Homework Assignment # 3

1. KFCS 2.4.1

2. KFCS 2.8.1, plus the following:
   (c) What is the particle acceleration of the element at $x = 0$ at $t = 0$?

3. KFCS 2.9.3

4. KFCS 2.9.4

5. KFCS 2.10.1

6. KFCS 2.11.2

7. KFCS 2.12.2

WAVE EQUATION SOLUTIONS 2.4.1

Note Title 9/20/2010

SHOW: $f_1(x-ct)$, $\ln[a(ct-x)]$, $a(ct-x)^2$ & $\cos[a(ct-x)]$ ARE SOLUTIONS

$a(ct-x^2)$ & $at(ct-x)$ ARE NOT SOLUTIONS

a. $y = f_1(x-ct)$ $\qquad u = x-ct$

$$\frac{\partial y}{\partial t} = \frac{\partial f_1}{\partial u}\frac{\partial u}{\partial t} = \frac{\partial f_1}{\partial u}(-c)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left(-c\frac{\partial f_1}{\partial u}\right) = \frac{\partial}{\partial u}\left(-c\frac{\partial f_1}{\partial u}\right)\frac{\partial u}{\partial t} = (\cancel{-}c)^2\frac{\partial^2 f_1}{\partial u^2}$$

$$\frac{\partial y}{\partial x} = \frac{\partial f_1}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial f_1}{\partial u}(1)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f_1}{\partial u}\right) = \frac{\partial}{\partial u}\left(\frac{\partial f_1}{\partial u}\right)\frac{\partial u}{\partial x} = \frac{\partial^2 f_1}{\partial u^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \underline{\underline{\frac{\partial^2 f_1}{\partial u^2}}} \overset{?}{=} \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{\cancel{c^2}}\cancel{c^2}\underline{\underline{\frac{\partial^2 f_1}{\partial u^2}}} \quad \text{Q.E.D.}$$

b. $y = \ln[a(ct-x)]$

$$\frac{\partial y}{\partial t} = [a(ct-x)]^{-1}ac$$

$$\frac{\partial^2 y}{\partial t^2} = -[\cancel{a}(ct-x)]^{-2}(\cancel{a}c)^2 = -(ct-x)^{-2}c^2$$

$$\frac{\partial y}{\partial x} = \left[a(ct-x)\right]^{-1}(-a)$$

$$\frac{\partial^2 y}{\partial x^2} = -\left[\cancel{a}(ct-x)\right]^{-2}(-\cancel{a})^2 = -(ct-x)^{-2}$$

$$\frac{\partial^2 y}{\partial x^2} = \underline{\underline{-(ct-x)^{-2}}} \stackrel{?}{=} \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{\cancel{c^2}}\left[\underline{\underline{-(ct-x)^2}}\,\cancel{c^2}\right]$$

Q.E.D.

c. $y = a(ct-x)^2$

$$\frac{\partial y}{\partial t} = 2a(ct-x)c \qquad \frac{\partial y}{\partial x} = 2a(ct-x)(-1)$$

$$\frac{\partial^2 y}{\partial t^2} = 2ac^2 \qquad \frac{\partial^2 y}{\partial x} = 2a(-1)^2 = 2a$$

$$\frac{\partial^2 y}{\partial x^2} = \underline{\underline{2a}} \stackrel{?}{=} \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{\cancel{c^2}}\,\underline{\underline{2a}}\,\cancel{c^2} \quad \text{Q.E.D.}$$

d. $y = \cos\left[a(ct-x)\right]$

$$\frac{\partial y}{\partial t} = -ac\sin\left[a(ct-x)\right] \qquad \frac{\partial y}{\partial x} = +a\sin\left[a(ct-x)\right]$$

$$\frac{\partial^2 y}{\partial t^2} = -(ac)^2\cos\left[a(ct-x)\right] \qquad \frac{\partial^2 y}{\partial x^2} = -a^2\cos\left[a(ct-x)\right]$$

$$\frac{\partial^2 y}{\partial x^2} = -a^2\underline{\underline{\cos\left[a(ct-x)\right]}} \stackrel{?}{=} \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = -\frac{1}{\cancel{c^2}}\underline{\underline{(a\cancel{c})^2\cos\left[a(ct-x)\right]}}$$

