58. GIVEN: $K = 2.00 \times 10^{-16}\ J$

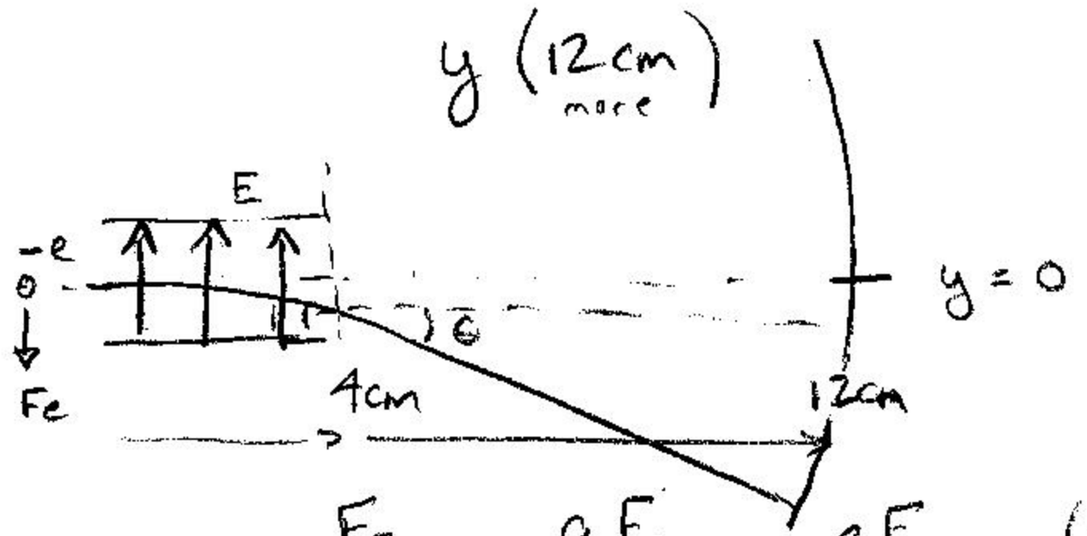
$\vec{E} = 2.00 \times 10^{4}\ N/C\ \hat{j}$ $\quad 0 \rightarrow 4\ cm$

$\vec{E} = 0$ $\quad 4cm \rightarrow 12\ cm$

FIND: $y\ (4cm)$

$\theta\ (4cm)$

$y\ (12cm)$



$$K = \frac{1}{2} m v^2$$

$$2.00 \times 10^{-16}\ J = \frac{1}{2}\left(9.11 \times 10^{-31}\ kg\right) v^2$$

$$v_{x_0} = 2.1 \times 10^{+7}\ m/s = v_x$$

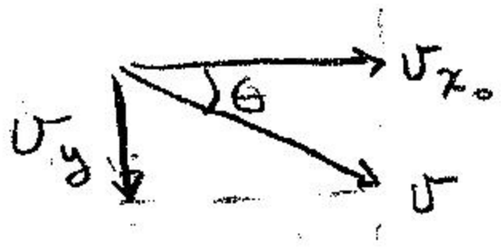
$$a_y = \frac{F_E}{m} = \frac{qE}{m} = \frac{eE}{m} = \frac{(1.6 \times 10^{-19}\ C)(20000\ N/C)}{9.11 \times 10^{-31}\ kg} = 3.52 \times 10^{15}\ \frac{m}{s^2}$$

$$t(4cm) = \frac{x}{v_{x_0}} = \frac{4 \times 10^{-2}\ m}{2.1 \times 10^{7}\ m/s} = 1.91 \times 10^{-9}\ s$$

$$y(4cm) = \cancel{y_0}^{\,0} + \cancel{v_{y_0} t}^{\,0} + \frac{1}{2} a t^2 = \frac{1}{2}\left(3.52 \times 10^{15}\ m/s^2\right)\left(1.91 \times 10^{-9}\ s\right)^2$$

$$= \boxed{6.41 \times 10^{-3}\ m}$$ BELOW AXIS

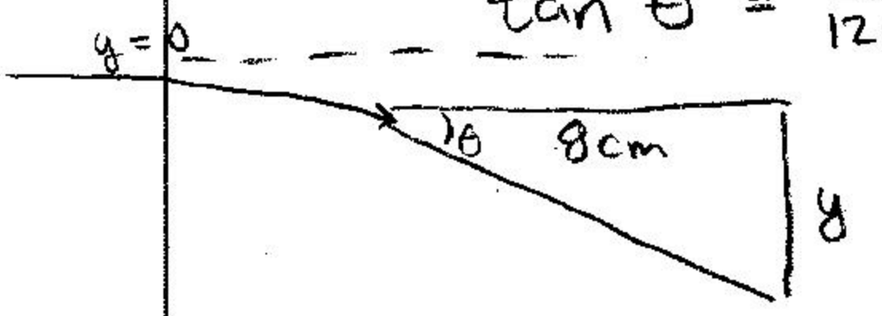
$$v_y = \cancel{v_{y_0}} + a_y t = \left(3.52 \times 10^{15}\ m/s^2\right)\left(1.91 \times 10^{-9}\ s\right) = 6.71 \times 10^{6}\ m/s$$



