1. Given the following function:

$$f(t)=\begin{cases} t & 0<t<\frac{T}{2} \\ t-T & \frac{T}{2}<t<T \end{cases}$$

   a. Draw a graph of the function over several cycles.
   b. Represent this function by a Fourier series

(Hint: You can integrate over any period you choose. It does not have to be 0->T.

2. A Triangle wave is represented by :

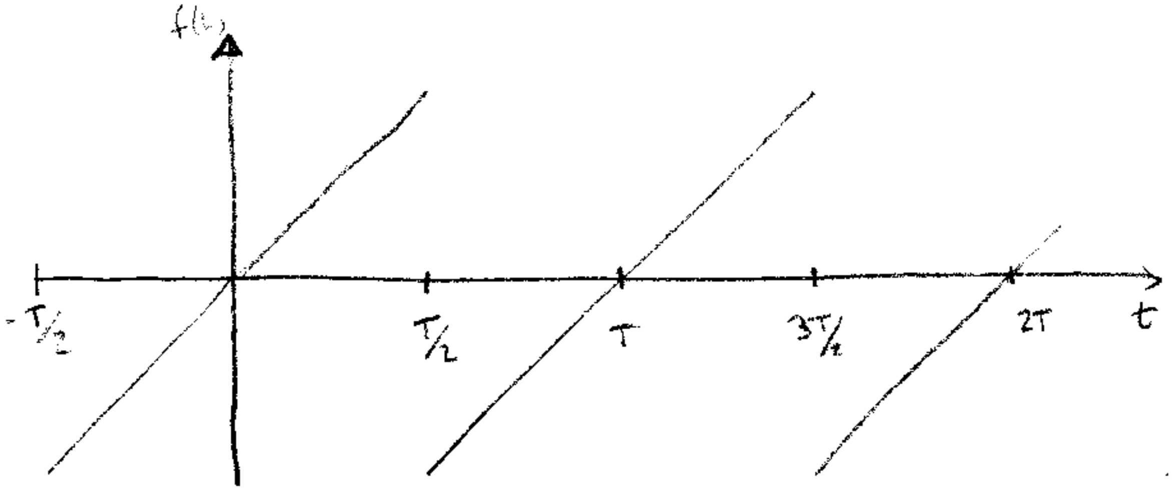
$$f(t)=\begin{cases} -t & -\frac{T}{2}<t<0 \\ t & 0<t<\frac{T}{2} \end{cases}$$

   a. Draw a graph of the function over several cycles.
   b. Represent this function by a Fourier series

3. KFCS #1.15.1

4. KFCS #1.15.7

1. $f(t) = \begin{cases} t & 0 < t < \frac{T}{2} \\ t - T & \frac{T}{2} < t < T \end{cases}$



Use Period From
$-T/2 \rightarrow T/2$

$$A_0 = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} t \, dt = 0$$

$$A_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} t \cos(n\omega t) \, dt = 0$$

$$B_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} t \sin(n\omega t) \, dt \qquad \begin{matrix} u = t & dv = \sin n\omega t \, dt \\ du = dt & v = -\frac{1}{n\omega} \cos n\omega t \end{matrix}$$

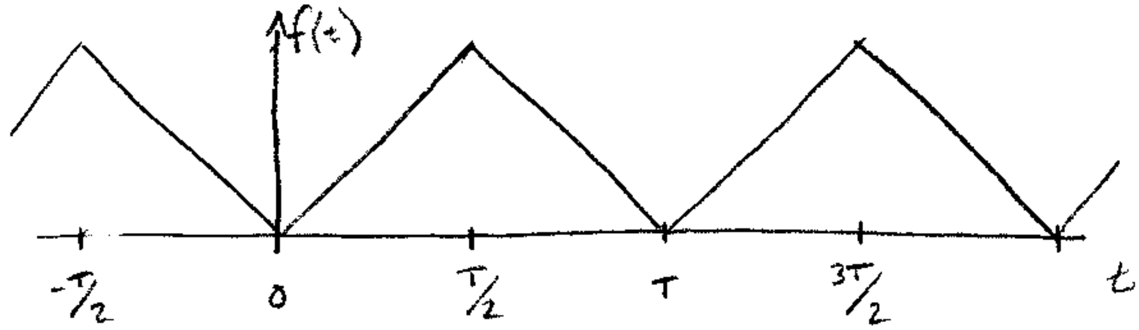
$$= \frac{2}{T} \left\{ \left[-\frac{t}{n\omega} \cos n \frac{2\pi}{T} t \right]_{-\frac{T}{2}}^{\frac{T}{2}} + \frac{1}{n\omega} \cancelto{0}{\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos n\omega t \, dt} \right\}$$

$$= \frac{2}{T} \left[-\frac{T}{2n\omega} \cos n \frac{2\pi}{\cancel{T}} \frac{\cancel{T}}{\cancel{2}} + \frac{-T}{2n\omega} \cos n \frac{2\pi}{\cancel{T}} \left(-\frac{\cancel{T}}{2} \right) \right]$$

$$= \frac{2}{\cancel{T}} \left[-\frac{\cancel{T}}{n\omega} \cos n\pi \right] = -\frac{2}{n\omega} \cos n\pi = -\frac{\cancel{2} T}{n \cancel{2} \pi} \cos n\pi$$

$$f(t) = \frac{T}{\pi} \left[\sin \omega t - \frac{\sin 2\omega t}{2} + \frac{\sin 3\omega t}{3} - \dots \right]$$

