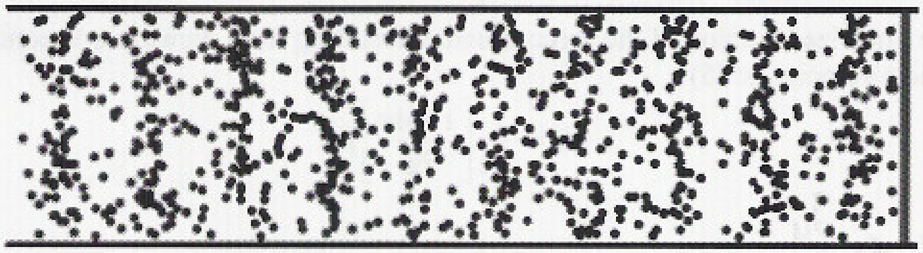
Name KEY SP436 Exam III

1. (20) A plane wave traveling through water with density, ρ, and phase speed, c, is modeled by the following equation:

$$\tilde{p} = P_o e^{j(\omega t + kx)}$$



a. What direction is this wave moving? (1) $(-x)$ DIRECTION

b. Using the linearized version of the force equation resulting from Newton's Second Law, develop an expression for the fluid particle velocity for this traveling wave. (5)

$$-\nabla \tilde{P} = \rho_o \frac{\partial \tilde{u}}{\partial t}$$

$$-jk P_o e^{j(\omega t+kx)}\hat{i} = j\omega \vec{\tilde{u}} \rho_o$$

$$\vec{\tilde{u}} = \frac{-\cancel{j}k P_o}{\cancel{j}\omega \rho_o} e^{j(\omega t+kx)}\hat{i}$$

$$\vec{\tilde{u}} = -\frac{P_o}{\rho_o c} e^{j(\omega t+kx)}\hat{i} \qquad \left(\frac{\omega}{k} = c\right)$$

c. Using the equation of state, develop an expression for the acoustic density condensation, s. (2)

$$\tilde{s} = \frac{\tilde{P}}{B} = \frac{P_o e^{j(\omega t+kx)}}{\rho_o c^2} \qquad \left(c^2 = \frac{B}{\rho_o}\right)$$

d. If angular frequency, **ω = 1885 rad/s** and the wavenumber, **k = 1.26 rad/m**, what is the phase speed? (2)

$$c = \frac{\omega}{k} = \frac{1885\ \text{r/s}}{1.26\ \text{r/m}} = 1496\ \text{m/s} \approx 1500\ \text{m/s}$$

e. If P_o is **150 mPa**, and the density is **1000 kg/m^3**, what is the effective or rms pressure? (2)

$$P_{rms} = \frac{P_o}{\sqrt{2}} = \frac{150\ \text{mPa}}{\sqrt{2}} = 106\ \text{mPa}$$

f. What is the maximum particle velocity, U_o? (2)

$$U_o = \frac{P_o}{\rho_o c} = \frac{150\ \text{mPa}}{(1000\ \text{kg/m}^3)(1500\ \text{m/s})} = 1\times 10^{-7}\ \text{m/s}$$

g. What is maximum density condensation, s_o? (2)

$$s_o = \frac{P_o}{\rho_o c^2} = \frac{150\ \text{mPa}}{(1000\ \text{kg/m}^3)(1500\ \text{m/s})^2} = 6.67\times 10^{-11}$$

h. What is the average intensity of this wave? (2)

$$I = \frac{P_o^2}{2\rho_o c} = \frac{(150\ \text{mPa})^2}{2(1000\ \text{kg/m}^3)(1500\ \text{m/s})} = 7.5\times 10^{-9}\ \frac{\text{W}}{\text{m}^2}$$

g. What is the intensity level or sound pressure level of this wave? (Use standard references for water) (2)

$$IL = 10 \log\left(\frac{I}{I_{ref}}\right) = 10 \log\left(\frac{7.5\times 10^{-9}\ \frac{\text{W}}{\text{m}^2}}{6.67\times 10^{-19}\ \frac{\text{W}}{\text{m}^2}}\right)$$

$$= 100.5\ \text{dB} \approx 100\ \text{dB}$$

2. (15) A wave emitted from a point source or a spherical source spreads out in all directions in air and can be modeled by the following approximate expression:

$$\tilde{p} = \frac{\tilde{A}}{r} e^{j(\omega t - kr)}$$

a. Using the linearized version of the force equation resulting from Newton's Second Law, show that the particle velocity is: (5)

