5.9.1 GIVEN: $\tilde{p} = P e^{j(\omega t - kx)}$

FIND: $\rho - \rho_0$, $\tilde{u}$, Φ, $\mathcal{E}_i$, I

$$\tilde{s} = \frac{\tilde{p}}{B} = \frac{\tilde{p}}{\rho_0 c^2}, \quad \rho - \rho_0 = \rho_0 s$$

$$\therefore \rho - \rho_0 = \frac{\tilde{p}}{c^2} = \frac{P e^{j(\omega t - kx)}}{c^2}$$

(ACOUSTIC DENSITY IS DIFF BETWEEN INSTANTANEOUS & STATIC DENSITY)

$$\tilde{u} = \frac{\tilde{p}}{Z} = \frac{P e^{j(\omega t - kx)}}{\rho_0 c}$$

$$\tilde{u} = \nabla \Phi = -jk\Phi \quad \therefore \quad \Phi = \frac{u}{-jk} = \frac{P e^{j(\omega t - kx)}}{-j\rho_0 c k}$$

$$= \frac{j P e^{j(\omega t - kx)}}{\rho_0 \omega}$$

$$\mathcal{E} = \frac{1}{2}\rho_0 \tilde{u}^2 + \frac{1}{2}\frac{\tilde{p}^2}{\rho_0 c^2}$$

$$= \frac{1}{2}\rho_0 \frac{P^2}{\rho_0^2 c^2}\cos^2(\omega t - kx) + \frac{1}{2}\frac{P^2}{\rho_0 c^2}\cos^2(\omega t - kx)$$

$$= \frac{P^2}{\rho_0 c^2}\cos^2(\omega t - kx)$$

$$\langle \mathcal{E} \rangle = \frac{1}{2}\frac{P^2}{\rho_0 c^2}$$

$$I = \langle \mathcal{E} \rangle c = \frac{1}{2}\frac{P^2}{\rho_0 c}$$

5.12.2 GIVEN: $f = 171\ \text{Hz}$ $\quad 40\ \text{dB re } 20\mu\text{Pa}$

FIND: $P_o,\ I,\ u_o,\ \rho - \rho_o,\ \xi_o,\ s_o$

$$SPL = 20 \log\left(\frac{P_{rms}}{P_{ref}}\right) = 20 \log\left(\frac{P_{rms}}{20\mu Pa}\right) = 40\ \text{dB}$$

$$\frac{P_{rms}}{20\mu Pa} = 10^2 \Rightarrow P_{rms} = 2000\ \mu Pa = 2\ mPa$$

$$P_o = \sqrt{2}\ P_{rms} = \sqrt{2}\ (2\ mPa) = \boxed{2.8\ mPa}$$

$$I = \frac{P_o^2}{2\rho_o c} = \frac{(2.8\ mPa)^2}{2\ 408\ \frac{Pa\text{-}s}{m}} = \boxed{9.8\times10^{-9}\ \frac{W}{m^2}}$$

$$u_o = \frac{P_o}{\rho_o c} = \frac{2.8\ mPa}{408\ \frac{Pa\text{-}s}{m}} = \boxed{6.86\times10^{-6}\ \frac{m}{s}}$$

$$\rho - \rho_o = \frac{P_o}{c^2} = \frac{2.8\ mPa}{(340\ m/s)^2} = 2.42\times10^{-8}\ \frac{kg}{m^3} = \rho_o s_o$$

$$s_o = \frac{2.42\times10^{-8}\ kg/m^3}{1.2\ kg/m^3} = \boxed{2\times10^{-8}}$$

$$\xi_o = \frac{u_o}{\omega} = \frac{6.86\times10^{-6}\ m/s}{2\pi\ 171\ Hz} = \boxed{6.4\ nm} \quad (\xi_{rms} = 4.5\ nm)$$

$$\rho_0 c = (1.2\ kg/m^3)(340\ m/s) = 408\ \frac{Pa\text{-}s}{m}$$

5.10 $\quad r = 10\ cm \qquad f = 10\ Hz,\ 100\ Hz,\ 1000\ Hz$

5.11.2

$$\tilde{Z} = \frac{jkr}{1+jkr}\rho_0 c = \frac{(1-jkr)\,jkr}{(1-jkr)(1+jkr)}\rho_0 c$$

$$= \frac{jkr + k^2r^2}{1+k^2r^2}\rho_0 c = \left[\frac{k^2r^2}{1+k^2r^2} + j\frac{kr}{1+k^2r^2}\right]\rho_0 c$$

$$= \frac{kr}{1+k^2r^2}(kr+j)\rho_0 c = z\,e^{i\Theta}$$

$$\tan\Theta = \frac{1}{kr} \Rightarrow \Theta = \tan^{-1}\left(\frac{1}{kr}\right)$$

$$z = \frac{kr}{1+k^2r^2}\left(k^2r^2+1\right)^{1/2}\rho_0 c = \frac{kr}{(1+k^2r^2)^{1/2}}\rho_0 c$$

