2.4.1. GIVEN: $y = \ln[a(ct - x)]$ $\qquad y = (ct - x^2)a$

$y = a(ct - x)^2$ $\qquad y = at(ct - x)$

SHOW: WHICH ARE SOLUTIONS TO THE WAVE EQUATION

a. see class notes

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

b. $y = \ln(a(ct - x))$

$\frac{\partial y}{\partial x} = [a(ct-x)]^{-1}(-a)$ $\qquad \frac{\partial y}{\partial t} = [a(ct-x)]^{-1}(ac)$

$\frac{\partial^2 y}{\partial x^2} = -[a(ct-x)]^{-2}(-a)^2$ $\qquad \frac{\partial^2 y}{\partial t^2} = -[a(ct-x)]^{-2}(ac)^2$

$$-[a(ct-x)]^{-2}a^2 \overset{\text{Yes}}{=} \frac{1}{c^2}\left\{-[a(ct-x)]^{-2}(ac)^2\right. \checkmark$$

c. $y = a(ct-x)^2$

$\frac{\partial y}{\partial x} = 2a(ct-x)(-1)$ $\qquad \frac{\partial y}{\partial t} = 2a(ct-x)c$

$\frac{\partial^2 y}{\partial x^2} = -2a(1)(-1) = 2a$ $\qquad \frac{\partial^2 y}{\partial t^2} = 2a(1)c^2$

$$2a \overset{\text{Yes}}{=} \frac{1}{c^2}\left\{2ac^2\right) \checkmark$$

$y = \cos[a(ct-x)]$

d) $\frac{\partial y}{\partial t} = -ac\sin(a(ct-x))$

$\frac{\partial^2 y}{\partial t^2} = -a^2c^2\cos(a(ct-x))$

$\frac{\partial y}{\partial x} = a\sin(a(ct-x))$

$\frac{\partial^2 y}{\partial x^2} = -a^2\cos(a(ct-x))$

$$-a^2\cos(a(ct-x)) \overset{\text{Yes}}{=} \frac{1}{c^2}(-a^2c^2\cos(ct-x))$$

e. $y = a(ct - x^2)$

$\frac{\partial y}{\partial x} = -2ax$ $\qquad \frac{\partial y}{\partial t} = ac$

$\frac{\partial^2 y}{\partial x^2} = -2a$ $\qquad \frac{\partial^2 y}{\partial t^2} = 0$

$$-2a \underset{\text{No}}{\neq} \frac{1}{c^2}(0) \checkmark$$

f. $y = at(ct - x)$

$\frac{\partial y}{\partial x} = -at$ $\qquad \frac{\partial y}{\partial t} = 2act - ax$

$\frac{\partial^2 y}{\partial x^2} = 0$ $\qquad \frac{\partial^2 y}{\partial t^2} = 2ac$

7.8.1 GIVEN: $y = 4\cos(3t - 2x)$ $\rho_L = 0.1\ g/cm$ y, x - [cm] t - [s]

FIND: $A,\ c,\ f,\ \lambda,\ k,\ u(x=0, t=0)$

$$A = \boxed{4\ cm}$$

$$c = \frac{\omega}{k} = \frac{3\ s^{-1}}{2\ cm^{-1}} = \boxed{1.5\ cm/s}$$

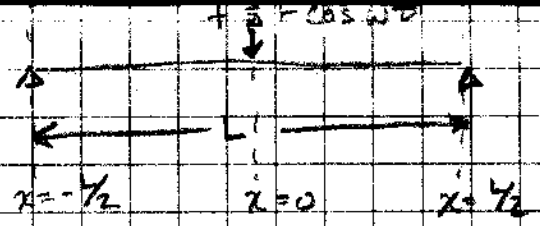
$$f = \frac{\omega}{2\pi} = \frac{3\ s^{-1}}{2\pi} = \boxed{0.477\ Hz}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\ cm^{-1}} = \boxed{\pi\ cm}$$

$$k = \boxed{2\ cm^{-1}}$$

$$u = \frac{\partial y}{\partial t} = (-4)(3)\sin(3t - 2x) = -12\sin(3t - 2x)\Big|_{\substack{x=0\\t=0}} = 0$$

