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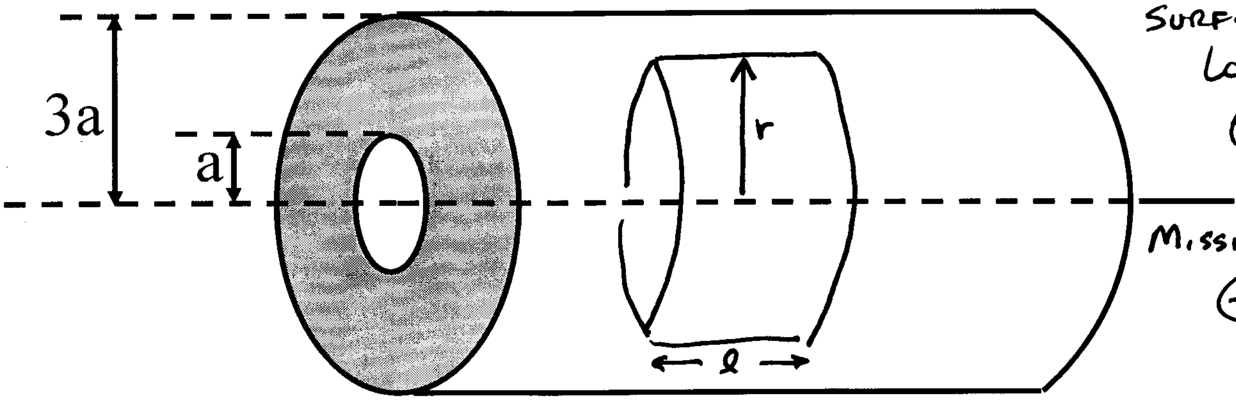
1. (25) A fairly long non-conducting cylindrical shell has inner radius "a" and outer radius "3a" as shown. The cylinder has uniform charge density, ρ, throughout the volume. For full credit, draw and label the Gaussian surface you are using for one of the parts below. Your answers should be in terms of the charge density, the dimensions on the drawing and appropriate constants.

GAUSSIAN SURF. TOO LONG (-.5)



OR INCONSIST. MISSING LABEL (-.5)

a. (5) Use Gauss' Law to solve for the **magnitude of the electric field** at a distance r from the longitudinal axis when r < a. $\Rightarrow q_{in} = 0 \Rightarrow E = 0$

ARE THERE HINTS?

WRONG OUTER RADIUS (-3)
VOLUME OF HOLE ONLY (-4)
NOT SUBT HOLE (-3)
$r^2 - a^2 = (r-a)^2$ (-2.5)
WRONG LENGTH (-2)

b. (10) Use Gauss' Law to solve for the **magnitude of the electric field** at a distance r from the longitudinal axis when a < r < 3a.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \quad (4)$$

WRONG RADIUS (-3)
WRONG LENGTH (-2)
SUBTRACTING AREAS (-4)
(4)

$$E(2\pi r l) = \frac{\rho(\pi r^2 l - \pi a^2 l)}{\epsilon_0} = \frac{\rho \pi l (r^2 - a^2)}{\epsilon_0}$$

$$E = \frac{\rho \cancel{\pi} \cancel{l}(r^2-a^2)}{2\cancel{\pi} r \cancel{l}\,\epsilon_0} = \boxed{\frac{\rho(r^2-a^2)}{2r\epsilon_0}}$$

(2)

c. (10) Use Gauss' Law to solve for the **magnitude of the electric field** at a distance r from the longitudinal axis when r > 3a.

IGNORING ADDING HOLE VOLUME (-3)
$(3a)^2 \neq 3a^2$ (-1)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{IN}}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\rho(\pi (3a)^2 l - \pi a^2 l)}{\epsilon_0}$$

$$E = \frac{\rho \cancel{\pi} \cancel{l}(9a^2 - a^2)}{2\cancel{\pi} r \cancel{l}\,\epsilon_0} = \frac{\rho\, 8a^2}{2r\epsilon_0} = \boxed{\frac{4\rho a^2}{\epsilon_0 r}}$$