$$|\tilde{u}| = \frac{1}{\rho_o c}\left[\frac{1+jkr}{jkr}\right]\tilde{p}$$

$$-\nabla\tilde{p} = \rho_o \frac{\partial \tilde{u}}{\partial t}$$

$$-\frac{\partial}{\partial r}\left[\left(\frac{A}{r}\right)e^{j(\omega t - kr)}\right]\hat{r} = -j\omega\rho_o \tilde{u}$$

$$\vec{\tilde{u}} = +\frac{1}{j\omega\rho_o}\left(\frac{A}{r}(jk)e^{j(\omega t-kr)} + \left(\frac{A}{r^2}\right)e^{j(\omega t-kr)}\right)\hat{r}$$

$$= \frac{1}{j\omega\rho_o}\left(jk + \frac{1}{r}\right)\tilde{p}\,\hat{r} = \frac{1}{\omega\rho_o}\left(k + \frac{1}{jr}\right)\tilde{p}\,\hat{r} = \frac{k}{\omega\rho_o}\left(1 + \frac{1}{jkr}\right)\tilde{p}\,\hat{r}$$

$$= \frac{1}{\rho_o c}\left(\frac{jkr+1}{jkr}\right)\tilde{p}\,\hat{r} \qquad \text{Q.E.D.}$$

b. Using the definition of specific acoustic impedance, show: (5)

$$\tilde{z}_{ac} = \rho_o c \frac{k^2r^2 + jkr}{1+k^2r^2}$$

$$\tilde{Z}_{Ac} = \frac{\tilde{p}}{\tilde{u}} = \frac{\tilde{p}}{\frac{1}{\rho_o c}\left(\frac{jkr+1}{jkr}\right)\tilde{p}} = \frac{\rho_o c \; jkr}{jkr+1}\cdot\frac{(1-jkr)}{(1-jkr)} = \frac{\rho_o c\left(k^2r^2 + jkr\right)}{1 + k^2r^2}$$

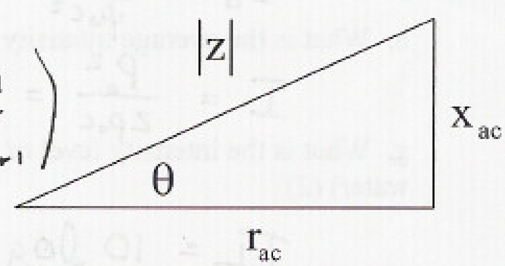
c. Identify the real and the imaginary parts of the specific acoustic impedance such that (2)

$$\tilde{z}_{ac} = r_{ac} + jx_{ac} = |z|e^{j\theta}$$

$$r_{ac} = \frac{\rho_o c\, k^2 r^2}{k^2r^2 + 1} \qquad x_{ac} = \frac{\rho_o c\, kr}{k^2r^2+1}$$

d. Using your knowledge of complex numbers and the triangle provided, show the phase angle is: (3)

$$\theta = \tan^{-1}\left(\frac{1}{kr}\right)$$

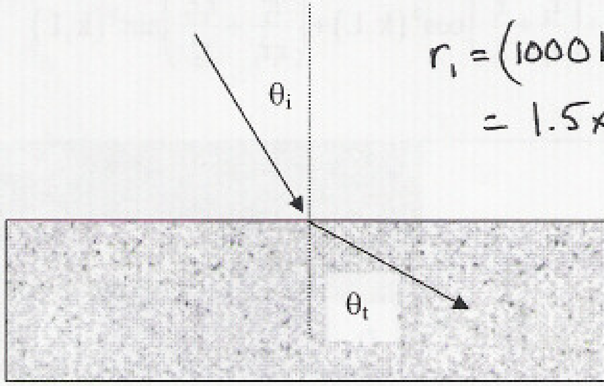


$$\theta = \tan^{-1}\left(\frac{x_{ac}}{r_{ac}}\right) = \tan^{-1}\left(\frac{\rho_o c\, kr / (k^2r^2+1)}{\rho_o c\, k^2r^2/(k^2r^2+1)}\right)$$

$$= \tan^{-1}\left(\frac{1}{kr}\right)$$

Name_______________________________ SP436 Exam III

3. (20) Sound from a sonar transducer in water (**ρ_1 = 1000 kg/m^3 and c_1 = 1500 m/s**) strikes a sandy bottom characterized by **ρ_2 = 1700 kg/m^3 and c_2 = 1600 m/s** as shown below.



