

A sheet poster 2.00 m by 4.00 m is stretched over a rigid square frame such that the density is homogeneous and the tension is isotropic, resulting in a wave speed of 10.0 m/s.

We showed that:

$$\tilde{y} = \tilde{A} \sin(k_x x) \sin(k_z z) e^{i\omega t}$$

where:

$$k_{mn}^2 = \frac{n^2 \pi^2}{L_x^2} + \frac{m^2 \pi^2}{L_z^2}$$

What are the four lowest frequencies of the normal modes this sheet poster vibrates (in Hz)?

$$\omega_{mn} = c k_{mn} = c \sqrt{\frac{n^2 \pi^2}{L_x^2} + \frac{m^2 \pi^2}{L_z^2}}$$

$$f_{mn} = \frac{\omega_{mn}}{2\pi} = \frac{c}{2\pi} \sqrt{\frac{n^2 \pi^2}{L_x^2} + \frac{m^2 \pi^2}{L_z^2}}$$

$$= \frac{c}{2} \sqrt{\frac{n^2}{L_x^2} + \frac{m^2}{L_z^2}} = \frac{10 \text{ m/s}}{2} \sqrt{\frac{n^2}{(2\text{m})^2} + \frac{m^2}{(4\text{m})^2}}$$

$$= 5 \text{ m/s} \sqrt{\frac{4n^2}{16\text{m}^2} + \frac{m^2}{16\text{m}^2}} = \frac{5 \text{ m}}{4 \text{ m}} \sqrt{4n^2 + m^2}$$

$$(m, n) \quad = 1.25 \text{ Hz} \sqrt{4n^2 + m^2}$$

$$1, 1 \quad f_{11} = 1.25 \text{ Hz} \sqrt{5} = 2.80 \text{ Hz}$$

$$2, 1 \quad f_{21} = 1.25 \text{ Hz} \sqrt{8} = 3.54 \text{ Hz}$$

$$3, 1 \quad f_{31} = 1.25 \text{ Hz} \sqrt{13} = 4.51 \text{ Hz}$$

$$1, 2 \quad f_{12} = 1.25 \text{ Hz} \sqrt{17} = 5.15 \text{ Hz}$$

$$\left. \begin{array}{l} 2, 2 \\ 4, 1 \end{array} \right\} \begin{array}{l} f_{22} = 1.25 \text{ Hz} \sqrt{20} = 5.59 \text{ Hz} \\ f_{41} = 1.25 \text{ Hz} \sqrt{20} = 5.59 \text{ Hz} \end{array} \quad \text{DEGENERATE}$$