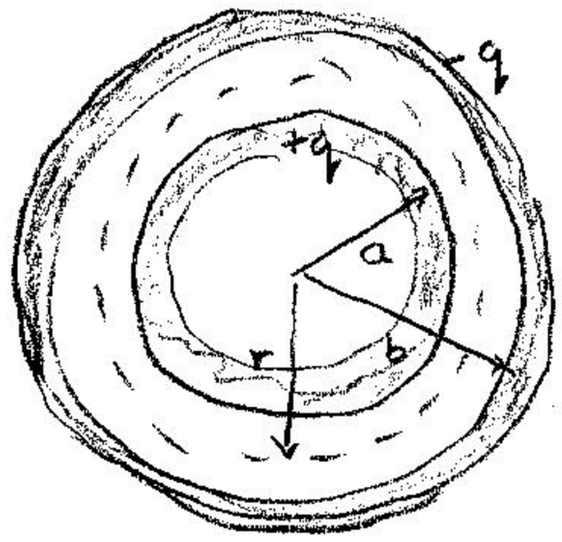
23-47 GIVEN:



FIND: $V_b - V_a$

$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{q_{IN}}{\epsilon_0} = E(4\pi r^2) = \frac{+q}{\epsilon_0}$$

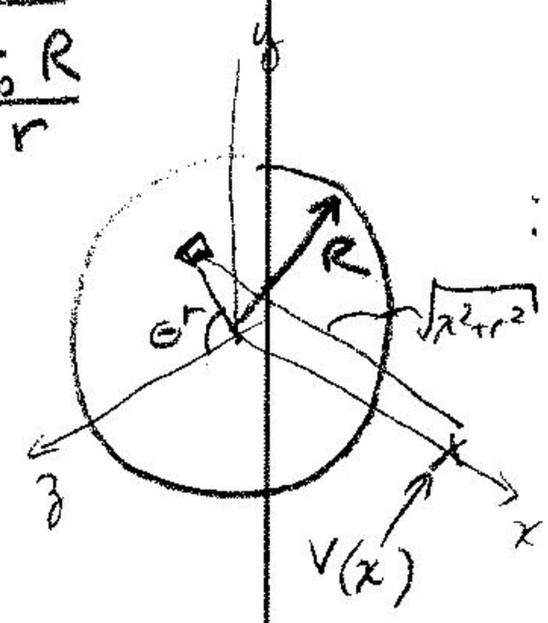
$$E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{kq}{r^2}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b \frac{kq}{r^2}\,dr = -kq\left(\frac{-1}{r}\right)\Big|_a^b$$

$$= \boxed{kq\left(\frac{1}{b}-\frac{1}{a}\right) = \frac{kq(a-b)}{ab}}$$

23-54 GIVEN: DISK, RADIUS = R $\quad \sigma(r) = \frac{\sigma_0 R}{r}$

FIND: Q_{TOTAL}, $V(x)$



$$dA = r\,dr\,d\theta \qquad dq = \sigma\, r\,dr\,d\theta$$

$$Q_{TOT} = \int_{\theta=0}^{2\pi}\int_{r=0}^{R} \frac{\sigma_0 R}{\cancel{r}}\,\cancel{r}\,dr\,d\theta$$

$$= 2\pi\sigma_0 R\int_0^R dr = \boxed{2\pi\sigma_0 R^2}$$

$$dV = \frac{k\,dq}{\sqrt{x^2+r^2}} = \frac{k\sigma r\,dr\,d\theta}{\sqrt{x^2+r^2}} = \frac{k\sigma_0 R\,\cancel{r}\,dr\,d\theta}{\cancel{r}\sqrt{x^2+r^2}}$$

$$V = \int_{\theta=0}^{2\pi}\int_{r=0}^{R} \frac{k\sigma_0 R\,dr\,d\theta}{\sqrt{x^2+r^2}} = 2\pi k\sigma_0 R\int_0^R \frac{dr}{\sqrt{x^2+r^2}}$$

TI-92+

$$= \boxed{2\pi k\sigma_0 R\,\ln\left(\frac{\sqrt{x^2+R^2}+R}{|x|}\right)}$$

$$= 2\pi\left(\frac{1}{4\pi\epsilon_0}\right)\sigma_0 R\,\ln\left(\frac{\sqrt{x^2+R^2}+R}{|x|}\right)$$