Q.E.D.

e. $y = a(ct - x^2)$

$\frac{\partial y}{\partial t} = ac$ $\qquad$ $\frac{\partial y}{\partial x} = -a2x$

$\frac{\partial^2 y}{\partial t} = 0$ $\qquad$ $\frac{\partial^2 y}{\partial x} = -2a$

$$\frac{\partial^2 y}{\partial x^2} = -2a \overset{?}{\neq} \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{c^2}(0)$$ <u>No</u> Q.E.D.

f. $y = at(ct - x)$

$\frac{\partial y}{\partial t} = 2act - ax$ $\qquad$ $\frac{\partial y}{\partial x} = -at$

$\frac{\partial^2 y}{\partial t^2} = 2ac$ $\qquad$ $\frac{\partial^2 y}{\partial x^2} = 0$

$$\frac{\partial^2 y}{\partial x^2} = 0 \overset{?}{\neq} \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{c^{\cancel{2}}} 2a\cancel{c}$$ <u>No</u> Q.E.D.

TRAVELING WAVE 2.8.1

Note Title 9/29/2010

GIVEN: $y(x,t) = 4\cos(3t - 2x)$ $\quad y, x \to [cm]$

$\rho_L = 0.1\ kg/m$ $\quad t \to [s]$

FIND: $A,\ c,\ f,\ \lambda,\ k,\ u(x=0, t=0),$

$\frac{\partial u}{\partial t}(x=0, t=0)$

$$\boxed{A = 4\,cm}$$

$$c = \frac{\omega}{k} = \frac{3\,s^{-1}}{2\,cm^{-1}} = \boxed{1.5\ cm/s}$$

$$f = \frac{\omega}{2\pi} = \frac{3\,s^{-1}}{2\pi} = \boxed{0.477\ Hz}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\,cm^{-1}} = \boxed{\pi\,cm}$$

$$k = \boxed{2\ cm^{-1}}$$

$$u = \frac{\partial y}{\partial t} = -(4\,cm)(3\,s^{-1})\sin(3t - 2x)$$

$$= -12\,cm/s\ \sin(3t - 2x)\Big|_{\substack{x=0\\t=0}} = \boxed{0}$$

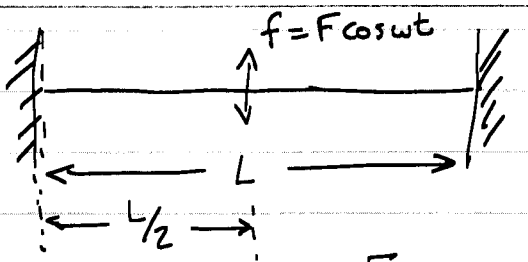
$$\frac{\partial u}{\partial t} = (-12\,cm/s)(3\,s^{-1})\cos(3t - 2x)$$

$$= -36\,\frac{cm}{s^2}\cos(3t - 2x)\Big|_{\substack{x=0\\t=0}} = \boxed{-36\,\frac{cm}{s^2}}$$

MIDPOINT DRIVEN STRING 2.9.3

Note Title 10/29/2010

GIVEN:



FIND: Z_{m_0}, $y\left(\frac{L}{2}\right) = \frac{F}{2kT}\tan\frac{kL}{2}$, $y\left(\frac{L}{4}\right)$

DECIDE ON A COORDINATE SYSTEM! (& STICK WITH IT)

$$\tilde{y} = \tilde{A}e^{j(\omega t - kx)} + \tilde{B}e^{j(\omega t + kx)}$$

$$\tilde{y}(x=0) = 0 = \tilde{A}e^{j\omega t} + \tilde{B}e^{j\omega t} \Rightarrow \tilde{A} = -\tilde{B}$$

$$\begin{aligned}\tilde{y}(x=L) = 0 &= \tilde{A}e^{j(\omega t - kL)} + \tilde{B}e^{j(\omega t + kL)} \\ &= \tilde{A}e^{j(\omega t - kL)} - \tilde{A}e^{j(\omega t + kL)} \\ &= \tilde{A}e^{j\omega t}\left[e^{-jkL} - e^{jkL}\right] \\ &= -\tilde{A}e^{j\omega t}\left[2j\sin kL\right]\end{aligned}$$

