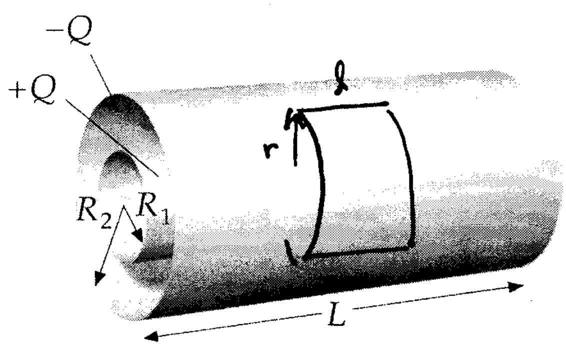
Name________________________________ SP222 Quiz

Section 1121/3321 Feb 2011

Two thin concentric conducting cylindrical shells with radii R_1 and R_2 and length L are arranged as shown. The center shell has a charge of +Q and the outer shell has a charge of –Q. By neglecting the end effects and using Gauss's Law, we can show that the electric field between the shells is:



$$|\vec{E}| = \frac{Q}{2\pi\varepsilon_o r L}$$

where r is the distance from the center axis in the range $R_1 < r < R_2$.

Use the definition of electric potential difference to find the potential difference between the inner and outer shells.

$$V_{ba} = V_b - V_a \equiv -\int_a^b \vec{E}\cdot d\vec{\ell}$$

For full credit, be sure to specify the direction of the electric field, the location of points a and b, and your displacement, $d\vec{\ell}$.

$$(2) \quad \vec{E} = \frac{Q}{2\pi\epsilon_o r L}\,\hat{r} \qquad (2) \quad d\vec{\ell} = dr\,\hat{r}$$

$$b \rightarrow r = R_1 \quad (1) \quad a \rightarrow r = R_2$$

(1)

$$\underset{\text{INNER}}{V_b} - \underset{\text{OUTER}}{V_a} = -\int_{R_2}^{R_1} \frac{Q}{2\pi\epsilon_o L r}\,\hat{r}\cdot dr\,\hat{r} = -\int_{R_2}^{R_1} \frac{Q}{2\pi\epsilon_o L}\,\frac{dr}{r} \quad (3)$$

$$= \frac{-Q}{2\pi\epsilon_o L}\,\ln r\Big|_{R_2}^{R_1} = -\frac{Q}{2\pi\epsilon_o L}\left(\ln R_1 - \ln R_2\right)$$

(1)

$$= \frac{Q}{2\pi\epsilon_o L}\,\ln\frac{R_2}{R_1}$$

Extra credit: (1 point) Use Gauss's Law to show that the electric field between the two shells is that given above.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{IN}}{\epsilon_o}$$

$$E(2\pi r \cancel{\ell}) = \frac{\frac{Q}{L}\cancel{\ell}}{\epsilon_o}$$

$$E = \frac{Q}{2\pi\epsilon_o L r}$$