2.9.3 GIVEN:



FIND: Z_{m_0}

$$y\left(\frac{L}{2}\right) = \frac{F}{2kT}\tan\frac{kL}{2}$$

$$y\left(\frac{L}{4}\right) = \ ?$$

a. $$\tilde{y} = \tilde{A}e^{j(\omega t - kx)} + \tilde{B}e^{j(\omega t + kx)}$$

$$y\left(x=\frac{L}{2}\right) = 0 = e^{j\omega t}\left(\tilde{A}e^{-jkL/2} + \tilde{B}e^{jkL/2}\right)$$

$$\tilde{B} = \frac{-\tilde{A}e^{-jkL/2}}{e^{jkL/2}} = -\tilde{A}e^{-jkL}$$

$$\tilde{f} + 2T\sin\theta\Big|_{x=0} = 0$$

$$Fe^{j\omega t} + 2T\frac{\partial\tilde{y}}{\partial x}\Big|_{x=0} = 0$$

$$Fe^{j\omega t} + 2T\left(-jk\tilde{A}e^{j(\omega t - kx)} + jk\tilde{B}e^{j(\omega t + kx)}\right)\Big|_{x=0} = 0$$

$$e^{j\omega t}\left[F + 2Tjk\left(-\tilde{A} + \tilde{B}\right)\right] = 0$$

$$F + 2jkT\left(-\tilde{A} - \tilde{A}e^{-jkL}\right) = 0$$

$$\frac{F}{2jkT} = \tilde{A}e^{-jkL/2}\left(e^{+jkL/2} + e^{-jkL/2}\right)$$

$$\frac{F}{2jkT} = \tilde{A}e^{-jkL/2}\,2\cos k\frac{L}{2}$$

$$\tilde{A} = \frac{Fe^{+jkL/2}}{4jkT\cos k\frac{L}{2}}$$

$$\tilde{B} = -\tilde{A}e^{-jkL} = \frac{-Fe^{-jk\frac{L}{2}}}{4jkT\cos k\frac{L}{2}}$$

$$\tilde{y} = \frac{F}{4jkT\cos k\frac{L}{2}}\left(e^{jkL/2}e^{j(\omega t+kx)} - e^{-jkL/2}e^{j(\omega t+kx)}\right)$$

$$= \frac{Fe^{j\omega t}}{4jkT\cos k\frac{L}{2}}\left(e^{jk\left(\frac{L}{2}-x\right)} - e^{-jk\left(\frac{L}{2}-x\right)}\right)$$

$$= \frac{Fe^{j\omega t}}{2\cancel{4}\cancel{j}kT\cos k\frac{L}{2}}\left(\cancel{2}\cancel{j}\sin k\left(\frac{L}{2}-x\right)\right)$$

$$= \boxed{\frac{Fe^{j\omega t}\sin\left(k\left(\frac{L}{2}-x\right)\right)}{2kT\cos\left(k\frac{L}{2}\right)}}$$

$$\tilde{u} = \frac{\partial\tilde{y}}{\partial t} = \frac{j\omega Fe^{j\omega t}\sin\left(k\left(\frac{L}{2}-x\right)\right)}{2kT\cos\left(k\frac{L}{2}\right)}$$

$$\tilde{Z}_{m_0} = \left.\frac{\tilde{f}}{\tilde{u}}\right|_{x=0} = \frac{\cancel{Fe^{j\omega t}}}{\frac{j\omega\cancel{Fe^{j\omega t}}\sin(kL/2)}{2kT\cos(kL/2)}}$$

$$= \frac{2kT\cot(kL/2)}{j\omega} \qquad T = \rho_L c^2$$

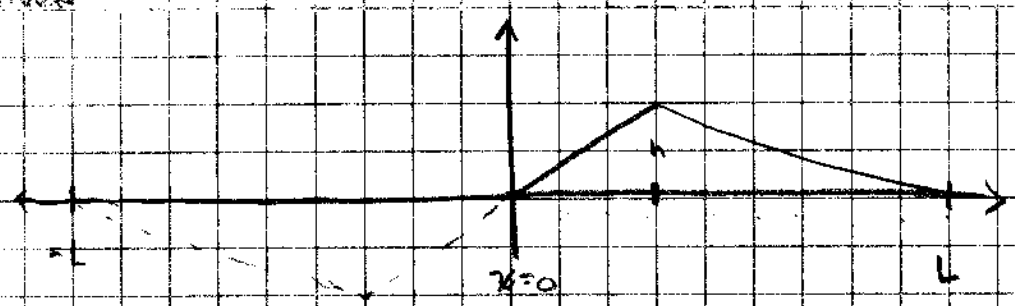
$$= -2j\frac{\rho_L c^{\cancel{2}}}{\cancel{c}}\cot\left(\frac{kL}{2}\right)$$