2. $f(t) = \begin{cases} -t & -\frac{T}{2} < t < 0 \\ t & 0 < t < \frac{T}{2} \end{cases}$



$$A_0 = \frac{2}{T}\left[\int_{-\frac{T}{2}}^{0} -t\,dt + \int_{0}^{T/2} t\,dt\right] = \frac{2}{T}\left[\left(-\frac{t^2}{2}\right)_{-T/2}^{0} + \left(\frac{t^2}{2}\right)_{0}^{T/2}\right]$$

$$= \frac{2}{T}\left(0 - \frac{-T^2/4}{2} + \frac{T^2/4}{2}\right) = \frac{\cancel{2}}{\cancel{T}}\,\frac{T^{\cancel{2}}}{4\cancel{2}} = \frac{T}{2}$$

$$A_n = \frac{2}{T}\left[-\int_{-\frac{T}{2}}^{0} t\cos n\omega t\,dt + \int_{0}^{T/2} t\cos n\omega t\,dt\right]$$

$$= \frac{2}{T}\left[2\int_{0}^{T/2} t\cos n\omega t\,dt\right] \qquad \begin{array}{ll} u = t & dv = \cos n\omega t\,dt \\ du = dt & v = \frac{1}{n\omega}\sin n\omega t \end{array}$$

$$= \frac{4}{T}\left[\frac{t}{n\omega}\sin n\frac{2\pi}{T}t\,\Big|_{0}^{T/2} - \frac{1}{n\omega}\int_{0}^{T/2}\sin n\omega t\,dt\right]$$

$$= \frac{4}{T}\left[0 - 0 - \frac{1}{n\omega}\left(-\frac{1}{n\omega}\right)\cos n\omega t\,\Big|_{0}^{T/2}\right]$$

$$= \frac{\cancel{4}}{\cancel{T}}\,\frac{1}{n^2\frac{(2\pi)^2}{T^2}}\left[\cos n\left(\frac{2\pi}{T}\right)\frac{T}{2} - 1\right] = \frac{T}{n^2\pi^2}\left[\cos n\pi - 1\right]$$

$n = 0, 2, 4, \ldots \qquad A_n = 0$

$n = 1, 3, 5, \ldots \qquad A_n = -\frac{2T}{n^2\pi^2}$

$B_n = 0$ BECAUSE $f(t)$ IS AN <u>EVEN</u> FUNCTION

$$f(t) = \frac{T}{4} - \sum_{n=\text{ODD}} \frac{2T}{n^2\pi^2}\cos n\omega t$$

1.15.1 GIVEN $\tilde{G}(w) = F[\delta(w - \omega)]$ FOR $f(t) = F e^{j\omega t}$

FIND: $\hat{u}(t) = \int_{-\infty}^{\infty} \frac{F \delta(w - \omega)}{\tilde{Z}_m} e^{jwt} \, dw$

$$= \frac{F}{\tilde{Z}_m} e^{j\omega t}$$

1.15.7 GIVEN: $F(t) = mg \; 1(t)$

FIND: $\tilde{u}(t)$, $\tilde{x}(t)$

$$\hat{G}(w) = mg\left(\frac{1}{2\pi}\right)\left(\frac{1}{jw}\right)$$

$$\tilde{Z}(w) = j\left(wm - \frac{s}{w}\right)$$

$$u(t) = \int_{-\infty}^{\infty} \frac{mg\left(\frac{1}{2\pi}\right)\left(\frac{1}{jw}\right)}{j\left(wm - \frac{s}{w}\right)} e^{jwt} \, dw$$

$$= \int_{-\infty}^{\infty} \frac{mg \, e^{jwt}}{2\pi(-1)w\left(wm - \frac{s}{w}\right)} \, dw = \int_{-\infty}^{\infty} \frac{-mg \, e^{jwt}}{2\pi(mw^2 - s)} \, dw$$

$$= -g\int_{-\infty}^{\infty} \frac{1}{2\pi} \frac{m e^{jwt}}{m\left(w^2 - \frac{s}{m}\right)} \, dw = -\frac{g}{\omega_0} \int_{-\infty}^{\infty} \frac{1}{2\pi} \frac{\omega_0}{w^2 - \omega_0^2} \, dw$$

$$= +\frac{g}{\omega_0} \int_{-\infty}^{\infty} \frac{1}{2\pi} \frac{\omega_0}{\omega_0^2 - w^2} \, dw = \boxed{\frac{g}{\omega_0} \sin \omega_0 t \;\; 1(t)}$$

$$X(t) = \frac{g}{\omega_0}\int_0^t \sin \omega_0 t \, dt \overset{1(t)}{=} -\frac{g}{\omega_0^2} \cos \omega_0 t \Big|_0^t \;\; 1(t)$$

$$= -\frac{g}{\omega_0^2}\left(\cos \omega_0 t - 1\right) \overset{1(t)}{=} \boxed{\frac{g}{\omega_0^2}\left(1 - \cos \omega_0 t\right)} \; 1(t)$$