Name KEY

SP436 Quiz 2

A long, Aluminum bar is fixed at both ends (x=0 and x = L)


The bar vibrates in the longitudinal mode subject to the wave equation,

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2}$$

where $c^2 = \frac{Y}{\rho}$. We showed that the complex harmonic solution of the wave equation is,

$$\tilde{\xi}(x,t) = \tilde{A}e^{j(\omega t - kx)} + \tilde{B}e^{j(\omega t + kx)}.$$

a. What are the boundary conditions for this system.

$\xi(x=0) = 0 \qquad \xi(x=L) = 0 \qquad$ (ANY TIME)

b. Use your boundary condition at x=0 to show that: $\tilde{\xi}(x,t) = -j2\tilde{A}e^{j\omega t}\sin kx$

(Hint: $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$)

$\xi(0,t) = \tilde{A}e^{j\omega t} + \tilde{B}e^{j\omega t} = 0$

$\tilde{A} = -\tilde{B}$

$\xi(x,t) = \tilde{A}e^{j(\omega t - kx)} - \tilde{A}e^{j(\omega t + kx)} = \tilde{A}e^{j\omega t}\left(e^{-jkx} - e^{+jkx}\right)$

$= Ae^{j\omega t}\left(-2j\sin kx\right)$

c. Use your boundary condition at x=L to show that: $f_n = \frac{nc}{2L}$

$\xi(L,t) = 0 = Ae^{j\omega t}(-2j)\sin kL$

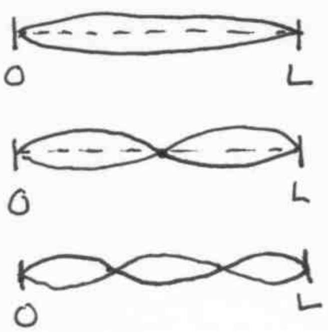
$kL = n\pi$

$k_n = \frac{n\pi}{L}$

$\omega_n = ck_n = \frac{n\pi c}{L}$

$f_n = \frac{\omega_n}{2\pi} = \frac{n\cancel{\pi}c}{L\,2\cancel{\pi}} = \frac{nc}{2L}$

d. Draw the first three normal modes in which the bar vibrates.




$\lambda_1 = 2L$

$\lambda_2 = L$

$\lambda_3 = \frac{2L}{3}$