1.8.2 GIVEN: $f = F \sin \omega t$ $\omega \neq \omega_0$ UNDAMPED

FIND: a) $u(t)$ FOR NO DAMPING

c) FOR SMALL DAMPING SHOW $u = \frac{\omega F}{s} \cos \omega t$

STEADY STATE:

$$m \frac{d^2 \tilde{x}}{dt^2} + s\tilde{x} = -jFe^{j\omega t}$$

$$x = \tilde{A} e^{j\omega t}$$

$$\ddot{x} = -\omega^2 \tilde{A} e^{j\omega t}$$

$$-m\omega^2 \tilde{A} e^{j\omega t} + s\tilde{A} e^{j\omega t} = -jFe^{j\omega t}$$

$$\left(-m\omega^2 \tilde{A} + s\tilde{A} + jF\right) e^{j\omega t} = 0$$

$$\tilde{A}\left(s - m\omega^2\right) + jF = 0$$

$$\tilde{A} = \frac{-jF}{s - m\omega^2} = \frac{F}{j\omega\left(\frac{s}{\omega} - m\omega\right)}$$

$$= \frac{-F}{j\omega\left(m\omega - \frac{s}{\omega}\right)}$$

$$x = \frac{-F e^{j\omega t}}{j\omega\left(m\omega - \frac{s}{\omega}\right)}$$

$$u_{ss} = \frac{-F \cancel{j\omega} e^{j\omega t}}{\cancel{j\omega}\left(m\omega - \frac{s}{\omega}\right)} = \frac{-Fe^{j\omega t}}{m\omega - \frac{s}{\omega}}$$

TRANSIENT SOLN:

$$m \frac{d^2 \tilde{x}}{dt^2} + s\tilde{x} = 0$$

$$\tilde{x} = \tilde{A} e^{j\omega_0 t}$$

$$\ddot{\tilde{x}} = -\omega^2 \tilde{A} e^{j\omega_0 t}$$

$$-m\omega^2 \tilde{A} e^{j\omega_0 t} + s\tilde{A} e^{j\omega_0 t} = 0$$

$$\omega^2 = \frac{s}{m}$$

$$\tilde{x} = A e^{j(\omega_0 t + \phi)}$$

$$\dot{\tilde{x}} = u_t = jA\omega_0 e^{j(\omega_0 t + \phi)}$$

$$\tilde{x} = A e^{j(\omega_0 t + \phi)} - \frac{F e^{+\omega t}}{j\omega\left(m\omega - \frac{s}{\omega}\right)}$$

$$Re(\tilde{x}(0)) = Re\left(A e^{j\phi} - \frac{F}{j\omega\left(m\omega - \frac{s}{\omega}\right)}\right) = 0$$

$$A\cos\phi = 0 \quad \Rightarrow \quad \boxed{\phi = -\frac{\pi}{2}}$$

$$\tilde{u} = jA\omega_0 e^{j(\omega_0 t + \phi)} - \frac{F e^{j\omega t}}{m\omega - \frac{s}{\omega}}$$

$$Re(\tilde{u}(0)) = -A\omega_0 \sin(\phi) - \frac{F}{m\omega - \frac{s}{\omega}} = 0$$

$$-A\omega_0(-1) = \frac{F}{m\omega - \frac{s}{\omega}}$$

$$A = \frac{F/\omega_0}{m\omega - \frac{s}{\omega}}$$

$$u = Re(\tilde{u}) = -\frac{F/\omega_0}{m\omega - \frac{s}{\omega}}\omega_0 \sin\left(\omega_0 t - \frac{\pi}{2}\right) - \frac{F}{m\omega - \frac{s}{\omega}}\cos\omega t$$

$$= \left(\frac{F}{m\omega - \frac{s}{\omega}}\right)\left[\cos\omega_0 t - \cos\omega t\right]$$

c. $\omega << \omega_0$ $\quad R_m =$ SMALL $\quad$ STEADY STATE

$$m\frac{d^2\tilde{x}}{dt^2} + R_m\frac{d\tilde{x}}{dt} + s\tilde{x} = -jFe^{j\omega t}$$