$$\theta = \tan^{-1}\left(\frac{v_y}{v_{x_0}}\right) = \tan^{-1}\left(\frac{6.71 \times 10^{6}\ m/s}{2.1 \times 10^{7}\ m/s}\right)$$

$$= \boxed{17.8^\circ}$$ DOWN

$$\tan\theta = \frac{y}{12cm} = \tan(17.8^\circ) \Rightarrow y = 3.84 \times 10^{-2}\ m$$



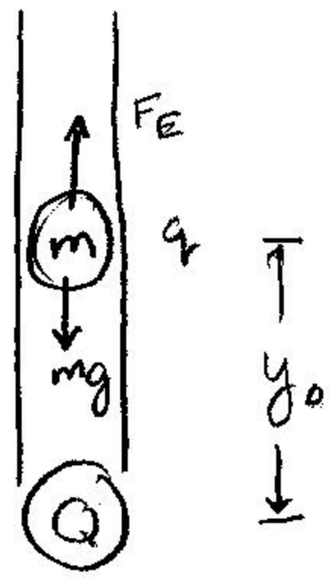
$$y(12\ cm) = 3.84 \times 10^{-2}\ m + 6.41 \times 10^{-3}\ m$$

$$= \boxed{4.48\ 10^{-2}\ m}$$ BELOW AXIS

81 GIVEN: m, q, Q

SMALL TUBE

SHOW: $y_0 = \sqrt{\frac{kqQ}{mg}}$, $\omega = \sqrt{\frac{2g}{y_0}}$

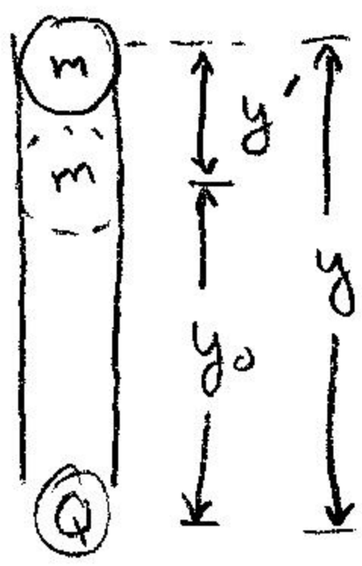


a. AT EQUILIBRIUM $F_e = mg$

$$\frac{kQq}{y_0^2} = mg$$

$$y_0^2 = \frac{kQq}{mg} \Rightarrow y_0 = \sqrt{\frac{kQq}{mg}} \quad \text{Q.E.D.}$$

b.



DISPLACE THE CHARGED MASS UPWARDS A SMALL AMOUNT y' SO FORCES ARE UNBALANCED

$$\Sigma F_y = F_E - mg = ma$$

$$\frac{kQq}{y^2} - mg = m\frac{d^2y}{dt^2}$$

WHERE

$y = y_0 + y'$

$\frac{d^2y}{dt^2} = \frac{dy'}{dt^2}$

$$\frac{kQq}{(y_0+y')^2} - mg = m\frac{d^2y'}{dt^2}$$

FACTOR OUT y_0^2

$$\frac{kQq}{y_0^2\left(1+\frac{y'}{y_0}\right)^2} - mg = m\frac{d^2y'}{dt^2}$$

NOTE $mg = \frac{kQq}{y_0^2}$

$$\cancel{m}g\left(1+\frac{y'}{y_0}\right)^{-2} - \cancel{m}g = \cancel{m}\frac{d^2y'}{dt^2}$$

IF y' IS SMALL

$$\left(1+\frac{y'}{y_0}\right)^{-2} = 1 - 2\frac{y'}{y_0}$$

(BINOMIAL EXPANSION)

$$g\left(\cancel{1} - 2\frac{y'}{y_0}\right) - \cancel{g} = \frac{d^2y'}{dt^2}$$

$$-\frac{2g}{y_0}y' = \frac{d^2y'}{dt^2}$$

$y' = A\cos(\omega t + \delta)$

$\frac{dy}{dt} = -A\omega\sin(\omega t+\delta)$

$\frac{d^2y}{dt^2} = -A\omega^2\cos(\omega t+\delta)$

$$+\frac{2g}{y_0}\left(\cancel{A}\cos(\omega t+\delta)\right) = +\cancel{A}\omega^2\cos(\omega t+\delta)$$

$$\omega^2 = \frac{2g}{y_0} \Rightarrow \omega = \sqrt{\frac{2g}{y_0}} \quad \text{Q.E.D.}$$