1.4.1 GIVEN: BAR OF LENGTH, L $\quad \eta(x=0) = 0$ FIXED

$\frac{\partial \eta}{\partial x}(x=L) = 0$ FREE

SHOW: a. ONLY ODD HARMONICS ARE ALLOWED

b. FIND f_1 (STEEL, $L = 0.5m$)

c. $\eta(x=0, t=0) = h \Rightarrow A_n = \frac{8h}{(n\pi)^2} \sin\frac{n\pi}{2}$

d. IF $F = 5000\,N$, $S = .00005\,m^2$, FIND A_n

BAR IS STEEL

$$\frac{\partial^2 \eta}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \eta}{\partial t^2} \qquad c = \sqrt{\frac{Y}{\rho}}$$

$$\tilde{\xi} = \tilde{A} e^{j(\omega t - kx)} + \tilde{B} e^{j(\omega t + kx)}$$

$$\tilde{\eta}(x=0) = 0 = \tilde{A} e^{j(\omega t)} + \tilde{B} e^{j(\omega t)} \Rightarrow \tilde{A} = -\tilde{B}$$

$$\tilde{\xi} = \tilde{A} e^{j\omega t}\left(e^{jkx} - e^{-jkx}\right) = -2j\tilde{A} e^{j\omega t} \sin kx$$

$$\left.\frac{\partial \tilde{\eta}}{\partial x}\right|_{x=L} = 0 = -2j\tilde{A} k e^{j\omega t} \cos kL$$

$$\therefore \cos kL = 0$$

kL = ODD MULTIPLES OF $\frac{\pi}{2}$

i.e. $k_n L = n\frac{\pi}{2} \qquad n = 1, 3, 5, 7, \ldots$

$$\boxed{k_n = \frac{n\pi}{2L}} \qquad n = \text{ODD ONLY} \quad (\text{Q.E.D.})$$

$$\omega_n = c k_n = \frac{n\pi c}{2L} \qquad n = \text{ODD}$$

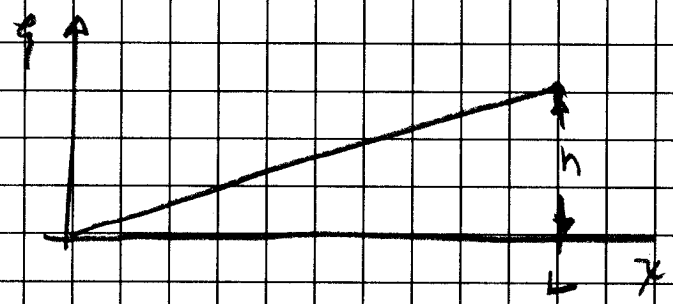
$$f_n = \frac{\omega_n}{2\pi} = \frac{nc}{4L} \qquad n = \text{ODD}$$

$$c_{STEEL} = \sqrt{\frac{19.5\times10^{10}\,Pa}{7700\,kg/m^3}} = 5032.36\,m/s$$

$$f_1 = \frac{(1)(5032.36\,m/s)}{4(.5\,m)} = \boxed{2516\,Hz}$$

$$\varphi(x,0) = \frac{h}{L}x \qquad u(x,0) = 0$$

$$\Rightarrow B_n = 0$$



$$\xi(x,t) = \left(A_n \cos\omega_n t + B_n \sin\omega_n t\right)\sin k_n x \qquad \underline{\underline{n = \text{ODD}}}$$

$$A_n = \frac{2}{L}\int_0^L \varphi(x,0)\sin k_n x\, dx = \frac{2}{L}\frac{h}{L}\int_0^L x \sin\frac{n\pi}{2L}x\, dx$$

$$= \frac{2h}{\cancel{L^2}}\left[-\frac{2\cancel{L^2}\cancel{\cos\frac{n\pi}{2}}^{\to 0}}{n\pi} + \frac{4\cancel{L^2}\sin\frac{n\pi}{2}}{n^2\pi^2}\right]$$

$$= \boxed{\frac{8h}{n^2\pi^2}\sin\frac{n\pi}{2}} \qquad \text{Q.E.D.}$$

$$F = YS\frac{d\xi}{dx} = 5000\,N = (19.5\times10^{10}\,Pa)(.00005\,m^2)\frac{h}{(.5\,m)}$$

$$h = .000256\ m$$

$$A_1 = \frac{8h}{\pi^2} = \frac{8(.000256\,m)}{\pi^2} = 0.000208\ m = \boxed{0.208\ mm}$$

$$A_2 = 0$$

$$A_3 = \frac{8h}{9\pi^2} = \boxed{-.0231\ mm}$$

$$A_4 = 0$$

$$A_5 = \frac{8h}{25\pi^2} = \boxed{8.31\ \mu m}$$

3.5.1 GIVEN: STEEL BAR $S = 0.0001\ m^2$ $m = .15\ kg$

$L = 0.25\ m$

FREE

$$\frac{\partial \xi(x=0)}{\partial x} = 0$$

MASS LOADED

$$-YS \left.\frac{\partial \xi}{\partial x}\right|_{x=L} = m \left.\frac{\partial^2 \xi}{\partial t^2}\right|_{x=L}$$