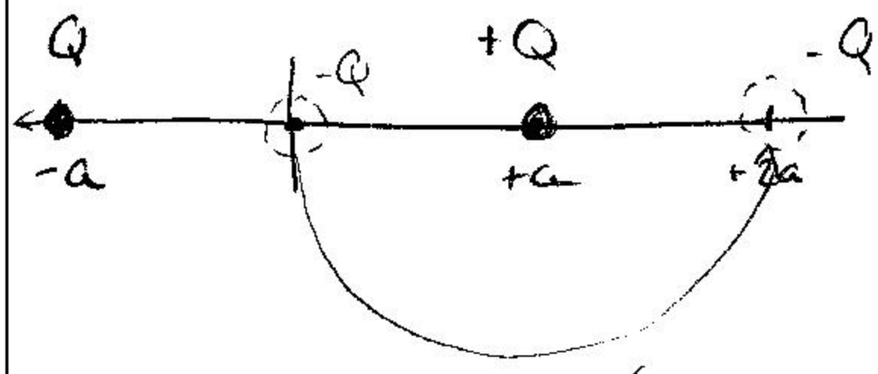
$$= \frac{\sigma_0 R}{2\epsilon_0}\,\ln\left(\sqrt{1+\frac{R^2}{x^2}}+\frac{R}{|x|}\right)$$

23-79 GIVEN: $+Q$ AT $x = -a$

FIND: WORK TO BRING $+Q$ TO $x = a$ (FROM ∞)

WORK TO BRING $-Q$ TO $x = 0$ (FROM ∞)

WORK TO MOVE $-Q$ TO $x = 2a$



$$W_{+Q \to a} = \frac{k q_1 q_2}{r} = \boxed{\frac{kQ^2}{2a}}$$

$$W_{-Q \to 0} = \left(\frac{kQ}{a}\right)(-Q) + \left(\frac{kQ}{a}\right)(-Q) = \boxed{-\frac{2kQ^2}{a}}$$

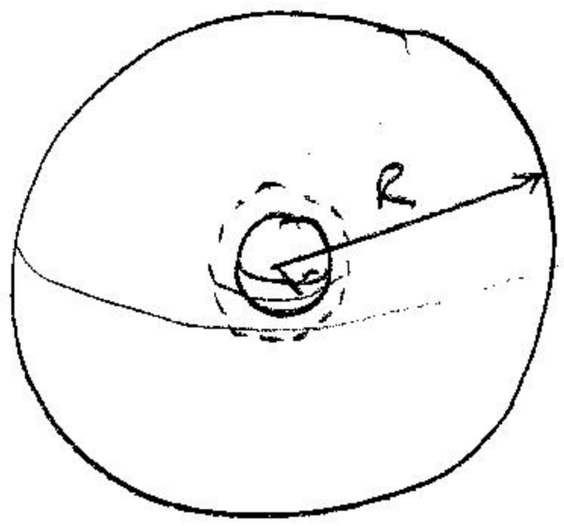
$$W_{-Q \to 2a} = \left(\frac{kQ}{a}\right)(-Q) + \frac{kQ}{3a}(-Q) = \frac{-3kQ^2}{3a} - \frac{kQ^2}{3a} = \frac{-4kQ^2}{3a}$$

$$W_{0 \to 2a} = \overset{(\text{final})}{W_{-Q \to 2a}} - \overset{(\text{initial})}{W_{-Q \to 0}} = \frac{-4kQ^2}{3a} - \left(\frac{-2kQ^2}{a}\right)$$

$$= \frac{-4kQ^2}{3a} + \frac{6kQ^2}{3a} = \boxed{\frac{2kQ^2}{3a}}$$

23-91 GIVEN: CHARGE Q, SPHERE, RADIUS $= R$

FIND: WORK TO ASSEMBLE SPHERE



POTENTIAL OF CHARGE ALREADY ASSEMBLED

$$dW = \left(\frac{kQ'}{r}\right) dq$$ ← ADDITIONAL CHARGE ADDED

$$Q' = \rho \tfrac{4}{3}\pi r^3 \qquad dq = \rho\, dV = \rho\, 4\pi r^2 dr$$

$$dW = \frac{k\left(\rho \frac{4}{3}\pi r^3\right)}{r}\left(\rho\, 4\pi r^2 dr\right) = k\,\frac{16\pi^2}{3}\rho^2 r^4 dr$$

$$W = k\,\frac{16\pi^2\rho^2}{3}\int_0^R r^4 dr = \frac{k\,16\pi^2\rho^2}{3}\,\frac{r^5}{5} = \frac{k\,16\pi^2\rho^2 R^5}{15}\,\frac{R}{R}\,\frac{9}{9}$$

$$Q = \rho \tfrac{4}{3}\pi R^3 \Rightarrow Q^2 = \rho^2 \tfrac{16}{9}\pi^2 R^6 \qquad = \frac{9kQ^2}{15R}$$

$$W = \frac{\cancel{9}^{\,3}}{4\pi\epsilon_0}\,\frac{Q^2}{\cancel{15}_{\,5}R} = \boxed{\frac{3Q^2}{20\pi\epsilon_0 R}}$$

(SINCE EQUIPOTENTIAL SURFACES ARE SPHERES, IT TAKES NO WORK TO MOVE ALONG A FIXED RADIUS)