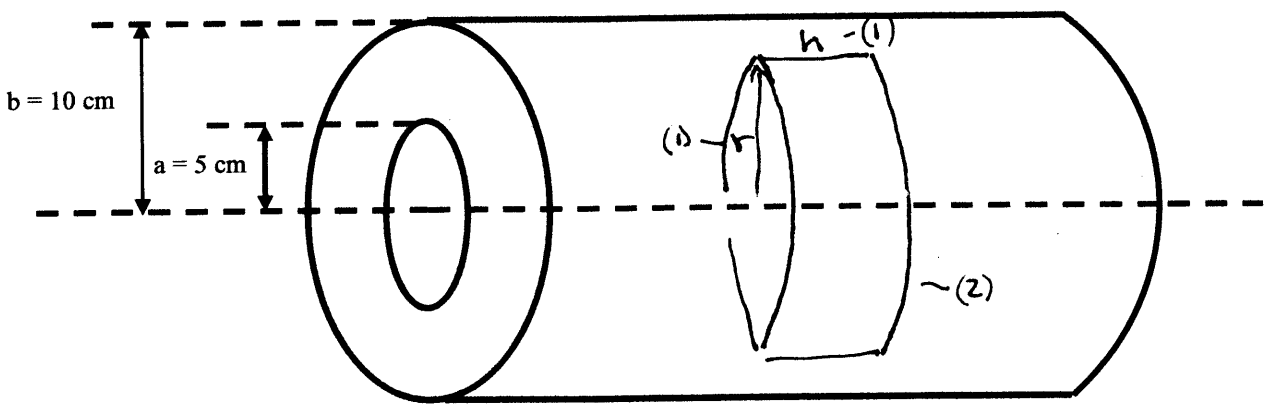
Name_____ SP222 Section 1121/3321

Exam 1 12 February 2010

1. A long non-conducting cylindrical shell has inner radius 0.050 m and outer radius 0.10 m as shown. The cylinder has uniform volume charge density, $\rho = +10\ \mu C/m^3$. Use Gauss's Law to solve for the magnitude of the electric field at a distance r = 0.075 m (7.5 cm) near the midpoint of the shell. (20 points)

For full credit:

a. Sketch and label the Gaussian surface. (4)
b. Identify the charge inside the Gaussian surface. (6)
c. Identify the value of $\oint \vec{E} \cdot d\vec{A}$ over the Gaussian surface. (5)
d. Solve for the magnitude of the electric field. (5)



(b) $Q_{IN} = \rho(\pi r^2 h - \pi a^2 h)$

(5) $\oint \vec{E} \cdot d\vec{A} = E(2\pi r h) = \frac{Q_{IN}}{\epsilon_0} = \frac{\rho(\pi r^2 h - \pi a^2 h)}{\epsilon_0}$

$$E = \frac{\rho(r^2 - a^2)}{2\epsilon_0 r}$$

$$= \frac{10\ \frac{\mu C}{m^3}\left[(.075\ m)^2 - (.05\ m)^2\right]}{2\left(8.85\times10^{-12}\ \frac{C^2}{N\text{-}m^2}\right)(.075\ m)}$$

$$= 23540.5\ N/C = 24000\ N/C$$

Extra credit: In the 1980 Winter Olympics, the USA hockey team defeated a highly favored USSR team in what was later termed the "miracle on ice." Who coached team USA? (no, it wasn't really Kurt Russell) (5 points)

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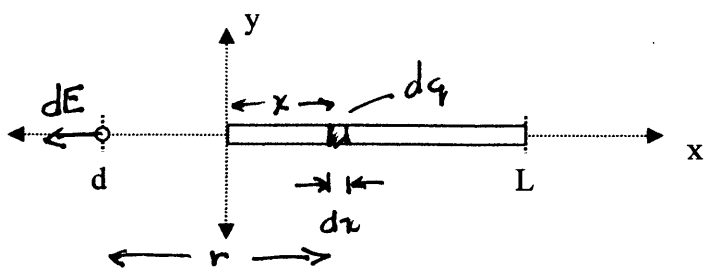
2. A charged stick with uniform charge density, λ, is located on the x axis with one end at the origin. The stick has a length, L as shown on the below diagram. Find an expression for the electric field at a point on the x-axis at x = -d. (25 points)

For full credit:

a. Draw and label the location and size of a charge element dq. (6)
b. Draw and label the electric field, dE caused by dq. (2)
c. Develop an expression for both dq and dE. (10)
d. Pay attention to the vector nature of dE. (1)
e. Integrate over appropriate limits to determine the electric field. (6)



$dq = \lambda\, dx$ (3) $\quad r = x + d$ (1)

$dE = \frac{k\, dq}{r^2} = \frac{k \lambda\, dx}{(x+d)^2}$ (3) TO THE LEFT (1)

$$\int dE = E = k\lambda \int_0^L \frac{dx}{(x+d)^2} = k\lambda \frac{L}{d(d+L)}$$

TI

$$= k\lambda \left(\frac{1}{d} - \frac{1}{(d+L)} \right)$$

$$= k\lambda \left(\frac{d+L-d}{d(d+L)} \right)$$

$$= \frac{k\lambda L}{d(d+L)}$$

$u = x + d$

$du = dx$

$$E = k\lambda \int_d^{L+d} \frac{du}{u^2} = -k\lambda \frac{1}{u}\Big|_d^{L+d} = k\lambda \left(\frac{1}{d} - \frac{1}{L+d} \right)$$

