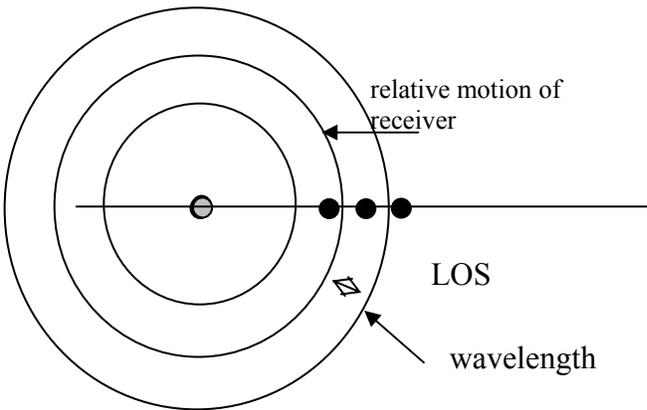


Doppler Effect

The Doppler Effect is the change in the **observed frequency** of a source due to the **relative motion** between the source and the receiver. The relative motion that affects the observed frequency is only the motion **in the Line-Of-Sight (LOS)** between the source and the receiver.

Relative motion of the receiver.

If a source is stationary, as the one below, it will emit sound waves that propagate out from the source as shown below.



As the receiver moves towards the source, it will detect the sound coming from the source but each successive sound wave will be detected earlier than it would have if the receiver were stationary, due to the motion of the receiver in the LOS. Thus the frequency that each successive wave front would be detected would be changed by this relative motion where:

$$\Delta f = \frac{v_r}{\lambda_0}$$

λ_0 is the original wavelength of the source

Δf is the change in the observed frequency

v_r is the velocity of the receiver in the LOS

Since the original frequency of the source can be expressed in terms of the wavelength where

$f_0 = \frac{c}{\lambda_0}$, the observed frequency becomes:

$$f' = f_0 + \Delta f$$

$$f' = \frac{c}{\lambda_0} + \frac{v_r}{\lambda_0}$$

$$f' = f_0 \left(\frac{c + v_r}{c} \right)$$

Note that this equation only works if the relative velocity of the receiver, v_r is towards the source. If the motion is away from the source, the relative velocity would be in the opposite direction and the equation would become:

$$f' = f_0 \left(\frac{c - v_r}{c} \right)$$

The two equations are usually combined and expressed as:

$$f' = f_0 \left(\frac{c \pm v_r}{c} \right)$$

Relative motion of the source

If the source is moving towards the receiver, the effect is slightly different. The spacing between the successive wave fronts would be less as seen in the diagram below. This would be expressed as:

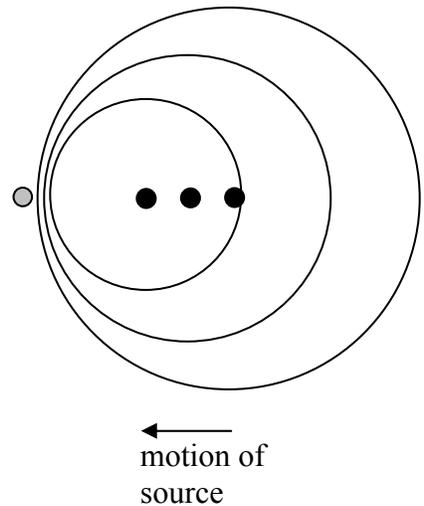
$$\Delta\lambda = \frac{v_s}{f_0}$$

v_s is the relative velocity of the source

To calculate the observed frequency:

$$f' = \frac{c}{(\lambda_0 + \Delta\lambda)}$$

$$f' = f_0 \left(\frac{c}{c - v_s} \right)$$



Note that this is only when the source is moving **towards** the receiver. If the source is moving away, the equation would be changed to:

$$f' = f_0 \left(\frac{c}{c + v_s} \right)$$

When combined with the previous result, the equation would be expressed as:

$$f' = f_0 \left(\frac{c}{c \mp v_s} \right)$$

Notice that this time, the plus/minus symbol is inverted because the sign on top is to be used for relative motion of the source **towards** the receiver.

Doppler Equation

By combining the previous results, we can derive one equation to use as the Doppler Equation. This is usually expressed as:

$$f' = f_0 \left(\frac{c \pm v_r}{c \mp v_s} \right)$$

The student must be careful that the quantities for the velocity of the receiver, v_r , and the velocity of the source, v_s , are only the **magnitudes** of the relative velocities **in (or along) the LOS**. In other words, the component of the velocity of the source and the receiver, that are perpendicular to the LOS do not change the received frequency. Secondly, the top sign in the numerator and the denominator are the sign convention to be used when the relative velocities are **towards** the other. If the source were moving towards the receiver, the sign to use in the denominator would be the minus sign. If the source were moving away from the receiver, the sign to use would be the plus sign.

Active Sonar Problem

One interesting Doppler problem is the active sonar problem. In this problem, one must define a “source” and “receiver” for both the outgoing active pulse and the returning signal.