$$\therefore \quad kL = n\pi \quad \Rightarrow \quad k_n = \frac{n\pi}{L}$$

$$\begin{aligned}y &= \tilde{A}e^{j(\omega t - kx)} - \tilde{A}e^{j(\omega t + kx)} \\ &= \tilde{A}e^{j\omega t}\left[e^{-jkx} - e^{jkx}\right] = \tilde{A}e^{j\omega t}\left(-2j\sin kx\right)\end{aligned}$$

$$\frac{\partial y}{\partial x} = -2\tilde{A}jk\,e^{j\omega t}\cos kx$$

$$\tilde{f} + 2T\frac{\partial \tilde{y}}{\partial x} = 0 \qquad \text{AT } x = \frac{L}{2}$$

$$F\cancel{e^{j\omega t}} + 2\rho_L c^2\left(-2\tilde{A}jk\right)\cancel{e^{j\omega t}}\cos kx = 0$$

$$F = j4\tilde{A}\rho_L c^2 k\cos\left(k\tfrac{L}{2}\right)$$

$$\tilde{A} = \frac{F}{j4\rho_L c^2 k\cos\left(k\frac{L}{2}\right)} = -\tilde{B}$$

$$\tilde{y} = \frac{-\cancel{2}\cancel{j}Fe^{j\omega t}\sin kx}{\cancel{j}\cancel{4}_{2}\rho_L c^2 k\cos\left(k\frac{L}{2}\right)} = \boxed{-\frac{Fe^{j\omega t}\sin kx}{2k\rho_L c^2\cos\left(k\frac{L}{2}\right)}}$$

$$\tilde{u} = \frac{\partial\tilde{y}}{\partial t} = \frac{-j\cancel{\omega}Fe^{j\omega t}\sin kx}{2\cancel{k}\rho_L c^{\cancel{2}}\cos\left(k\frac{L}{2}\right)} = \frac{-jFe^{j\omega t}\sin kx}{2\rho_L c\cos k\frac{L}{2}}$$

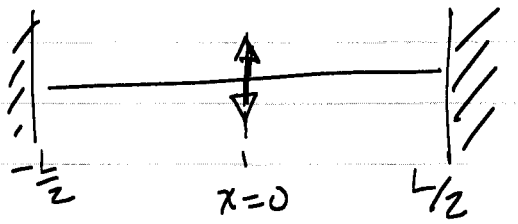
$$\tilde{Z}_{m\frac{L}{2}} = \left.\frac{\tilde{f}}{\tilde{u}}\right|_{x=\frac{L}{2}} = \left.\frac{\cancel{Fe^{j\omega t}}\,2\rho_L c\cos\left(\frac{kL}{2}\right)}{-j\cancel{Fe^{j\omega t}}\sin kx}\right|_{x=\frac{L}{2}}$$

$$= 2\rho_L c\,\frac{\cos\frac{kL}{2}}{-j\sin\frac{kL}{2}} = \boxed{2j\rho_L c\cot\frac{kL}{2}}$$

$$y\left(\frac{L}{2}\right) = \frac{-Fe^{j\omega t}\sin k\frac{L}{2}}{2k(\rho_L c^2)\cos\frac{kL}{2}} = \underbrace{\frac{-F}{2kT}\tan\frac{kL}{2}}_{\text{DISPLACEMENT AMPL (Q.E.D)}}e^{j\omega t}$$

$$y\left(\frac{L}{4}\right) = \frac{-Fe^{j\omega t}\sin k\frac{L}{4}}{2k(\rho_L c^2)\cos k\frac{L}{2}} = \underbrace{-\frac{F}{2kT}\,\frac{\sin k\frac{L}{4}}{\cos k\frac{L}{4}}}_{\text{DISPL. AMPLITUDE}}e^{j\omega t}$$