$$\tilde{x} = \tilde{A}e^{j\omega t}$$

$$\dot{\tilde{x}} = j\omega\tilde{A}e^{j\omega t}$$

$$\ddot{\tilde{x}} = -\omega^2\tilde{A}e^{j\omega t}$$

$$-m\omega^2\tilde{A}e^{j\omega t} + j\omega R_m\tilde{A}e^{j\omega t} + s\tilde{A}e^{j\omega t} = -jFe^{j\omega t}$$

$$\tilde{A}\left(-m\omega^2 + j\omega R_m + s\right) + jF = 0$$

$$\tilde{A}\,j\omega\left(j\omega m + R_m - j\frac{s}{\omega}\right) = -jF$$

$$\tilde{A} = \frac{-\cancel{j}F}{\cancel{j}\omega\left[R_m + j\left(\omega m - \frac{s}{\omega}\right)\right]}$$

$$\tilde{x} = \frac{-F\,e^{j\omega t}}{\omega\left[R_m + j\left(\omega m - \frac{s}{\omega}\right)\right]}$$

$$\tilde{u} = \frac{-j\cancel{\omega}F\,e^{j\omega t}}{\cancel{\omega}\left[R_m + j\left(\omega m - \frac{s}{\omega}\right)\right]}$$

$R_m \to$ small $\qquad \tilde{u} \approx \dfrac{-jFe^{j\omega t}}{j\left(\omega m - \frac{s}{\omega}\right)} \approx \dfrac{-Fe^{j\omega t}}{\omega m - \frac{s}{\omega}}$

$\omega << \omega_0 \qquad \tilde{u} \approx \dfrac{-Fe^{j\omega t}}{-\frac{s}{\omega}}$

$\omega^2 << \omega_0^2 \qquad \Rightarrow \qquad \approx \dfrac{F\omega}{s}e^{j\omega t}$

$\omega^2 << \dfrac{s}{m}$

$\omega m << \dfrac{s}{\omega}$

$$\boxed{u = Re(\tilde{u}) = \frac{F\omega}{s}\cos\omega t}$$

Q.E.D.

1.10.1 SHOW $Z_m = \omega_0 m\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2 + \frac{1}{Q^2}\right]^{1/2}$

$$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

$$Q = \frac{\omega_0}{2\beta} = \frac{m\omega_0}{R_m}$$

$$Z_m = \left[\left(\omega m - \frac{s}{\omega}\right)^2 + R_m^2\right]^{1/2}$$

$$= \omega_0 m\left[\frac{\left(\omega m - \frac{s}{\omega}\right)^2}{(\omega_0 m)^2} + \frac{R_m^2}{(\omega_0 m)^2}\right]^{1/2}$$

$$= \omega_0 m\left[\left(\frac{\omega \cancel{m}}{\omega_0 \cancel{m}} - \frac{s}{\omega\,\omega_0 m}\right)^2 + \frac{1}{Q^2}\right]^{1/2}$$

$$= \omega_0 m\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0^{\cancel{2}}}{\omega\,\cancel{\omega_0}}\right)^2 + \frac{1}{Q^2}\right]^{1/2}$$

$$= \omega_0 m\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2 + \frac{1}{Q^2}\right]^{1/2} \quad \text{Q.E.D.}$$

