1.3.2 GIVEN: $\omega = 5\ \text{rad/s}$ $\quad x(t=0) = .03\text{m}$

$v(t=0) = 0\ \text{m/s}$

FIND: $a(t=0)$, A, v_{max}

$$x = A_1 \cos\omega t + A_2 \sin\omega t$$

$$v = -A_1\omega \sin\omega t + A_2\omega \cos\omega t$$

$$x(t=0) = \boxed{.03\text{m} = A_1}$$

$$v(t=0) = A_2\omega = 0 \Rightarrow A_2 = 0$$

$$x = .03\text{m} \cos\omega t$$

$$v = -.03\text{m}\ \omega \sin\omega t$$

$$a = -.03\text{m}\omega^2 \cos\omega t$$

$$a(t=0) = (-.03\text{m})(5\ \text{rad/s})^2 = \boxed{-.75\ \text{m/s}^2}$$

$$v_{max} = A\omega = (.03\ \text{m})(5\ \text{rad/s}) = \boxed{.15\ \text{m/s}}$$

1.5.3 GIVEN: $\tilde{A} = A e^{j(\omega t + \theta)}$

$\tilde{B} = B e^{j(\omega t + \phi)}$

a. $$Re(\tilde{A}\tilde{B}) = Re\left(AB e^{j(2\omega t + \theta + \phi)}\right)$$

$$= \boxed{AB \cos(2\omega t + \theta + \phi)}$$

b. $$Re\left(\frac{\tilde{A}}{\tilde{B}}\right) = Re\left(\frac{A e^{j(\omega t + \theta)}}{B e^{j(\omega t + \phi)}}\right) = Re\left(\frac{A}{B} e^{j(\theta - \phi)}\right)$$

$$= \boxed{\frac{A}{B} \cos(\theta - \phi)}$$

c. $$Re(\tilde{A})\, Re(\tilde{B}) = A \cos(\omega t + \theta)\, B \cos(\omega t + \phi)$$

$$= \boxed{AB \cos(\omega t + \theta) \cos(\omega t + \phi)}$$

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD

d. Θ OF $\tilde{A}\tilde{B}$ IS $\boxed{2\omega t + \Theta + \phi}$

e. Θ OF $\frac{\tilde{A}}{B}$ IS $\boxed{\Theta - \phi}$

1.6.1. GIVEN: $m = .5\,kg$ $\Delta m = .2\,kg$

$\Delta x = .04\,m$

$\tau = 1.0\,s$

FIND: R_m, ω_d, A & ϕ

$$m\frac{d^2\tilde{x}}{dt^2} + R_m\frac{d\tilde{x}}{dt} + s\tilde{x} = 0$$

$$\tilde{x} = \tilde{A}e^{\gamma t}$$

$$\dot{\tilde{x}} = \tilde{A}\gamma e^{\gamma t}$$

$$\ddot{\tilde{x}} = \tilde{A}\gamma^2 e^{\gamma t}$$

$$m\tilde{A}\gamma^2 e^{\gamma t} + R_m\tilde{A}\gamma e^{\gamma t} + s\tilde{A}e^{\gamma t} = 0$$

$$m\cancel{\tilde{A}}\gamma^2 + R_m\cancel{\tilde{A}}\gamma + s\cancel{\tilde{A}} = 0$$

$$m\gamma^2 + R_m\gamma + s = 0$$

$$\gamma = \frac{-R_m \pm \sqrt{R_m^2 - 4ms}}{2m}$$

$$= -\frac{R_m}{2m} \pm \sqrt{\left(\frac{R_m}{2m}\right)^2 - \frac{\cancel{4}\cancel{m}s}{\cancel{4}m^{\cancel{2}}}}$$

$\beta = \frac{R_m}{2m}$

$\omega_0^2 = \frac{s}{m}$

$$= -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

$$\tilde{x} = \tilde{A}e^{-\beta t}\left(e^{+\gamma(\omega_0^2-\beta^2)^{1/2}t} + e^{-\gamma(\omega_0^2-\beta^2)^{1/2}t}\right)$$

$$\tau = \frac{1}{\beta} = 1\ \text{sec}$$

$$\beta = 1\,s^{-1} = \frac{R_m}{2m}$$

$$R_m = 2m\beta = 2(.5\,kg)(1\,s^{-1}) = \boxed{1\ kg/s}$$

$$s = \frac{\Delta f}{\Delta x} = \frac{(.2\,kg)(9.8\,m/s^2)}{.04\,m}$$

$$= 49\ N/m$$

$$\omega_0^2 = \frac{s}{m} = \frac{49\,N/m}{.5\,kg} = 98\ s^{-2}$$

$$\omega_D^2 = \omega_0^2 - \beta^2 = 98\,s^{-2} - 1\,s^{-2} = 97\,s^{-2}$$

$$\omega_d = \sqrt{97\,s^{-2}} = \boxed{9.85\ s^{-1}}$$

$$\tilde{x} = e^{-\beta t}\left(\tilde{A}_1 e^{+j\omega_d t} + \tilde{A}_2 e^{-j\omega_d t}\right)$$

$$= A e^{j\phi} e^{-\beta t}\left(e^{j\omega_d t} + e^{-j\omega_d t}\right)$$

$$= A e^{-\beta t}\cos(\omega t + \phi)$$

$$\tilde{x}(0) = .04\,m = A(1)\cos(\phi)$$

$$\dot{x} = -A\omega e^{-\beta t}\sin(\omega t + \phi) - A\beta e^{-\beta t}\cos(\omega t + \phi)$$

$$\dot{x}(0) = 0 = -A\omega_d \sin\phi - A\beta\cos\phi$$

$$\sin\phi = -\frac{\beta}{\omega_d}\cos\phi$$

$$\frac{\sin\phi}{\cos\phi} = \tan\phi = \frac{-\beta}{\omega_d}$$

$$\phi = \tan^{-1}\left(\frac{-1\,s^{-1}}{9.85\,s^{-1}}\right) = \boxed{-.101\ rad}$$

$$= \boxed{-5.8^\circ}$$

$$A = \frac{.04\,m}{\cos(-.101\,rad)} = \boxed{.0402\,m}$$