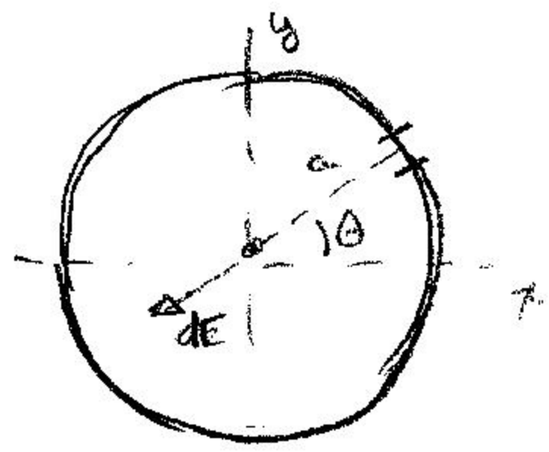
22-22 GIVEN: RING OF CHARGE RADIUS = a

$\lambda(\theta) = \lambda_0 \sin\theta$

FIND: $\vec{E}$ AT CENTER



$$dE_x = -\frac{k}{a^2}\cos\theta\, dq$$

$$dE_y = -\frac{k}{a^2}\sin\theta\, dq$$

$$dq = \lambda\, ds = \lambda a\, d\theta = \lambda_0 \sin\theta\, a\, d\theta$$

x COMPONENT

$$E_x = \frac{-k\lambda_0 a}{a^2}\int_0^{2\pi} \sin\theta\cos\theta\, d\theta \qquad u = \sin\theta,\quad du = \cos\theta\, d\theta$$

$$= \frac{-k\lambda_0}{a}\int_0^0 u\, du = 0$$

y COMPONENT

$$E_y = \frac{-k\lambda_0 a}{a^2}\int_0^{2\pi} \sin^2\theta\, d\theta = \frac{-k\lambda_0}{a}\int_0^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$$

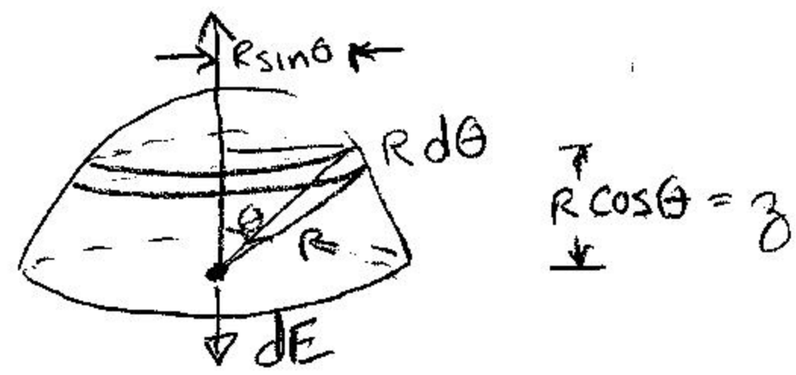
$$= \frac{-k\lambda_0}{a}\pi + 0 = -\frac{k\lambda_0\pi}{a}$$

FIELD IS POINTED IN $-\hat{j}$ DIRECTION

WITH A MAGNITUDE $E = \dfrac{k\lambda_0\pi}{a}$

22-26 GIVEN: THIN HEMISPHERICAL SHELL OF RADIUS = R
UNIFORM SURFACE CHARGE DENSITY = σ

FIND: $\vec{E}$ AT CENTER OF BASE



FOR A RING:

$$dE = \frac{k z \, dQ}{(z^2 + r^2)^{3/2}} \qquad dQ = \sigma \underbrace{(2\pi R \sin\theta)}_{\text{CIRCUMFERENCE}} \underbrace{R\,d\theta}_{\text{WIDTH}}$$

$$dE = \frac{k (R\cos\theta)\sigma(2\pi R\sin\theta) R\,d\theta}{(R^2\cos^2\theta + R^2\sin^2\theta)^{3/2}}$$

$$= \frac{k\, 2\pi R^3 \sigma \sin\theta \cos\theta \, d\theta}{(R^2(\cos^2\theta + \sin^2\theta))^{3/2}}$$

$$= k\, 2\pi \sigma \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$$

$$u = \sin\theta$$
$$du = \cos\theta \, d\theta$$

$$= k 2\pi\sigma \int_0^1 u\,du = k 2\pi\sigma \left.\frac{u^2}{2}\right|_0^1$$

$$= \frac{k 2\pi\sigma}{2} = k\pi\sigma$$

POINTED DOWN IN $-z$ DIRECTION