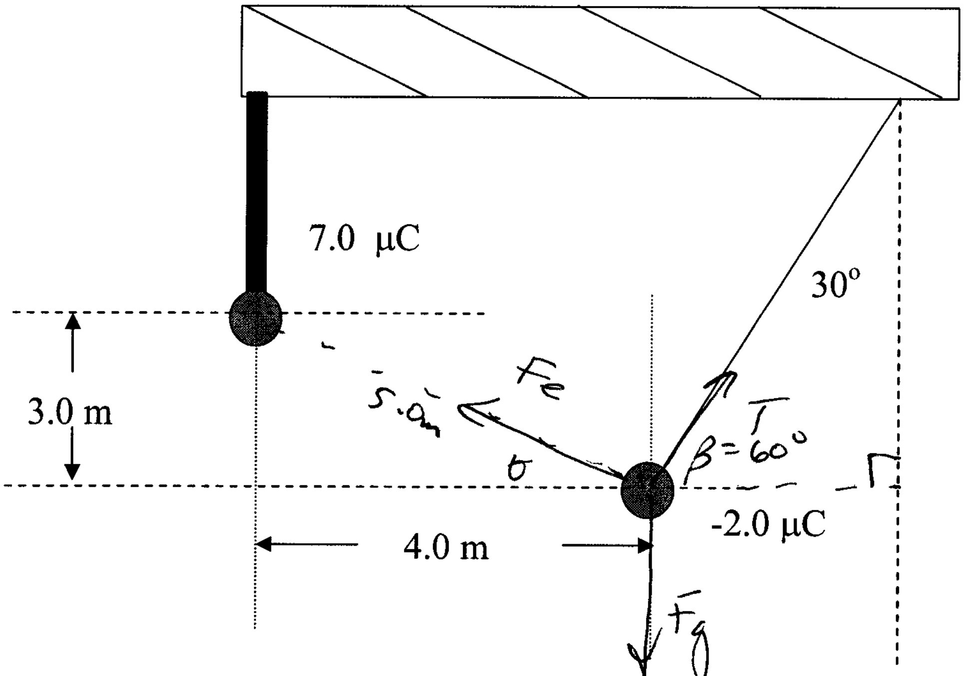
$$\Rightarrow \boxed{E = \frac{\rho (r^2 - a^2)}{2\varepsilon_0 r}}$$

f. Use Gauss's Law to solve for the magnitude of the electric field at a distance r from the longitudinal axis when r < a. (3)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0} = \boxed{0}$$

$$\Rightarrow \vec{E} = \boxed{0}$$

4. A particle of with an unknown mass, m, and with charge –2.0 μC is suspended in the presence of gravity at rest by a mass-less string as shown. A fixed charge of 7.0 μC is mounted as shown. (15 points)



a. Draw and label the forces acting on the –2.0 μC charge. (3)

b. What is the magnitude of the electrostatic force acting on the –2.0 μC charge? (3) Calculate the horizontal (x) and vertical (y) components (2 each) of this electrostatic force.

$$\vec{F}_e = -k\frac{q_1 q_2}{r^2}\hat{r}$$

$$|\vec{F}_e| = \frac{(8.988\times10^{9}\,\frac{Nm^2}{C^2})(7.0\times10^{-6}C)(2.0\times10^{-6}C)}{(5.0m)^2} = \boxed{5.03\times10^{-3}N}$$

$$F_{e_x} = |\vec{F}_e|\cos\theta = (5.03\times10^{-3}N)\left(\frac{4}{5}\right) = \boxed{4.03\times10^{-3}N}$$

$$F_{e_y} = |\vec{F}_e|\sin\theta = (5.03\times10^{-3}N)\left(\frac{3}{5}\right) = \boxed{3.02\times10^{-3}N}$$

c. What is the tension in the string? (2)

$$F_{e_x} - T\cos\beta = 0$$

$$\Rightarrow T = \frac{F_{e_x}}{\cos\beta} = \frac{4.03\times10^{-3}N}{\cos 60^\circ} = \boxed{8.05\times10^{-3}N}$$