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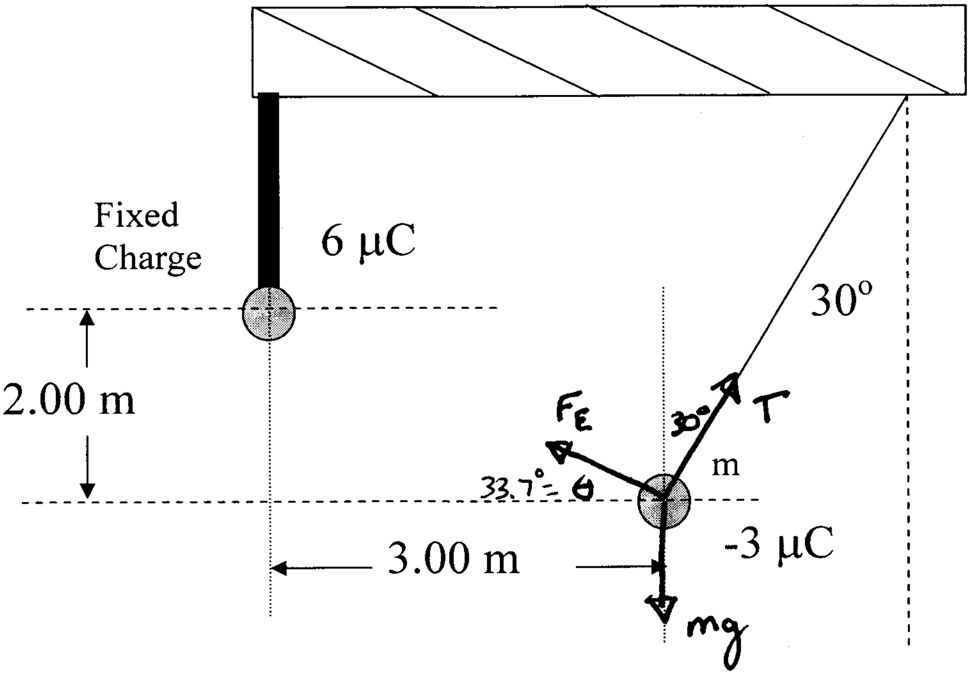
2. (25) A particle of mass, m, and charge –3.00 μC is suspended in the presence of gravity at rest by a 5.00 m long massless string at an angle of 30.0° with respect to the vertical as shown. A fixed point charge of 6.00 μC is mounted as shown. ($k = 8.99 \times 10^9$ N-m²/C²)



a. (6) Draw and label the forces acting on the –3 μC charge.

b. (9) What is the magnitude of the electrostatic force acting on the –3 μC charge? Calculate ~~are~~ the horizontal (x) and vertical (y) components of this electrostatic force.

$$F_E = \frac{k q_1 q_2}{r^2} = \frac{(9\times10^9 \frac{N\cdot m^2}{C^2})(6\times10^{-6}C)(3\times10^{-6}C)}{4m^2 + 9m^2} = \boxed{.0125\ N}$$

NEGATIVE

θ = 30° (-2)

UNITS -.5 @

$$F_{E_x} = .0125\ N\cos\theta = (.0125\ N)\frac{3}{\sqrt{13}} = \boxed{.0104\ N}$$

$$F_{E_y} = .0125\ N\sin\theta = (.0125\ N)\frac{2}{\sqrt{13}} = \boxed{.0069\ N}$$

c. (6) What is the tension in the string?

WRONG TRIG FUNCTION (-3)

$$T\sin 30 = F_{Ex} = .0104\ N$$

$$T = \frac{.0104\ N}{.5} = \boxed{.0208\ N}$$

d. (4) What is the mass of the –3 μC charge?

TRIG (-2)

WRONG SIGN (-2)

$$F_{E_y} + T\cos 30 = mg = .0069\ N + (.0208\ N)(.866)$$

$$m(9.8\ m/s^2) = .0249$$

$$m = \boxed{2.54\ gms} = 2.54\times10^{-3}\ kg$$

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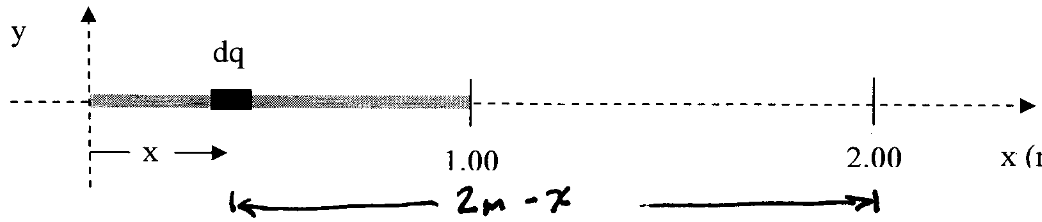
3. (20) A non-conducting charged stick has a non-uniform positive linear charge density

$$\lambda(x) = \left(5.00\frac{\mu C}{m^2}\right)x$$

and length 1.00 m as shown below. The charged stick is set on the x axis with the left end on the origin and the x coordinate as shown.



