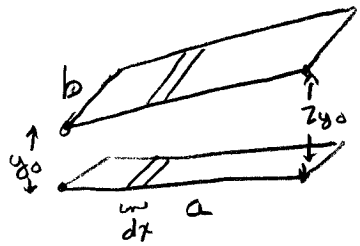
24-93 GIVEN: RECTANGULAR PLATES $a \times b$
TOP PLATE INCLINED AT SMALL ANGLE
$d$ VARIES FROM $y_0 \rightarrow 2y_0$ ($\ll a$ OR $b$)

FIND: CAPACITANCE




$$dC = \frac{\epsilon_0 \, dA}{y}$$

$$y = y_0 + \frac{2y_0 - y}{a} x = y_0 + \frac{y_0}{a} x$$

$$dA = b \, dx$$

$$dC = \frac{\epsilon_0 b \, dx}{y_0 + \frac{y_0}{a} x}$$

$$C = \epsilon_0 b \int_0^a \frac{dx}{y_0 + \frac{y_0}{a} x} = \epsilon_0 b \int_{y_0}^{2y_0} \frac{\frac{a}{y_0} du}{u}$$

(SINCE CAPACITORS ARE PARALLEL)

$$u = y_0 + \frac{y_0}{a} x$$

$$du = \frac{y_0}{a} dx$$

$$= \frac{\epsilon_0 b a}{y_0} \int_{y_0}^{2y_0} \frac{du}{u} = \frac{\epsilon_0 b a}{y_0} \ln u \Big|_{y_0}^{2y_0}$$

$$= \frac{\epsilon_0 b a}{y_0} \ln\left(\frac{2y_0}{y_0}\right) = \boxed{\frac{\epsilon_0 b a \ln(2)}{y_0}}$$