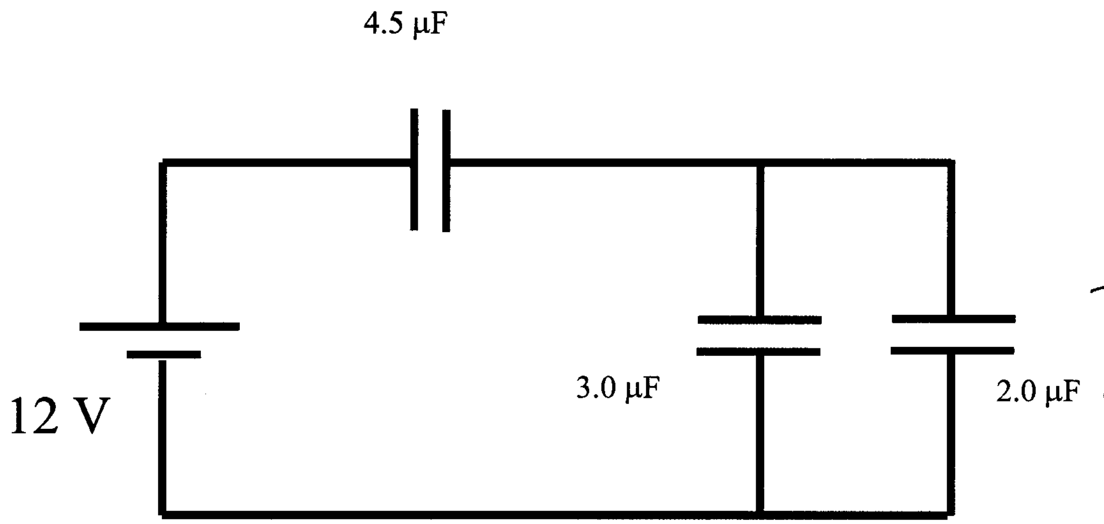
d. What is the mass of the particle? (3)

$$F_{e_y} + T\sin\beta - mg = 0$$

$$\Rightarrow m = \frac{F_{e_y} + T\sin\beta}{g} = \frac{3.02\times10^{-3}N + (8.05\times10^{-3}N)\sin 60^\circ}{9.81\,m/s^2}$$

$$\boxed{m = 1.02\times10^{-3}\,kg}$$

5. Three capacitors are connected to a battery in the circuit shown below. (15 points)



$\Big\} \; C_{\parallel} = 5.0\,\mu F$

a. What single capacitor would equivalently replace the three capacitors above? (3)

$$\frac{1}{C_{eq}} = \frac{1}{4.5\mu F} + \frac{1}{5.0\mu F} \Rightarrow C_{eq} = \underline{2.37\,\mu F}$$

b. What is the charge on the 4.5 μF capacitor? (3)

$$Q_{tot} = C_{eq}\,\Delta V_{batt} = (2.37\times10^{-6}F)(12.0V) = \boxed{2.84\times10^{-5}C}$$
$$= Q_{4.5\mu F} \text{ (series)}$$

c. What is the voltage across the 4.5 μF capacitor? (3)

$$\Delta V = \frac{Q}{C} = \frac{2.84\times10^{-5}C}{4.5\times10^{-6}F} = \boxed{6.3\,V}$$

d. What is the voltage across the 3.0 μF capacitor? (3)

$$\Rightarrow \Delta V_{\parallel} = \Delta V_{batt} - \Delta V = 12 - 6.3 = \boxed{5.7V}$$

e. What is the charge on the 3.0 μF capacitor? (3)

$$Q = C\Delta V = (3.0\times10^{-6}F)(5.7V) = \boxed{1.7\times10^{-5}C}$$

6. A parallel plate capacitor is constructed of two metal plates of area 25 cm^2 separated by a distance of 5.0 mm. (10 pts)

a. Calculate the capacitance (2 pts):

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ C^2/N\cdot m^2)(25\ cm^2)\left(\frac{1\ m}{100\ cm}\right)^2}{5.0\times10^{-3}\ m}$$

$$\boxed{C = 4.4\ pF}$$

The capacitor is immersed in a jar filled with distilled water (κ = 80.0) and connected to a 9.0 V battery.

b. Calculate the charge on the plates (after the battery has been connected for a long time) (2 pts):

$$Q = (\kappa C)(\Delta V) = (80.0)(4.4\times10^{-12}\ F)(9.0\ V)$$

$$= \boxed{3.2\ nC}$$

c. Calculate the electrostatic potential energy stored in the capacitor (2 pts):

$$U = \frac{1}{2} C \Delta V^2 = \frac{1}{2}(80.0)(4.4\times10^{-12}\ F)(9.0\ V)^2$$

$$= \boxed{1.4\times10^{-8}\ J}$$

The capacitor is **disconnected from the battery** and then the **water is drained out.** (*Hint: Think about what happens to the voltage across an isolated capacitor if you put a dielectric between the plates. You are now doing the REVERSE of that)*

d. What is the voltage across the capacitor plates now (2 pts):

$$\Delta V_{wet} = \frac{\Delta V_{dry}}{\kappa} \Rightarrow \Delta V_{dry} = \kappa \Delta V_{wet}$$

$$= (80.0)(9.0\ V)$$

$$= \boxed{\cancel{270\ V}}\ 720\ V$$

e. Calculate the electrostatic potential energy stored in the capacitor now (2 pts):

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2}(3.2\times10^{-9}\ C)(\cancel{270}\ 720\ V)$$

$$= \boxed{\cancel{4.3\times10^{-7}}\ J}$$

$$1.15\times10^{-6}$$