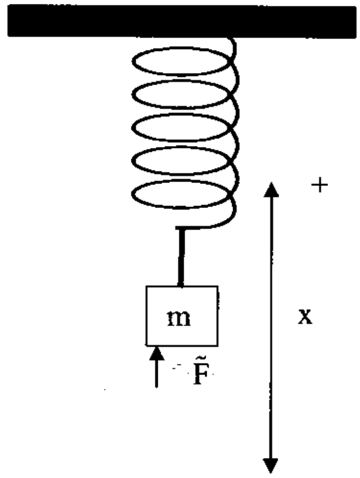
Name $\underline{\quad KEY \quad}$ SP436 Exam I



1. (35 points) A 2.00 kg mass on the end of an ideal spring (no damping) with spring constant 72.0 N/m is driven by a force:

$$\tilde{F} = Fe^{j\omega t} = (10.0\ \mathrm{N})e^{j\left(15.0\frac{\mathrm{rad}}{\mathrm{sec}}\right)t}$$

a. (2) What is the driving frequency? What is the period?

$$\omega = 15\ r/s \qquad f = \frac{15\ r/s}{2\pi} = 2.39\ \mathrm{Hz}$$

$$T = \frac{1}{f} = \frac{1}{2.39\,\mathrm{Hz}} = .419\ s$$

b. (4) What is the equation of motion for this system?

$$m\frac{\partial^2 \tilde{x}}{\partial t^2} + s\tilde{x} = Fe^{j\omega t}$$

$$(2\,kg)\,\ddot{\tilde{x}} + (72\,\tfrac{N}{m})\,\tilde{x} = 10N\,e^{j(15\frac{r}{s})t}$$

c. (3) What is the numerical value of the complex impedance for this system at the frequency of the driver?

$$\tilde{Z}_m = j\left(\omega m - \frac{s}{\omega}\right) = j\left(15\tfrac{r}{s}\cdot 2\,kg - \frac{72\ N/m}{15\ r/s}\right) = j\ 25.2\ kg/s$$

d. (3) What is the numerical value of the magnitude and phase of the complex impedance at this frequency?

$$|\tilde{Z}_m| = 25.2\ kg/s$$

$$\Theta = \frac{\pi}{2}$$

e. (3) What is the steady state complex particle velocity?

$$\tilde{u}_{ss} = \frac{\tilde{f}}{\tilde{Z}_m} = \frac{10N\ e^{j(15\,r/s)t}}{j\ 25.2\ kg/s} = .397(-j)\,e^{j(15\,r/s)t}\ m/s$$

f. (3) What is the steady state complex position? What is the real part?

$$\tilde{x}_{ss} = \frac{1}{j15\frac{r}{s}}\left((.397\tfrac{m}{s})(-j)\right)e^{j(15\,r/s)t} = -.0265\,m\ e^{j(15\,r/s)t}$$

$$Re(\tilde{x}_{ss}) = -.0265\,m\ \cos[(15\,r/s)t]$$

g. (2) What is natural frequency of the system?

$$\omega_0 = \sqrt{\frac{s}{m}} = \sqrt{\frac{72\ N/m}{2\,kg}} = \sqrt{36\ \frac{m/s^2}{m}} = 6\ \frac{rad}{s}$$

h. (3) What is the real part of the transient solution for the position of the mass? Remember, you must have two arbitrary constants in this solution.

$$x_{trans} = A\cos(\omega_0 t + \phi) = A\cos\left(6\tfrac{r}{s}t + \phi\right)$$

i. (3) What is the real part of the complete solution for the position of the mass at any time?

$$x = -2.65\times10^{-2}\,m\ \cos\left[\left(15\,r/s\right)t\right] + A\cos\left(6\tfrac{r}{s}t + \phi\right)$$

j (3)What is the real part of the complete solution for the velocity of the mass at any time?