THE PROBLEM CAN ALSO BE WORKED WITH A DIFF. COORDINATE SYSTEM (& THE RESULTS SHOULD BE THE SAME)



$$\tilde{y} = \tilde{A} e^{j(\omega t - kx)} + \tilde{B} e^{j(\omega t + kx)}$$

$$\tilde{y}\left(-\frac{L}{2}\right) = 0 = \left(\tilde{A} e^{+jk\frac{L}{2}} + \tilde{B} e^{-jk\frac{L}{2}}\right) e^{j\omega t}$$

$$\tilde{B} = -\tilde{A} \frac{e^{+jk\frac{L}{2}}}{e^{-jk\frac{L}{2}}} = -\tilde{A} e^{+jkL}$$

$$\tilde{y} = e^{j\omega t}\left[\tilde{A} e^{-jkx} - \tilde{A} e^{+jkL} e^{+jkx}\right]$$

$$= \tilde{A} e^{j\omega t}\left[e^{-jk\frac{L}{2}} e^{-jkx} - e^{+jk\frac{L}{2}} e^{jkx}\right] e^{+jk\frac{L}{2}}$$

$$= \tilde{A} e^{j\omega t}\left[e^{-jk\left(\frac{L}{2}+x\right)} - e^{+jk\left(\frac{L}{2}+x\right)}\right] e^{+jk\frac{L}{2}}$$

$$= -\tilde{A} e^{j\omega t}\left(2j \sin\left(k\left(\frac{L}{2}+x\right)\right)\right) e^{+jk\frac{L}{2}}$$

$$\frac{\partial \tilde{y}}{\partial x} = -2j\tilde{A} e^{j\omega t}(k)\cos\left(k\left(\frac{L}{2}+x\right)\right) e^{+jk\frac{L}{2}}$$

$$f + 2T \left.\frac{\partial \tilde{y}}{\partial x}\right|_{x=0} = 0$$

$$0 = F\cancel{e^{j\omega t}} - 2\rho_L c^2 k\, 2j\, \tilde{A}\, \cancel{e^{j\omega t}}\, e^{+jk\frac{L}{2}} \cos\left(\frac{kL}{2}\right)$$

$$\tilde{A} = \frac{F e^{-jk\frac{L}{2}}}{4j\rho_L c^2 k \cos\left(\frac{kL}{2}\right)}$$

$$\tilde{y} = \frac{-F\cancel{e^{-jk\frac{L}{2}}}\, e^{j\omega t}\, \cancel{2j} \sin\left(k\left(\frac{L}{2}+x\right)\right) \cancel{e^{+jk\frac{L}{2}}}}{\cancel{4j}\, 2\rho_L c^2 k \cos\left(k\frac{L}{2}\right)}$$

$$= -\frac{F}{2\rho_L c^2 k} \frac{\sin\left(k\left(\frac{L}{2}+x\right)\right)}{\cos\frac{kL}{2}} e^{j\omega t}$$

$$\tilde{u} = -\frac{j\omega F}{2\rho_L c^2 k} \frac{\sin\left(k\left(\frac{L}{2}+x\right)\right)}{\cos\frac{kL}{2}} e^{j\omega t}$$

$$\tilde{Z} = \frac{\tilde{f}}{\tilde{u}} = \frac{2\rho_L c^{\cancel{2}} \cancel{k} \cos\frac{kL}{2}}{-j\cancel{\omega} \sin\frac{kL}{2}} = \boxed{2j\rho_L c \cot\frac{kL}{2}}$$

$$y(0) = -\frac{F}{2kT} \frac{\sin k\frac{L}{2}}{\cos\frac{kL}{2}} e^{j\omega t} = \underline{\underline{-\frac{F}{2kT} \tan\frac{kL}{2}}}\, e^{j\omega t}$$

$$y\left(-\frac{L}{4}\right) = \underline{\underline{-\frac{F}{2kT} \frac{\sin\frac{kL}{4}}{\cos\frac{kL}{2}}}}\, e^{j\omega t}$$

STANDING WAVE ON A STRING 2.9.4.