For the outgoing active pulse, the Doppler shifted frequency of the active pulse when it hits the target would be:

$$f' = f_0 \left(\frac{c \pm v_r}{c \mp v_s} \right)$$

For the return pulse, there would be a similar shift but now the “source” would be the target, the “receiver” would be the ship sending out the original active pulse and the base frequency, f_0 would be the Doppler shifted frequency from above. If we redefine the velocity of the target as, v_t , and the velocity of the source of the active pulse as v_s , we get:

$$f'' = f' \left(\frac{c \pm v_s}{c \mp v_t} \right)$$

Substituting in the equation from above for f' and changing the subscripts for the appropriate terms:

$$f'' = f_0 \left(\frac{c \pm v_t}{c \mp v_s} \right) \left(\frac{c \pm v_s}{c \mp v_t} \right)$$

Again, the velocities are only the magnitudes of the velocity **in the LOS** and one must take care to pick the correct sign to use in front of each velocity.

Problems:

1. A surface ship is traveling on a course of $045^\circ T$ and is conducting an active sonar search. The sonar frequency is 4 kHz. At the same time a submarine is located on a bearing of $060^\circ T$ from the surface ship. The submarine is traveling due north ($000^\circ \tau$) at a speed of 10 kts. The surface ship is traveling at 15 kts. (Useful information: $c=2950$ kts)
 - a) What is the frequency that the submarine receives?
 - b) What is the frequency of the active return from the submarine?
 - c) If the submarine emits a strong tonal at 415 Hz, what is the frequency received at the surface ship?
 - d) Assuming that there is a large school of shrimp near the submarine, what is the frequency of the active return from the school of shrimp? (Assume that the shrimp are not moving.)

2. A US submarine is trailing a new Afghani diesel submarine to gain tonal intelligence on the Afghani sub. Both subs were going 5 knots on course north when the Afghani sub turns due west.
 - a) If the tonal the US sub was originally tracking was 250Hz, what is the received frequency after the Afghani sub turns?
 - b) If the US sub goes active with a frequency of 18,000 Hz to get an exact range on the Afghani sub, what is the frequency of the received return?

3. A sound source emits a sound frequency of 1000 Hz on a day when the speed of sound in air is 340 m/s and there is no wind. What is the frequency you will receive if:
 - a) You move toward the source at 34 m/s?
 - b) You are stationary and the source moves towards you at 34 m/s?
 - c) Repeat part a) with a speed of 68 m/s instead of 34 m/s.
 - d) Repeat part b) with a speed of 68 m/s instead of 34 m/s.

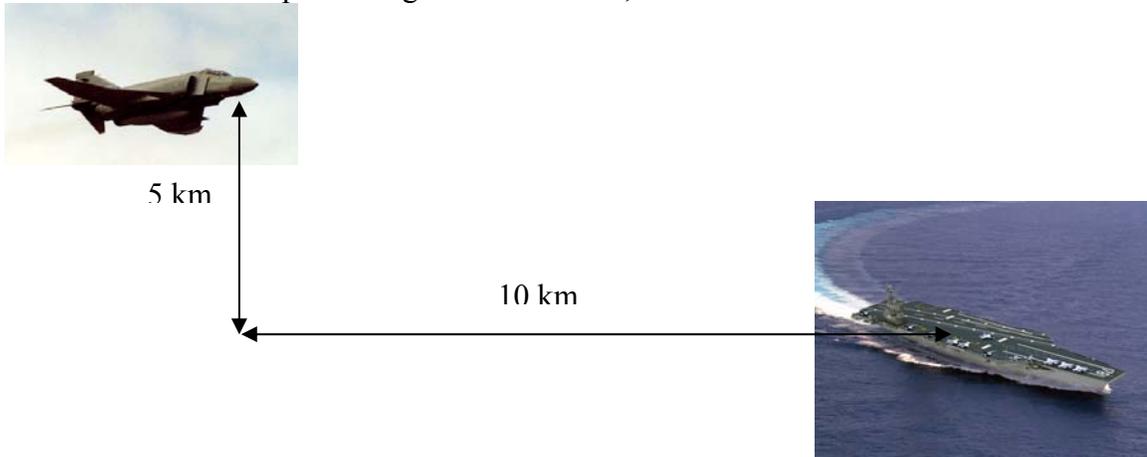
4. Two submarines are moving as shown in the figure, where the speeds are in knots. The speed of sound in knots is 2912 kts. Sub A is pinging on B with an active sonar frequency of 10 kHz.