$$u = (2.65\,cm)(15\,r/s)\sin(15\,r/s\,t) - A(6\,r/s)\sin(6\,r/s\,t + \phi)$$

$$= .397\sin\left(15\tfrac{r}{s}t\right) - A\left(6\tfrac{r}{s}\right)\sin\left(6\tfrac{r}{s}t + \phi\right)$$

j. (6) At t = 0 sec, the mass is at rest at +0.030 m. Evaluate the arbitrary constants.

$$u(0) = 0 - A(6\,r/s)\sin\phi \quad\Rightarrow\quad \boxed{\phi = 0}, \pi, 2\pi \ldots$$

$$x(0) = +.030\,m = -2.65\,cm + A\cos(0)$$

$$A = 5.65\,cm$$

$$= \boxed{.0565\ m}$$

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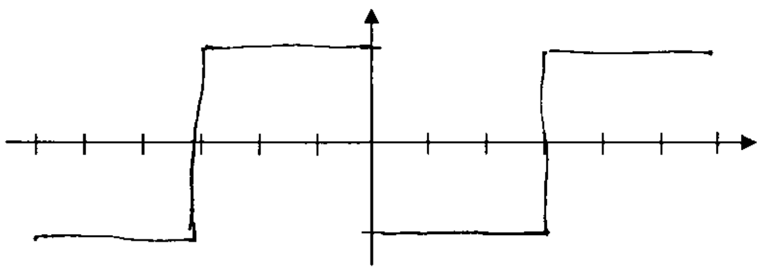
2. (20 points) A square wave periodically repeating electrical signal is sent to the oscilloscope according to the following mathematical pattern:

$$v(t) = \begin{cases} +1\text{v} & \text{for } -3\text{s} \le t \le 0\text{s} \\ -1\text{v} & \text{for } 0\text{s} \le t \le 3\text{s} \end{cases}$$

a. (3) Draw and label this voltage pattern from -6 s to +6 s.



b. (3) What is the period and fundamental angular frequency, ω?

$$T = 6\,s \qquad \omega = \frac{2\pi}{6s} = \frac{\pi}{3}\ \text{r/s} = 1.05\ \text{r/s}$$

b. (3) Is this signal odd or even? How does this affect your expected Fourier coefficients?

$$\text{ODD} \Rightarrow A_n = 0$$

c. (3) What is the average value of the signal. How does this affect your expected Fourier coefficients?

$$\langle v(t) \rangle = 0 \Rightarrow A_o = 0$$

d. (5) What are the Fourier coefficients A_o, A_n, and B_n ?

$$B_n = \frac{2}{T}\int_0^T v(t)\sin n\omega t\,dt = \frac{2}{6s}\left[\int_0^{3s}(-1)\sin n\left(\frac{\pi}{3}\frac{r}{s}\right)t\,dt + \int_{3s}^{6s}(1)\sin n\left(\frac{\pi}{3}\right)t\,dt\right]$$

$$= \frac{1}{3s}\left[\left.\frac{\cos\left(n\frac{\pi}{3}t\right)}{n\frac{\pi}{3s}}\right|_{0s}^{3s} + \left.\frac{-\cos\left(n\frac{\pi}{3}t\right)}{n\frac{\pi}{3s}}\right|_{3s}^{6s}\right]$$

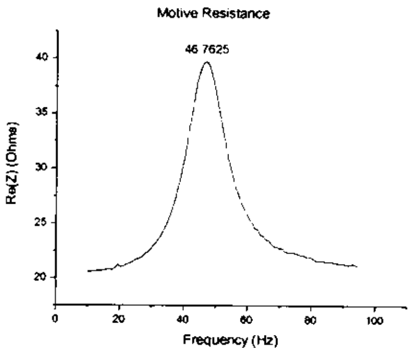
$$= \frac{1}{\cancel{3s}}\left[\frac{\cos n\pi - 1}{n\frac{\pi}{\cancel{3s}}} + \frac{-\cos n2\pi + \cos n\pi}{n\frac{\pi}{\cancel{3s}}}\right] = \frac{2}{n\pi}\left(\cos n\pi - 1\right)$$