$10\ Hz \Rightarrow \lambda = \frac{340\ m/s}{10\ s} = 34\ m \qquad k = \frac{2\pi}{\lambda} = .185\ m^{-1}$

$kr = \boxed{.0185}$

$$\Theta = \tan^{-1}\left(\frac{1}{.0185}\right) = 88.9^\circ = \boxed{89^\circ} = 1.55\ RAD$$

$$z = \frac{.0185}{\sqrt{1+(.0185)^2}}\left(408\ \frac{Pa\text{-}s}{m}\right) = \boxed{7.5\ \frac{Pa\text{-}s}{m}}$$

$100\ Hz \Rightarrow \lambda = 3.4\ m \qquad k = \frac{2\pi}{3.4} = 1.85\ m^{-1} \qquad kr = \boxed{.185}$

$$\Theta = \tan^{-1}\left(\frac{1}{.185}\right) = 79.5^\circ = \boxed{80^\circ} = 1.396\ RAD$$

$$z = \frac{.185}{\sqrt{1+(.185)^2}}\left(408\ \frac{Pa\text{-}s}{m}\right) = \boxed{74\ \frac{Pa\text{-}s}{m}}$$

$1000\ Hz = \lambda = .34\ m \qquad k = \frac{2\pi}{.34\ m} = 18.5\ m^{-1} \qquad kr = \boxed{1.85}$

$$\Theta = 28.4^\circ = \boxed{28^\circ} = .488\ RAD$$

$$z = 358.8\ \frac{Pa\text{-}s}{m^2} \quad \boxed{360\ \frac{Pa\ s}{m}}$$

10000 Hz $\quad \Theta = 3.12^\circ$

$z = \rho_0 c = 408\ \frac{Pa\text{-}s}{m}$

5.12.3 Given: $f = 100\ Hz$ $P_o = 2\ Pa$ AIR

FIND: I, IL, ξ_o, u_o, P_{rms}, SPL re 20 dB

$$I = \frac{1}{2}\frac{P_o^2}{\rho_o c} = \frac{1}{2}\frac{(2\ Pa)^2}{410\ \frac{Pa\text{-}s}{m}} = \boxed{4.88\times10^{-3}\ \frac{W}{m^2}}$$

$$IL = 10\log\frac{I}{I_{ref}} = 10\log\left(\frac{4.88\times10^{-3}\ \frac{W}{m^2}}{10^{-12}\ \frac{W}{m^2}}\right) = \boxed{97dB}$$

$$u_o = j\omega\xi_o = \frac{P_o}{Z} \Rightarrow \xi_o = \frac{P_o}{j\omega Z}$$

$$|\xi_o| = \frac{2\ Pa}{2\pi(100\ Hz)(410\ \frac{Pa\text{-}s}{m})} = \boxed{7.8\times10^{-6}\ m}$$

$$|u_o| = \omega\xi_o = \frac{2\ Pa}{410\ \frac{Pa\text{-}s}{m}} = \boxed{4.88\times10^{-3}\ \frac{m}{s}}$$

$$P_{rms} = \frac{P_o}{\sqrt{2}} = \frac{2\ Pa}{\sqrt{2}} = \boxed{1.41\ Pa}$$

$$SPL = 20\log\left(\frac{P_{rms}}{P_{ref}}\right) = 20\log\left(\frac{1.41\ Pa}{20\mu Pa}\right) = \boxed{97dB}$$

5.12.6 Given: 70 dB

FIND: $\mathcal{E}$, P_{rms} FOR re $10^{-12}\frac{W}{m^2}$ ($20\mu Pa$) AIR

re $1\mu Bar$ WATER

AIR re $10^{-12}\frac{W}{m^2}$

$$IL = 10\log\frac{I}{I_{ref}} = 70 \Rightarrow I = I_{ref}(10^7) = 10^{-12}\frac{W}{m^2}10^7 = 10^{-5}\frac{W}{m^2}$$

$$\frac{P_{rms}^2}{\rho c} = I \Rightarrow P_{rms}^2 = 415\ \frac{Pa\text{-}s}{m}\ 10^{-5}\frac{W}{m^2}$$

$$P_{rms} = \sqrt{4.15\times10^{-3}\ Pa^2} = \boxed{.064\ Pa}$$

$$\mathcal{E} = \frac{P_{rms}^2}{\rho c^2} = \frac{(.064\ Pa)^2}{1.42\times10^5\ Pa} = \boxed{2.9\times10^{-8}\ \frac{J}{m^3}}$$

WATER re $1\mu Bar$

$$SPL = 20\log\frac{P_{rms}}{P_{ref}} = 70 \quad P_{rms} = (1\mu Bar)10^{3.5} = 3.16\times10^{-3}\ Bar = \boxed{316\ Pa}$$

$$\mathcal{E} = \frac{P_{rms}^2}{\rho c^2} = \frac{(316.2\ Pa)^2}{2.25\times10^9\ Pa} = \boxed{4.44\times10^{-5}\ \frac{J}{m^3}}$$