a. (3) Write an expression for the charge element dq (you can drop the units if you like).

$$\lambda = 5x \qquad dq = \lambda\,dx = 5x\,dx$$

b. (~~5~~) 1 What is the total charge on the stick?

$$Q_{TOT} = \int_0^{1m} dq = \int_0^{1m} \left(5\frac{\mu C}{m^2}\right)x\,dx = 5\frac{\mu C}{m^2}\,\frac{x^2}{2}\Big|_0^{1m} = 5\frac{\mu C}{m^2}\left(\frac{(1m)^2}{2}\right)$$

$$= 2.5\,\mu C$$

LIMITS -1.5

MULT -3.5

c. (2) What is the direction of the electric field at x = 2.00 m?

RIGHT (+x)

d. (~~5~~) 6 What is the magnitude of the electric field at x = 2.00 m? (be sure to tell me what you put in your calculator in case your final answer is wrong)

$$dE = \frac{k\,dq}{r^2} = \frac{k\left(5\frac{\mu C}{m^2}\right)x\,dx}{(2m - x)^2}$$

$$E = \left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\left(5\,\frac{\mu C}{m^2}\right)\int_0^{1m}\frac{x\,dx}{(2-x)^2}$$

-3

$$= 45\times10^3\,\frac{N}{C}\,(.306) = 13800\,\frac{N}{C}$$

PT CHARGE -6
⊥ LINE CHARGE

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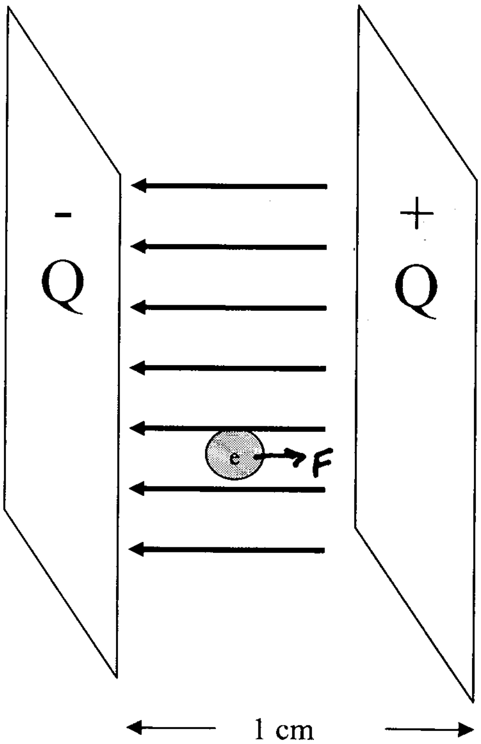
4. (25) Two parallel plates are separated by 1.00 cm. The electric field between the plates is a constant 10.0 N/C to the left as shown. An electron is placed near the negative plate. You may ignore the gravitational force on the electron. ($e = 1.60 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg)



a. (4) What is the electric force on the electron? Draw and label it.

$$F = qE = (1.6\times10^{-19}\,C)(10\,N/C) = \boxed{1.6\times10^{-18}\,N}$$

b. (3) What is the acceleration of the electron?

$$a = \frac{F}{m} = \frac{1.6\times10^{-18}\,N}{9.11\times10^{-31}\,kg} = \boxed{1.76\times10^{12}\,\frac{m}{s^2}}$$

c. (3) If the electron starts from rest at one plate, use this acceleration to find the velocity of the electron just before it strikes the other plate?

$$v^2 = 2ax = 2(1.76\times10^{12}\,\tfrac{m}{s^2})(.01\,m)$$
$$= 3.52\times10^{10}\,\frac{m^2}{s^2} \Rightarrow v = \boxed{1.87\times10^5\,\frac{m}{s}}$$

d. (3) How long does it take for the electron to travel from one plate to the other?

$$x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}}$$
$$t = \sqrt{\frac{2(.01\,m)}{1.76\times10^{12}\,m/s^2}} = 1.07\times10^{-7}\,s$$

e. (4) What is the potential difference between the plates?