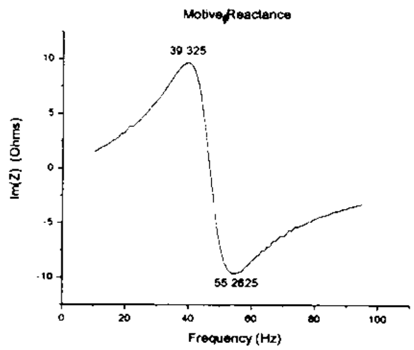
$$B_n = 0 \quad n = \text{EVEN} \qquad B_n = \frac{-4}{n\pi} \quad n = \text{ODD}$$

e. (3) Write the first four lowest frequency terms in the Fourier series that approximates this signal.

$$v(t) = -\frac{4}{\pi}\sin\frac{\pi}{3}t - \frac{4}{3\pi}\sin\pi t - \frac{4}{5\pi}\sin\frac{5\pi}{3}t - \frac{4}{7\pi}\sin\frac{7\pi}{3}t$$

3. (4 points) A laboratory study of the impedance of a system (in this case a loudspeaker) shows the below results. We have made the case that the upper and lower half power points are near to the extremes of the imaginary part of the complex impedance.



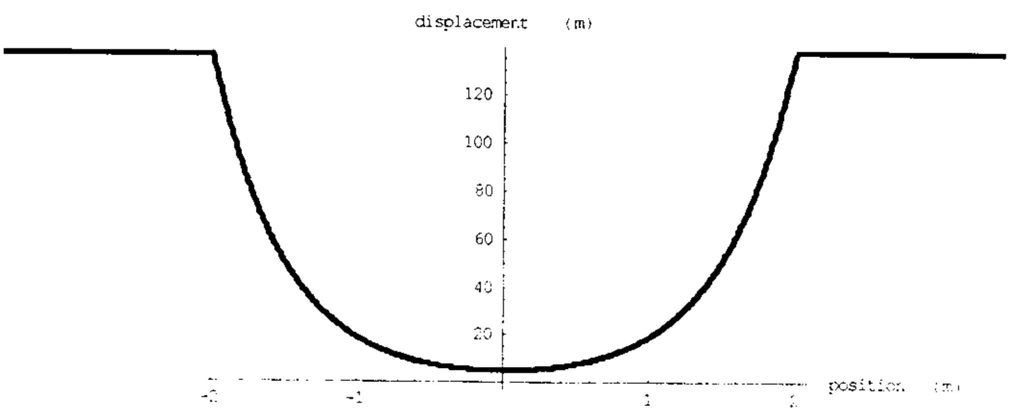


From this data, estimate the resonance Q factor for this system.

$\omega_0 = 46.76 \, r/s$ $\Delta\omega = (55.26 - 39.32) r/s = 15.94 \, r/s$

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{46.76 \, r/s}{15.94 \, r/s} = \boxed{2.93}$$

4. (6 points) The transverse wave equation for a wave traveling on a string is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.
Show by direct substitution that $y = \cosh[\omega t - kx]$ is a solution to the wave equation.



$y = \cosh(\omega t - kx)$

$\frac{\partial y}{\partial t} = \omega \sinh(\omega t - kx)$ $\frac{\partial y}{\partial x} = -k \sinh(\omega t - kx)$

$\frac{\partial^2 y}{\partial t^2} = \omega^2 \cosh(\omega t - kx)$ $\frac{\partial^2 y}{\partial x^2} = (-k)^2 \cosh(\omega t - kx)$

$$(-k)^2 \cosh(\omega t - kx) = \frac{1}{c^2}\omega^2 \cosh(\omega t - kx)$$

$$k^2 = \frac{\omega^2}{c^2} = k^2 \checkmark$$