$$r_1 = \left(1000\ \tfrac{kg}{m^3}\right)\left(1500\ \tfrac{m}{s}\right) = 1.5\times10^6\ \frac{kg}{m^2 s}$$

$$r_2 = \left(1700\ \tfrac{kg}{m^3}\right)\left(1600\ \tfrac{m}{s}\right) = 2.72\times10^6\ \frac{kg}{m^2 s}$$

a. What angle does the transmitted sound in the sand make with respect to the normal if the sound in water makes an incident angle (w.r.t. the normal) of **30.0 degrees**? (5)

$$\frac{\sin\theta_i}{c_1} = \frac{\sin\theta_t}{c_2}$$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i = \frac{1600\ m/s}{1500\ m/s}\sin(30^\circ) \Rightarrow \theta_t = 32.2^\circ$$

b. What is the power reflection coefficient for this incident angle? (5)

$$R_\Pi = |\tilde{R}|^2 = \left(\frac{r_2\sin\theta_i - r_1\sin\theta_t}{r_2\sin\theta_i + r_2\sin\theta_t}\right)^2 = \left(\frac{2.72\cos 30 - 1.5\cos 32.2}{2.72\cos 30 + 1.5\cos 32.2}\right)^2$$

$$= .089$$

c. What is the critical angle of incidence corresponding to total reflection off the sand? (5)

$$\theta_c = \sin^{-1}\left(\frac{c_1}{c_2}\right) = \sin^{-1}\left(\frac{1500\ m/s}{1600\ m/s}\right) = 69.6^\circ$$

d. At a different location the bottom is red clay (no sand) characterized by **ρ_2 = 1340 kg/m^3 and c_2 = 1460 m/s.** What incident angle would result in perfect transmission of sound into the red clay? (5)

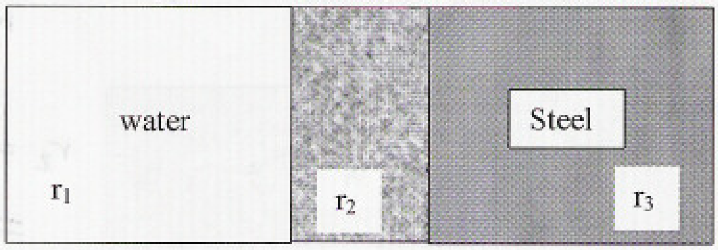
$$r_2 = 1340\ \frac{kg}{m^3}\left(1460\ m/s\right) = 1.96\times10^6\ \frac{kg}{m^2 s}$$

$$\sin\theta_I = \sqrt{\frac{1-\left(\frac{r_1}{r_2}\right)^2}{1-\left(\frac{\rho_1}{\rho_2}\right)^2}} \Rightarrow \theta_I = \sin^{-1}\sqrt{\frac{1-\left(\frac{1.5}{1.96}\right)^2}{1-\left(\frac{1000}{1340}\right)^2}}$$

$$= 75.2^\circ$$

4. (10) The intensity transmission coefficient for a three layer problem is given by:

$$T_I = \frac{4}{2+\left(\frac{r_3}{r_1}+\frac{r_1}{r_3}\right)\cos^2(k_2L)+\left(\frac{r_2^2}{r_1r_3}+\frac{r_1r_3}{r_2^2}\right)\sin^2(k_2L)}$$



$$r_3 = \left(7700\,\frac{kg}{m^3}\right)\left(6100\,\frac{m}{s}\right) = 4.697\times10^7\,\frac{kg}{m^2 s}$$

a. What must be the speed of sound in the intermediate (r_2) layer of a certain material having a density of **2700 kg/m³**, if it is to completely transmit normally-incident plane waves at a single frequency of **10.0 kHz** from water to steel with no reflection? (**ρ_{steel} = 7700 kg/m³, c_{steel} = 6100 m/s**) Hint: This is special case 4. (5)

$$r_2 = \sqrt{r_1 r_3} = \sqrt{1.5\times10^6\,\frac{kg}{m^2 s}\cdot 4.697\times10^7\,\frac{kg}{m^2 s}}$$

$$= 8.39\times10^6\,\frac{kg}{m^2 s}$$

$$c_2 = \frac{r_2}{\rho_2} = \frac{8.39\times10^6\,\frac{kg}{m^2 s}}{2700\,kg/m^3} = 3108\,m/s = 3100\,m/s$$

b. How thick must this intermediate layer be for complete transmission of this **10.0 kHz** plane wave. There are many answers, give me the thinnest. (5)

$$(n=1) \qquad f = 10\,kHz = \frac{(n-\frac{1}{2})\,c_2}{2L} = \frac{c_2}{4L}$$

$$L = \frac{c_2}{4f} = \frac{3100\,m/s}{4(10000\,Hz)} = 0.078\,m$$