$$= \boxed{-2j\rho_L c\cot\left(\frac{kL}{2}\right)}$$

b. $$\tilde{y}(x=0) = \frac{Fe^{j\omega t}\sin\left(k\frac{L}{2}\right)}{2kT\cos\left(k\frac{L}{2}\right)} = \underbrace{\frac{F}{2kT}\left(\tan\frac{kL}{2}\right)}_{\text{AMPLITUDE, Q.E.D.}}e^{j\omega t}$$

c. $$\tilde{y}\left(x=\frac{L}{4}\right) = \frac{F\sin\left(k\left(\frac{L}{2}-\frac{L}{4}\right)\right)e^{j\omega t}}{2kT\cos\left(k\frac{L}{2}\right)} = \underbrace{\frac{F}{2kT}\,\frac{\sin\left(k\frac{L}{4}\right)}{\cos\left(k\frac{L}{2}\right)}}_{\text{AMPLITUDE}}e^{j\omega t}$$

2.10.1 Given



$k_n L = n\pi$

$k_n = \frac{n\pi}{L}$

$-L \to -L/3 \qquad y_n(x,0) = -\frac{3h}{2} - \frac{h}{\frac{2}{3}L}x = -\frac{3h}{2} - \frac{3h}{2L}x$

$-L/3 \to L/3 \qquad y_n(x,0) = \frac{h}{\frac{1}{3}L}x = \frac{3h}{L}x$

$L/3 \to L \qquad y_n(0,x) = +\frac{3h}{2} - \frac{h}{\frac{2}{3}L}x = +\frac{3h}{2} - \frac{3h}{2L}x$

$u_n(x,0) = 0 \Rightarrow B_n = 0$

$$A_n = \frac{2}{2L}\left\{\int_{-L}^{-L/3}\left(-\frac{3h}{2} - \frac{3h}{2L}x\right)\sin\frac{n\pi x}{L}\,dx + \int_{-L/3}^{L/3} +\frac{3h}{L}x\sin\frac{n\pi x}{L}\,dx + \int_{L/3}^{L}\left(\frac{3h}{2} - \frac{3h}{2L}x\right)\sin\frac{n\pi x}{L}\,dx\right\}$$

$$= \frac{12h}{n^2\pi^2}\left[\sin^3\left[\frac{n\pi}{3}\right]\right]$$

$A_1 = .790h$

$A_2 = .197h$

$A_3 = 0$

$A_4 = -.049h$

$A_5 = -.032h$

$A_6 = 0$

$$y_n = (A_n \cos \omega_n t) \sin \frac{n\pi}{L} x$$

$t = 0$ $$\boxed{y_n = A_n \sin \frac{n\pi}{L} x}$$

$$t = \frac{L}{c} \Rightarrow \omega t = ck\frac{L}{c} = kL = n\pi \quad (n=1)$$

$$\boxed{y_n = A_n \cos(n\pi) \sin \frac{n\pi x}{L}}$$

2.12.2 GIVEN: FIXED-FIXED STRING $L = 31.4\ cm$ $\rho_L = 0.1\ g/cm$

$$y = 2\sin\left(\frac{x}{5}\right)\cos(3t) \qquad y, x - [cm] \quad t - [s]$$

FIND: $c,\ f,\ k$

$y(L/2),\ u(L/2)$ $\qquad \mathcal{E}(L/2),\ \mathcal{E}(L/4)$

$y(L/4),\ u(L/4)$ $\qquad E_{TOTAL}$

$$c = \frac{\omega}{k} = \frac{3\ s^{-1}}{1/5\ cm^{-1}} = \boxed{15\ cm/s}$$

$$f = \frac{\omega}{2\pi} = \frac{3}{2\pi}\ Hz \approx \boxed{.477\ Hz}$$

$$k = \frac{1}{5}\ cm^{-1} = \boxed{0.2\ cm^{-1}}$$

$$y(L/2) = 2\sin\left(\frac{31.4/2}{5}\right)\cos 3t = \boxed{(3.19\times10^{-3}\ cm)}\cos 3t$$