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1.10.4 SHOW $\omega_{1/2} \approx \omega_0 \pm \frac{R_m}{2m}$

$$\Pi = \frac{F^2 R_m}{2 Z_m^2}$$

WITH $Z_m^2 = R_m^2 + \left(\omega m - \frac{s}{\omega}\right)^2$

AT RESONANCE $Z_m^2 = R_m^2$

AT ½ PWR POINT $Z_m^2 = 2R_m^2$

$$\therefore R_m^2 = X_m^2$$

$$\pm R_m = X_m = \omega m - \frac{s}{\omega}$$

$$\pm R_m \omega = \omega^2 m - s$$

$$m\omega^2 \mp R_m \omega - s = 0$$

$$\omega = \frac{\pm R_m \pm \sqrt{R_m^2 + 4ms}}{2m}$$

$$\omega = \pm \frac{R_m}{2m} \pm \sqrt{\frac{R_m^2}{4m^2} + \frac{4ms}{4m^2}}$$

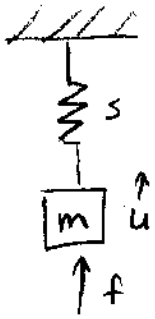
$$= \pm \frac{R_m}{2m} \pm \sqrt{\beta^2 + \omega_0^2}$$

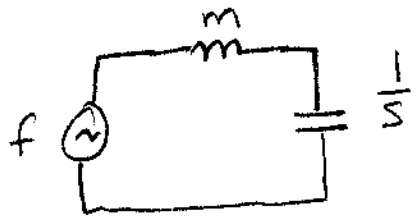
$$= \sqrt{\omega_0^2 + \beta^2} \pm \frac{R_m}{2m}$$

$\omega_0 \gg \beta$ $\approx \omega_0 \pm \frac{R_m}{2m}$ Q.E.D.

1.12.2

a)





$$m\frac{d^2x}{dt^2} + sx = f$$

$$-m\omega^2\tilde{A}e^{j\omega t} + s\tilde{A}e^{j\omega t} = fe^{j\omega t}$$

$$(-m\omega^2 + s)\tilde{A} = F$$

$$\tilde{A} = \frac{F}{-m\omega^2 + s}$$

$$x = \left(\frac{F}{-m\omega^2+s}\right)e^{j\omega t}$$

$$\tilde{u} = \frac{j\omega F}{-m\omega^2+s}e^{j\omega t}$$

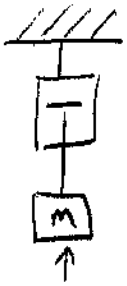
$$\frac{1}{\tilde{Z}} = \frac{j\omega}{-m\omega^2+s} = \frac{j\cancel{\omega}}{\cancel{\omega}\left(-m\omega + \frac{s}{\omega}\right)} = \frac{-1}{j\left(-m\omega + \frac{s}{\omega}\right)}$$

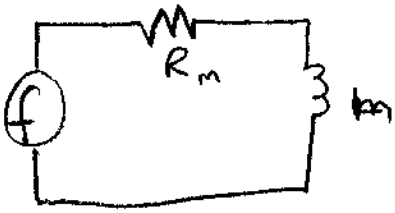
$$\boxed{\tilde{Z} = j\left(m\omega - \frac{s}{\omega}\right)}$$

$$\boxed{\omega_0^2 = \frac{s}{m}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{s}{m}}$$

b)



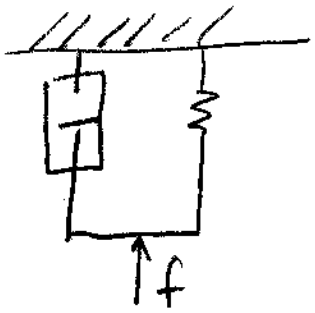


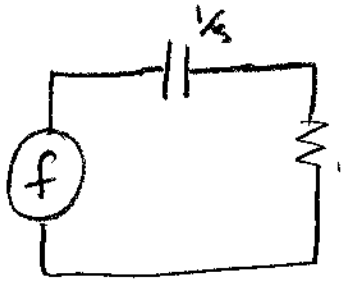
$$\boxed{Z_m = R_m + jm\omega}$$

THIS TIME, JUST WRITE THE IMPEDANCE BY INSPECTION

LR CIRCUIT DOES NOT VIBRATE $(\omega \to 0)$

c)





$$Z_m = R_m + \frac{s}{j\omega}$$

$$= R_m - j\frac{s}{\omega} \qquad (\omega \to \infty)$$

RC CIRCUIT DOES NOT VIBRATE

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