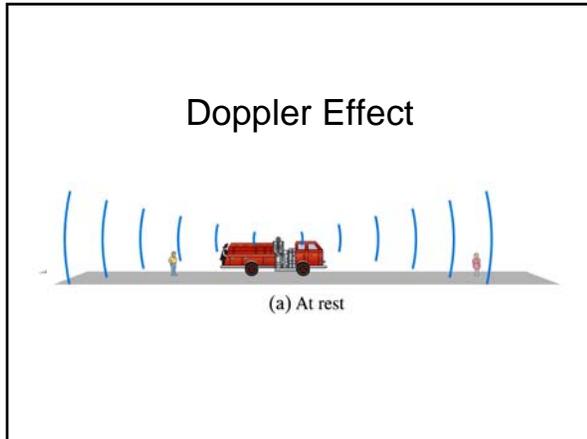
- a) What frequency will B receive from A's sonar?
 - b) What is the frequency of the echo A receives from B?
5. Ship S is on course $045^\circ T$ with speed 30 kts. Target T is on course $330^\circ T$ with speed 10 kts. S uses active sonar to ping on T with frequency 10 kHz. The speed of sound is 3000 kts. When T is due east of S (as shown in the below sketch):



- a) What frequency will T receive from S?
 - b) What will be the echo frequency that S receives back from T?
 - c) What is the frequency of the reverberation received by S?
6. A phantom jet flying at an altitude of 5 km is directly behind and closing at a horizontal range of 10 km from the carrier. The jet is tracking the ship with an active radar unit of source frequency 400 MHz. The jet's speed is 200 m/s parallel to the ground. The ship cruises at 10 m/s. If the speed of light is 3×10^8 m/s,



- a) Compute the radar frequency detected by the aircraft carrier.
- b) Compute the echo frequency detected by the jet.
- c) Compute the Doppler shift between the echo and source frequency.



Doppler Effect – 4 cases

- Source moving toward receiver
- Source moving away from receiver
- Receiver (observer) moving towards source
- Receiver (observer) moving away from source.

Source moving case

(a)

(b)

$\lambda' = \lambda - v_s T$

Away: $c = \frac{\lambda}{T}$ $\lambda' = \lambda - v_s \frac{\lambda}{c} = \lambda \left(1 - \frac{v_s}{c}\right)$

Towards: $f' = f \left(\frac{1}{1 + \frac{v_s}{c}} \right)$ $f' = \frac{c}{\lambda'} = \frac{c}{\lambda \left(1 - \frac{v_s}{c}\right)} = f \left(\frac{1}{1 - \frac{v_s}{c}} \right)$

Receiver (observer) moving case

Towards:

$$f' = \frac{c + v_r}{\lambda} = f \left(1 + \frac{v_r}{c} \right)$$

Away:

$$f' = \frac{c - v_r}{\lambda} = f \left(1 - \frac{v_r}{c} \right)$$

Source and receiver moving

$$f' = f \left(\frac{1 \pm \frac{v_r}{c}}{1 \mp \frac{v_s}{c}} \right) = f \left(\frac{c \pm v_r}{c \mp v_s} \right)$$

- Numerator – Receiver (observer)
 - Toward +
 - Away -
- Denominator – Source
 - Toward -
 - Away +

Doppler Example

- Intelligence tells you that a particular piece of machinery in the engine room of a Soviet Victor III submarine emits a frequency of 320 Hz. Your sonar operator hears the machinery but reports the frequency is 325 Hz. Assume you have slowed to a negligible speed in order to better hear the Russian.
 - Is the VIII coming toward you or moving away from you?
 - Assuming the Victor is either moving directly toward or away from you, what is his speed in m/s?

Lesson 19

Line of sight diagrams

$f' = f \left(\frac{1 \pm \frac{v_r}{c}}{1 \mp \frac{v_s}{c}} \right) = f \left(\frac{c \pm v_r}{c \mp v_s} \right)$

$v_s = 12 \text{ kts} \sin 30 = 6 \text{ kts}$

$v_r = 10 \text{ kts} \sin 45 = 7.07 \text{ kts}$

$1 \text{ knot} = 0.5144 \text{ m/s}$

Active Case

$f_B = f_A \left(\frac{1 + \frac{v_r}{c}}{1 - \frac{v_s}{c}} \right)$

$\Delta f = 2f_A \left(\frac{v_r + v_s}{c} \right)$

$f_{B \rightarrow A} = f_B \left(\frac{1 + \frac{v_s}{c}}{1 - \frac{v_r}{c}} \right) = f_A \left(\frac{1 + \frac{v_r}{c}}{1 - \frac{v_s}{c}} \right) \left(\frac{1 + \frac{v_s}{c}}{1 - \frac{v_r}{c}} \right) = f_A + 2f_A \left(\frac{v_r + v_s}{c} \right)$

Example

- A ship moving north at 20 kts pings on a target (that bears 030 and is) moving east at 10 kts. The source frequency is 4000 Hz and the sound speed is 3000 kts.
 - What is the frequency received by the target?
 - What is the echo frequency received by the ship?
 - What is the doppler shift of the echo received by the ship relative to the ship's source?
 - What is the frequency heard by reverberation near the target
 - What is the echo frequency received by the ship from reverberation?
 - What is the doppler shift of the echo received by the ship from reverberation relative to the ships source?