Note Title 9/29/2010

GIVEN: $\rho_\ell = 0.01\ kg/m$ $T = 5N$

$L = .44m$ DIST BETWEEN NODES

$y_{max} = 0.02m$ $= 0.1\ m$

FIND: $Fe^{j\omega t}$ $(F\ \&\ \omega)$

FOLLOWING DISCUSSION IN CLASS:

$$\tilde{y} = \tilde{A}e^{j(\omega t - kx)} + \tilde{B}e^{j(\omega t + kx)}$$

BC: $y(x=L) = 0$ $Fe^{j\omega t} = -T\frac{\partial \tilde{y}}{\partial x}$

$$\tilde{y}(x=L) = 0 = \left(\tilde{A}e^{-jkL} + \tilde{B}e^{jkL}\right)e^{j\omega t}$$

$$\tilde{B} = \frac{-\tilde{A}e^{-jkL}}{e^{+jkL}} = -\tilde{A}e^{-2jkL}$$

$$\tilde{y} = \left(\tilde{A}e^{-jkx} - \tilde{A}e^{-2jkL}e^{+jkx}\right)e^{j\omega t}$$

$$= \tilde{A}e^{-jkL}\left[e^{jkL}e^{-jkx} - e^{-jkL}e^{+jkx}\right]e^{j\omega t}$$

$$= \tilde{A}e^{-jkL}\left[e^{jk(L-x)} - e^{-jk(L-x)}\right]e^{j\omega t}$$

$$= \boxed{\tilde{A}e^{-jkL}\ 2j\sin\left[k(L-x)\right]e^{j\omega t}}$$

$$\frac{\partial y}{\partial x} = \tilde{A}e^{-jkL}\ 2j(-k)\cos\left[k(L-x)\right]e^{j\omega t}$$

$$F e^{j\omega t} = -T \frac{\partial \tilde{y}}{\partial x}\bigg|_{x=0} = T\tilde{A}\, 2jk e^{-jkL} \cos\left[k(L-\cancel{x})\right] e^{j\omega t}$$

$$F = \tilde{A}\, 2jkT\, e^{-jkL} \cos(kL)$$

$$\tilde{A} = \frac{F e^{jkL}}{2jkT\cos(kL)}$$

$$\tilde{y} = \frac{F \cancel{e^{jkL}}}{\cancel{2j}kT\cos(kL)} \cancel{e^{-jkL}}\, \cancel{2j} \sin\left[k(L-x)\right] e^{j\omega t}$$

$$= \frac{F \sin\left[k(L-x)\right]}{kT \cos kL} e^{j\omega t}$$

$\tilde{y} = 0$ WHEN $k(L-x) = n\pi \Rightarrow \frac{2\cancel{\pi}}{\lambda}(L-x) = n\cancel{\pi}$

(NODES)

$$L - x = \frac{n\lambda}{2} \Rightarrow x = L - \frac{n\lambda}{2}$$

$$.1m = x_1 - x_2 = \left(4 - \frac{\lambda}{2}\right) - \left(4 - \frac{2\lambda}{2}\right) = \frac{\lambda}{2}$$

$$\lambda = \boxed{.2m} \Rightarrow k = \frac{2\pi}{\lambda} = \frac{2\pi}{.2m} = \boxed{10\pi \frac{r}{m}}$$

$$c = \sqrt{\frac{T}{\rho_s}} = \sqrt{\frac{5N}{.01\,kg/m}} = \boxed{22.4\,m/s}$$

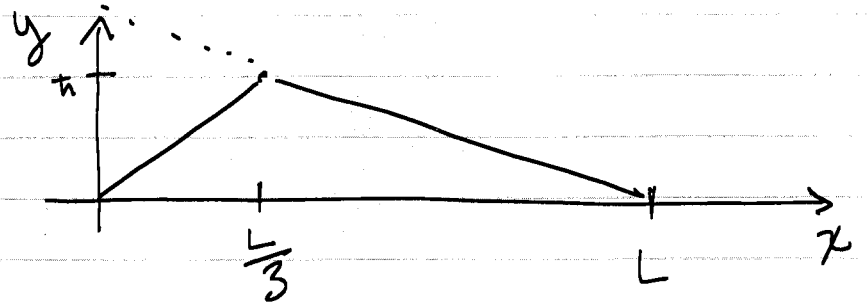
$$f = \frac{c}{\lambda} = \frac{22.4\,m/s}{.2m} = \boxed{112\,Hz} \quad \left(704\,\frac{r}{s}\right)$$

$$y_{max} = \frac{F}{kT\cos kL} = .02m$$

$$F = \left(10\pi \, r/s\right)\left(5N\right)\cos\left(10\pi\,(.44)\right)\left(.02\,s\right) = \boxed{0.97\,N}$$

