

A simple oscillator whose natural frequency is 5 rad/sec is displaced a distance 0.030 m from its equilibrium position and released with an initial push that gives it a speed of .10 m/s at t = 0 sec. What is the amplitude of the resulting motion?

$$x = A \cos(\omega t + \phi)$$

$$x(t=0) = .03 \text{ m} = A \cos \phi$$

$$u(t=0) = .10 \text{ m/s} = -A\omega \sin(\omega t + \phi) = -A\omega \sin \phi$$

$$A = \frac{.03 \text{ m}}{\cos \phi}$$

$$.10 \text{ m/s} = -\frac{(.03 \text{ m})}{\cos \phi} (5 \text{ rad/s}) \sin \phi$$

$$\tan \phi = -.667 \Rightarrow \phi = -.588 \text{ rad} \quad (-33.7^\circ)$$

$$A = \frac{.03 \text{ m}}{\cos(-.588 \text{ rad})} = \boxed{0.036 \text{ m}}$$

OR

$$x = A_1 \cos \omega t + A_2 \sin \omega t$$

$$x(t=0) = .03 = A_1$$

$$u(t=0) = -A_1 \omega \sin \omega t + A_2 \omega \cos \omega t = .10 \text{ m/s}$$

$$A_2 = \frac{.10 \text{ m/s}}{5 \text{ rad/s}} = .02 \text{ m}$$

$$A = \sqrt{A_1^2 + A_2^2} = \sqrt{.03 \text{ m}^2 + .02 \text{ m}^2}$$

$$= \boxed{0.036 \text{ m}}$$

$$\phi = \tan^{-1} \left( \frac{.02 \text{ m}}{.03 \text{ m}} \right) = .588 \text{ rad} \quad (33.7^\circ)$$