$$y(L/4) = 2\sin\left(\frac{31.4/2}{5}\right)\cos 3t = \boxed{(2\ cm)}\cos 3t$$

$$u = \frac{\partial y}{\partial t} = -2(3)\sin\left(\frac{x}{5}\right)\sin(3t)$$

$$= \boxed{-6\ cm/s\ \sin\left(\frac{x}{5}\right)\sin(3t)}$$

$$u(L/2) = -6\ cm/s\ \sin\left(\frac{31.4/2}{5}\right)\cos 3t = -\boxed{9.56\times10^{-3}\ cm/s}\cos 3t$$

$$u(L/4) = -6\ cm/s\ \sin\left(\frac{31.4/4}{5}\right)\cos 3t = -\boxed{6\ cm/s}\cos 3t$$

$$\mathcal{E} = \frac{dE}{dx} = \frac{1}{2}\rho_L c^2\left[\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{1}{c}\frac{\partial y}{\partial t}\right)^2\right]$$

$$\frac{\partial y}{\partial x} = 2\left(\frac{1}{5}\right)\cos\frac{x}{5}\cos 3t = \boxed{\frac{2}{5}\cos\frac{x}{5}\cos 3t}$$

$$= \frac{1}{2}\rho_L c^2\left[\frac{4}{25}\cos^2\left(\frac{x}{5}\right)\cos^2(3t) + \frac{1}{c^2}36\sin^2\left(\frac{x}{5}\right)\sin^2(3t)\right]$$

$$= \frac{1}{2}\left(.1\ \frac{g}{cm}\right)\left(225\ cm^2/s^2\right)\left(.16\right)\left[\cos^2\left(\frac{x}{5}\right)\cos^2(3t) + \sin^2\left(\frac{x}{5}\right)\sin^2(3t)\right]$$

$$= 1.8 \ \mathrm{g\,cm/s^2} \left[\cos^2\left(\frac{x}{5}\right)\cos^2(3t) + \sin^2\left(\frac{x}{5}\right)\sin^2(3t) \right]$$

$$\mathcal{E}\left(x = \frac{L}{2}\right) = \left(1.8 \ \mathrm{Dyne}\right)\left[\cancel{\cos^2\left(\frac{31.4/2}{5}\right)}^{\,1} \cos^2(3t) + \cancel{\sin^2\left(\frac{31.4/2}{5}\right)}^{\,0} \sin^2(3t) \right]$$

$$= \boxed{\left(1.8 \ \mathrm{Dyne}\right) \cos^2(3t)}$$

$$\mathcal{E}\left(x = \frac{L}{4}\right) = \left(1.8 \ \mathrm{Dyne}\right)\left[\cancel{\cos^2\left(\frac{31.4/4}{5}\right)}^{\,0} \cos^2(3t) + \cancel{\sin^2\left(\frac{31.4/4}{5}\right)}^{\,1} \sin^2(3t) \right]$$

$$= \boxed{\left(1.8 \ \mathrm{Dyne}\right) \sin^2(3t)}$$

$$E_{TOT} = \int_0^L \mathcal{E} \, dx$$

At Any Time, $E = \mathrm{Const}$

$$\langle \sin^2(3t) \rangle = \langle \cos^2(3t) \rangle = \frac{1}{2}$$

Since Energy Is On Average Split Between The Potential & Kinetic Forms, Pick A Time When It Is Only Potential, $3t = 0$

$$E_{TOT} = 1.8 \ \mathrm{Dynes} \int_0^{31.4\,\mathrm{cm}} \cos^2 \frac{x}{5} \, dx \qquad \cos^2\theta = \frac{1}{2} + \cos 2\theta$$

$$= 1.8 \ \mathrm{Dynes} \left(31.4 \ \mathrm{cm}\right)\left(\frac{1}{2}\right) = \boxed{28.3 \ \mathrm{Ergs}} \left(\frac{1 \ \mathrm{J}}{10^7 \ \mathrm{Erg}}\right)$$

$$= \boxed{2.83 \ \mu\mathrm{J}}$$