PLUCKED STRING 2.10.1

Note Title 11/1/2010



$$k_n L = n\pi$$

$$k_n = \frac{n\pi}{L}$$

$$y_n(x,0) = \begin{cases} \dfrac{h}{\frac{1}{3}L}\,x = \dfrac{3h}{L}\,x \\[2ex] \dfrac{3h}{2} - \dfrac{h}{\frac{2}{3}L}\,x = \dfrac{3h}{2} - \dfrac{3h}{2L}\,x \end{cases}$$

$$u_n(x,0) = 0 \qquad (\text{AT REST}) \Rightarrow B_n = 0$$

$$y_n(x,t) = \left(A_n \cos\omega_n t + B_n \sin\omega_n t\right)\sin k_n x =$$

$$\boxed{y_n(x,0) = A_n \cancelto{1}{\cos\omega_n t}\,\sin k_n x} \qquad k_n = \frac{n\pi}{L}$$

$$A_n = \frac{2}{L}\int_0^L y_n(x,0)\sin k_n x\,dx$$

$$= \frac{2}{L}\left[\int_0^{L/3}\left(\frac{3h}{L}x\right)\sin\left(\frac{n\pi x}{L}\right)dx + \int_{L/3}^{L}\left(\frac{3h}{2} - \frac{3h}{2L}x\right)\sin\frac{n\pi x}{L}\,dx\right]$$

MATHE MATICA

$$= \frac{12h}{n^2\pi^2}\sin^3\frac{n\pi}{3} = \frac{12h}{n^2\pi^2}\sin^2\frac{n\pi}{3}\sin\frac{n\pi}{3}$$

$$= \frac{12h}{n^2\pi^2}\left(\frac{\sqrt{3}}{2}\right)^2\sin\frac{n\pi}{3} = \frac{9h}{n^2\pi^2}\sin\frac{n\pi}{3}.$$

$$A_1 = \frac{9h}{1^2\pi^2}\frac{\sqrt{3}}{2} = .790h$$

$$A_2 = \frac{9h}{2^2\pi^2}\frac{\sqrt{3}}{2} = .197h$$

$$A_3 = 0$$

$$A_4 = \frac{9h}{4^2\pi^2}\left(-\frac{\sqrt{3}}{2}\right) = -.049h$$

$$A_5 = \frac{9h}{5^2\pi^2}\left(-\frac{\sqrt{3}}{2}\right) = -.032h$$

$$A_6 = 0$$

$$y_n(x, t=0) = A_n \sin\frac{n\pi}{L}x$$

$$t = \frac{L}{c} \Rightarrow \omega t = \omega\frac{L}{c} = kL = m\pi \qquad (m=1)$$

$$y_n\left(x, t=\frac{L}{c}\right) = A_n(\cos\pi)\sin\frac{n\pi x}{L}$$

$$= -A_n \sin\frac{n\pi x}{L}$$

$$\frac{1}{L}\left(\int_{-L}^{-L/3}\left(\left(\frac{-3h}{2}-\frac{3hx}{2L}\right)\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\right)dx+\int_{-L/3}^{L/3}\left(\frac{3hx}{L}\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\right)dx+\int_{L/3}^{L}\left(\left(\frac{3h}{2}-\frac{3hx}{2L}\right)\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\right)dx\right)$$

$$\frac{-\frac{2hL\mathrm{Cos}[\frac{n\pi}{3}]}{n\pi}+\frac{6hL\mathrm{Sin}[\frac{n\pi}{3}]}{n^2\pi^2}+\frac{2hLn\pi\mathrm{Cos}[\frac{n\pi}{3}]+3hL\mathrm{Sin}[\frac{n\pi}{3}]-3hL\mathrm{Sin}[n\pi]}{n^2\pi^2}}{L}$$