FIND: FUNDAMENTAL FREQ, f_1

NODE POINTS FOR FUNDAMENTAL

$y_1(x=0)$

$y_1(x=L)$

f_2

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} \qquad c^2 = \sqrt{\frac{Y}{\rho}}$$

$$\xi = \tilde{A} e^{j(\omega t - kx)} + \tilde{B} e^{j(\omega t + kx)}$$

$$\frac{\partial \xi}{\partial x} = -jk\tilde{A} e^{j(\omega t - kx)} + jk\tilde{B} e^{j(\omega t + kx)}$$

$$\left.\frac{\partial \xi}{\partial x}\right|_{x=0} = 0 = -jk\tilde{A} e^{j\omega t} + jk\tilde{B} e^{j\omega t}$$

$$\boxed{\tilde{A} = \tilde{B}}$$

$$\xi = \tilde{A} e^{j\omega t}\left(e^{-jkx} + e^{+jkx}\right) = \boxed{2\tilde{A} e^{j\omega t} \cos kx}$$

$$\boxed{\frac{\partial^2 \xi}{\partial t^2} = (j\omega)^2\, 2\tilde{A} e^{j\omega t} \cos kx}$$

$$\boxed{\frac{\partial \xi}{\partial x} = -2\tilde{A} k e^{j\omega t} \sin kx}$$

$$+YS\, \cancel{2}\tilde{A} k \underline{e^{j\omega t}} \sin kL = -m\omega^2\, \cancel{2}\tilde{A} \underline{e^{j\omega t}} \cos kL$$

$$\tan kL = -\frac{m\omega^2}{YSk} = -\frac{m\omega^2}{\rho c^2 S k}$$

$$= -\frac{m k^2}{m_B/L\, k} = -\frac{m}{m_B} kL$$

$$\tan kL = -\frac{.15\ kg}{.1925\ kg}(kL) = (-.779)\,kL$$

$$\left(m_s = \rho S L = (7700\ kg/m^3)(.25\ m)(.0001\ m^2) = .1925\ kg\right)$$

$$kL = 2.11,\ 4.97,\ 51.9,\ 67.6 \qquad (\text{TI-92+})$$

$$k(.25\ m) = 2.116$$

$$k_1 = \frac{2.116}{.25\ m} = \boxed{8.46\ m^{-1}}$$

$$c = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{19.5\times10^{10}\ Pa}{7700\ kg/m^3}} = 5032\ m/s$$

$$\omega_1 = c k_1$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{c k_1}{2\pi} = \frac{(5032\ m/s)(8.46\ m^{-1})}{2\pi}$$

$$= \boxed{6780\ Hz}$$

Node Pts $\quad \xi = 0 = 2\tilde{A} e^{j\omega t} \cos\left[(8.46\ m^{-1})x\right]$

$$(8.46\ m^{-1})x = \frac{\pi}{2}$$

$$\boxed{x = 0.186\ m}$$

$$\frac{y_1(x=0)}{y_1(x=L)} = \frac{\cancel{2\tilde{A}}\ \cancel{e^{j\omega t}}}{\cancel{2\tilde{A}}\ \cancel{e^{j\omega t}}\ \cos kL} = \frac{1}{\cos\left[(8.46\ m^{-1})(.25\ m)\right]}$$

$$\uparrow 2.12$$

$$= \boxed{-1.93} \quad (\text{OUT OF PHASE } 180^\circ)$$

$$k_2 L = k_2(.25\ m) = 4.965$$

$$k_2 = 19.86\ m^{-1}$$

$$f_2 = \frac{c k_2}{2\pi} = \boxed{15900\ Hz}$$

3.9.1. GIVEN: $y = \left[A\cosh gx + B\sinh gx + C\cos gx + D\sin gx\right]\cos(\omega t+\phi)$

$$\frac{\partial^2 y}{\partial t^2} = -K^2c^2\frac{\partial^4 y}{\partial x^4}$$

$$\longrightarrow c^2 = \frac{Y}{\rho}$$

$$\frac{\partial y}{\partial t} = -\left[\quad\right]\omega\sin(\omega t+\phi)$$

$$\frac{\partial^2 y}{\partial t^2} = -\left[\quad\right]\omega^2\cos(\omega t+\phi)$$

$$\frac{\partial y}{\partial x} = \left[Ag\sinh gx + Bg\cosh gx - Cg\sin gx + Dg\cos gx\right]\cos(\,)$$

$$\frac{\partial^2 y}{\partial x^2} = \left[Ag^2\cosh gx + Bg^2\sinh gx - Cg^2\cos gx - Dg^2\sin gx\right]\cos(\,)$$

$$\frac{\partial^3 y}{\partial x^3} = \left[Ag^3\sinh gx + Bg^3\cosh gx + Cg^3\sin gx - Dg^3\cos gx\right]\cos(\,)$$

$$\frac{\partial^4 y}{\partial x^4} = \left[Ag^4\cosh gx + Bg^4\sinh gx + Cg^4\cos gx + Dg^4\sin gx\right]\cos(\,)$$

$$-\omega^2\left[A\cosh gx + B\sin gx + C\cos gx + D\sin gx\right]\cos(\,)$$

$$= -K^2c^2g^4\left[A\cosh gx + B\sinh gx + C\cos gx + D\sin gx\right]\cos(\,)$$

$$+\omega^2 = +K^2c^2g^4$$

$$g^4 = \frac{\omega^2}{K^2c^2}\left(\frac{\omega^2}{\omega^2}\right) = \frac{\omega^4}{v^4}$$

$$g = \frac{\omega}{v} \quad \checkmark$$