$$\Delta V = (10\,N/C)(.01\,m) = \boxed{0.10\,V}$$

f. (3) What is the change in the potential energy of the electron as it travels from one plate to the other?

$$\Delta U = q\,\Delta V = (1.6\times10^{-19}\,C)(.10\,V) = \boxed{1.6\times10^{-20}\,J}$$

e. (2) What is the kinetic energy of the electron just before it hits the positive plate assuming again that it is released from rest?

$$K = \boxed{1.6\times10^{-20}\,J}$$

f. (3) Using this kinetic energy, calculate the final velocity of the electron just before it strikes the negative plate.

$$\frac{1}{2}mv^2 = 1.6\times10^{-20}\,J = \frac{1}{2}(9.11\times10^{-31}\,kg)\,v^2$$
$$v = \sqrt{\frac{(1.6\times10^{-20}\,J)(2)}{9.11\times10^{-31}\,kg}} = \boxed{1.87\times10^5\,m/s}$$

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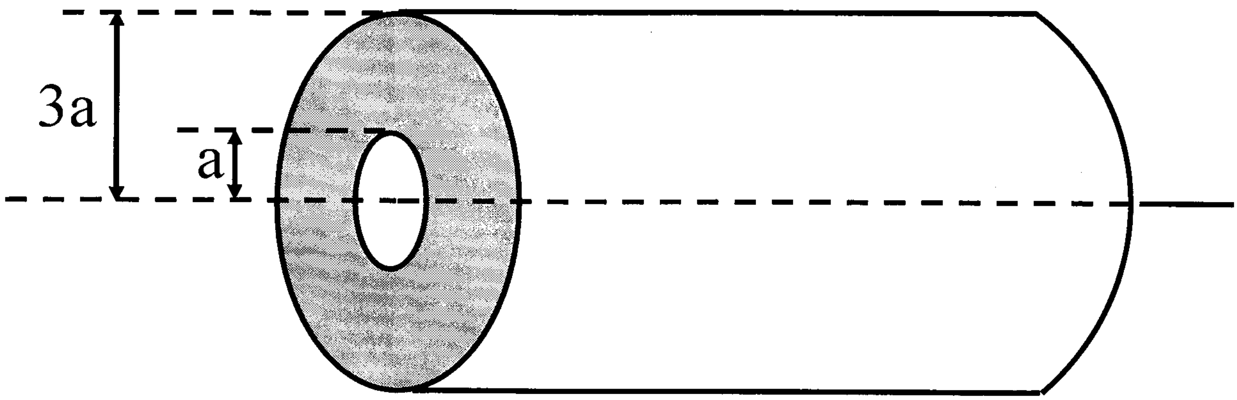
5. (15) The non-conducting hollow cylinder from Question 1 has inner and outer radius a and 3a as shown.



Suppose you found the electric field inside the cylinder (Question 1.b.) was of the form:

$$\vec{E} = \frac{C_1 r^2 - C_2}{r}\hat{r}$$

Where C_1 and C_2 are some constants composed of the charge density, dimensions, and appropriate physical constants. (in fact, this would be correct. Hopefully your answer was of this form and you got the constants correct)

We defined potential difference between two points as:

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\int_a^b \vec{E}\bullet d\vec{\ell}$$

3

a. (10) What is the potential difference between the inner and outer surfaces of the non conducting hollow cylinder (in terms of this constant, C, and appropriate dimensions)? Make sure you specify what you use for $d\ell$. 4 $\rightarrow d\vec{\ell} = dr\,\hat{r}$ $a \Rightarrow r = a$ $b \Rightarrow r = 3a$

1

$$V_{ba} = -\int_{r=a}^{r=3a} \frac{C_1 r^2 - C_2}{r}\hat{r}\cdot dr\,\hat{r} = -\int_a^{3a}\left(C_1 r - \frac{C_2}{r}\right)dr$$

$$= -C_1\frac{r^2}{2}\Big|_a^{3a} + C_2 \ln r\Big|_a^{3a} = -\frac{C_1}{2}\left(9a^2 - a^2\right) + C_2 \ln\left(\frac{3a}{a}\right)$$

$$= -\frac{C_1\, 8a^2}{2} + C_2 \ln 3 = \boxed{-C_1 4a^2 + C_2 \ln 3}$$

b. (5) What is the difference in potential energy of an electron (charge = -e) at the inner and outer surfaces?

$$U_{ba} = (-e)\,V_{ba} =$$