$$\mathrm{Simplify}\left[\frac{1}{L}\left(\int_{-L}^{-L/3}\left(\left(\frac{-3h}{2}-\frac{3hx}{2L}\right)\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\right)dx+\int_{-L/3}^{L/3}\left(\frac{3hx}{L}\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\right)dx+\int_{L/3}^{L}\left(\left(\frac{3h}{2}-\frac{3hx}{2L}\right)\mathrm{Sin}\left[\frac{n\pi x}{L}\right]\right)dx\right)\right]$$

$$\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}$$

$$n=1;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

0.78972 h

$$n=2;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

0.19743 h

$$n=3;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

0.

$$n=4;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

-0.0493575 h

$$n=5;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

-0.0315888 h

$$n=6;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

0.

$$n=7;\ N\left[\frac{12h\,\mathrm{Sin}[\frac{n\pi}{3}]^3}{n^2\pi^2}\right]$$

0.0161167 h

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Plot[.79 Sin[π x], {x, 0, 1}]
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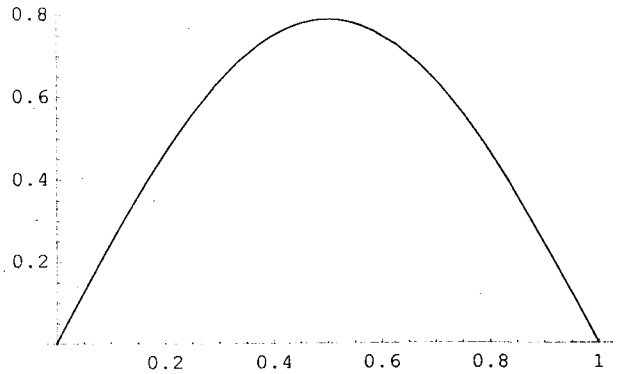

- Graphics -

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Plot[.198 Sin[2 π x], {x, 0, 1}]
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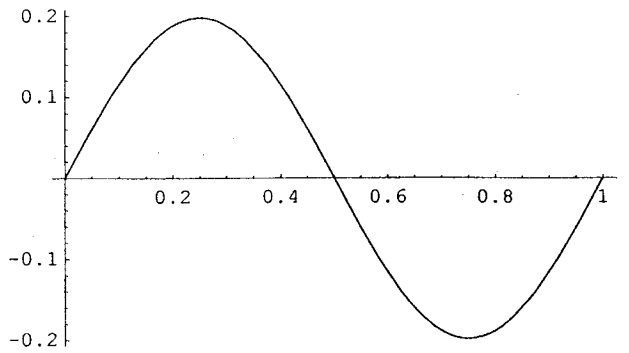

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Plot[-.049 Sin[4 π x], {x, 0, 1}]
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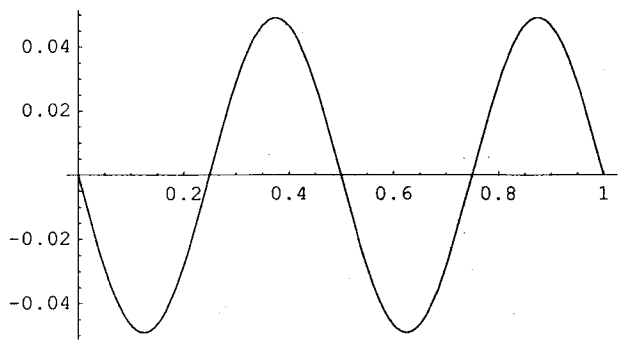

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Plot[.79 Sin[π x] + .198 Sin[2 π x] - .037 Sin[4 π x] - .0 Sin[5 π x], {x, 0, 1}]
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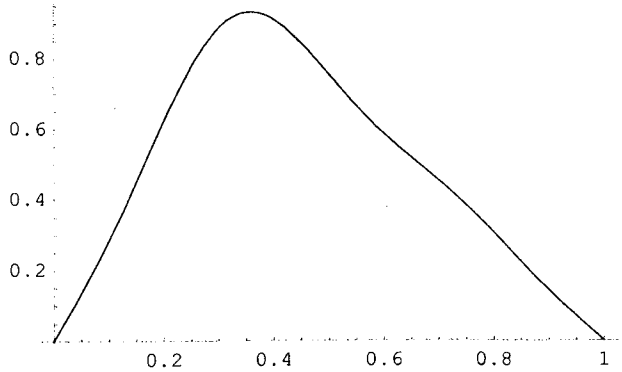

