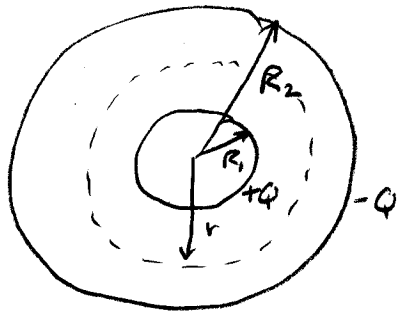
24 - 49 GIVEN : SPHERICAL CAPACITOR



SHOW : $C = \dfrac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$

WHERE $R_1 \approx R_2$ $\quad C = \dfrac{\epsilon_0 A}{d}$, $d = R_2 - R_1$

(1) $\vec{E}$ $\quad R_1 < r < R_2$

$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{q_{IN}}{\epsilon_0}$$

$$|\vec{E}|(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0}$$

(2) $$V_{R_1} - V_{R_2} = -\int_{R_2}^{R_1} \vec{E} \cdot d\vec{l} = -\int_{R_2}^{R_1} \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \cdot dr\,\hat{r} = -\int_{R_2}^{R_1} \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2}$$

$$= \frac{Q}{4\pi\epsilon_0} \left.\frac{1}{r}\right|_{R_2}^{R_1} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{R_2 - R_1}{R_1 R_2}$$

(3) $$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon_0}\frac{R_2 - R_1}{R_1 R_2}} = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1} \quad \text{QED}$$

IF $R_1 \approx R_2$ $\quad 4\pi R_1 R_2 = 4\pi R^2 = A$

$$d = R_2 - R_1$$

$$C = \frac{\epsilon_0 A}{d}$$