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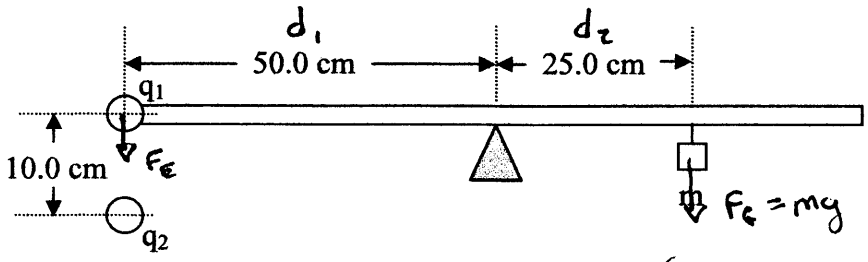
3. A rigid (but massless) meter stick is pivoted about its center as shown. A charge $q_1 = 5.00 \times 10^{-7}$ C is placed at one end of the meter stick, and a charge $q_2 = -q_1$ is placed a distance d = 10.0 cm below it. (15 points)

a. What is the torque on the meter stick about the pivot due to the force between the charges? (10)

b. What mass hung 25.0 cm from the pivot will keep the meter stick balanced? (5)



$$F_E = \frac{k q_1 q_2}{r^2} = \frac{\left(9 \times 10^9 \frac{N\text{-}m^2}{C^2}\right)\left(5 \times 10^{-7}\, C\right)^2}{(.1\, m)^2}$$

$$= .225\ N$$

$$\vec{\tau}_E = \vec{r} \times \vec{F} \Rightarrow |\vec{\tau}_E| = (.5\, m)(.225\, N) = .1125\ Nm$$

$$= .113\ N \cdot m \quad \text{CCW} \quad (1)$$

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$$|\vec{\tau}_M| = d_2 m g = (.25\, m)\, m\, (9.8\, m/s^2) = |\vec{\tau}_E| = .113\ N \cdot m$$

$$m = \frac{.113\ N \cdot m}{(.25\, m)(9.8\, m/s^2)} = .0459\ kg$$

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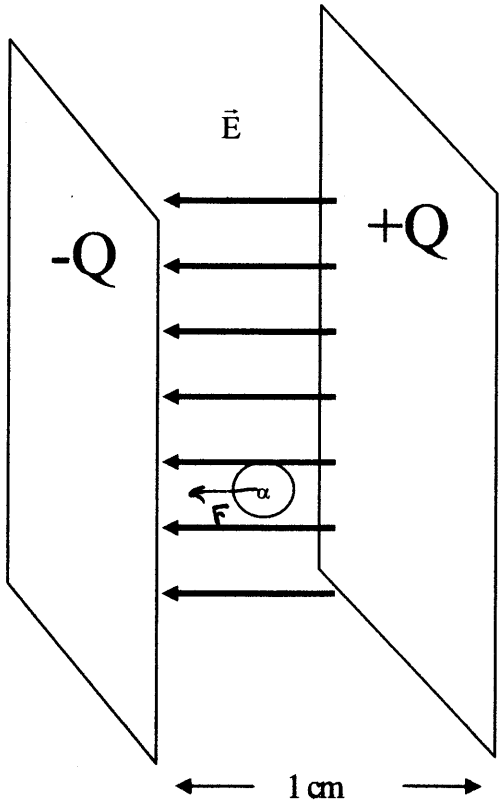
4. Two parallel plates are separated by 1.0 cm. The electric between the plates is 5.0 N/C as shown. An alpha particle is released from rest just in front of the positive plate. An alpha particle is made up of two protons and two neutrons. (25 points)



a. What is the potential difference between the plates? (3)

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\Delta V = (5\ N/C)(.01\ m) = .05\ V$$

b. What is the change in potential energy of the alpha particle as it travels from the positive plate to the negative plate? (4)

$$\Delta U = q\,\Delta V = 2(1.6\times10^{-19}\ C)(.05\ V)$$

$$= 1.6\times10^{-20}\ J$$

c. What is the kinetic energy of the alpha particle just before it hits the negative plate? (3)

$$K_f = 1.6\times10^{-20}\ J$$

d. What is the velocity of the alpha particle just before it strikes the negative plate? (3)

$$K_f = \tfrac{1}{2}mv_f^2 = 1.6\times10^{-20}\ J = \tfrac{1}{2}(4\cdot1.67\times10^{-27}\ kg)\,v^2$$

$$v = 2190\ m/s$$

e. What is the force on the alpha particle? Include direction. (3)

$$F = qE = 2(1.6\times10^{-19}\ C)(5\ N/C) = 1.6\times10^{-18}\ N$$

LEFT

f. What is the acceleration of the alpha particle? (3)

$$a = \frac{F}{m} = \frac{1.6\times10^{-18}\ N}{4(1.67\times10^{-27}\ kg)} = 2.395\times10^{8}\ m/s^2$$

g. Use the acceleration to verify you get the same velocity as part d. (3)

$$v^2 = 2a\,\Delta x \Rightarrow v = \sqrt{2(2.395\times10^{8}\ m/s^2)(.01\ m)} = 2190\ m/s$$

h. How long does it take for the alpha particle to travel from one plate to the other? (3)

$$x = \tfrac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\,(.01\ m)}{2.395\times10^{8}\ m/s^2}}$$

$$= 9.14\times10^{-6}\ s$$