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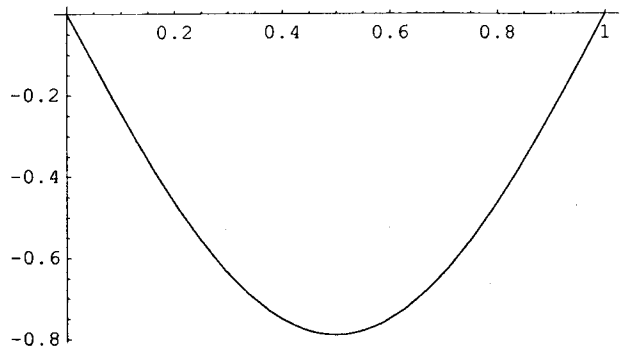
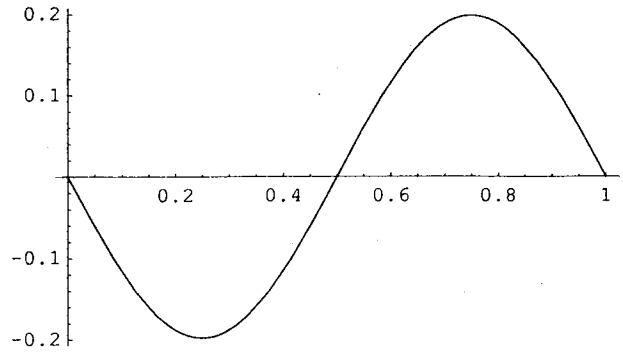
Plot[-.79 Sin[π x], {x, 0, 1}]
```



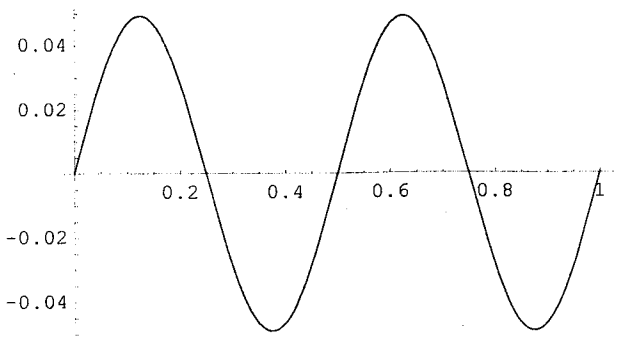
- Graphics -

```
Plot[-.198 Sin[2 π x], {x, 0, 1}]
```



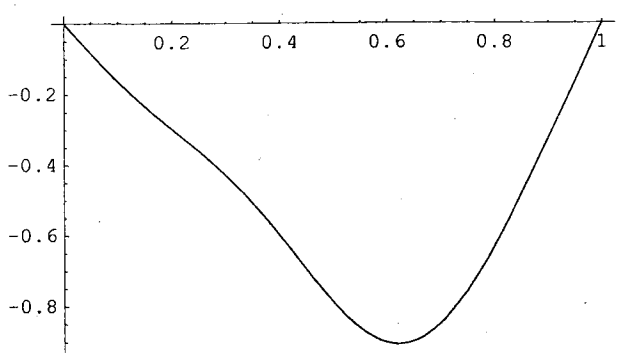
- Graphics -

```
Plot[.049 Sin[4 π x], {x, 0, 1}]
```



- Graphics -

```
Plot[-.79 Sin[π x] + .198 Sin[2 π x] - .037 Sin[4 π x] + .0 Sin[5 π x], {x, 0, 1}]
```



- Graphics -

$$n = .;\ \text{Simplify}\left[\frac{2}{L}\left(\int_0^{L/3}\left(\frac{3\,h\,x}{L}\,\text{Sin}\left[\frac{n\,\pi\,x}{L}\right]\right)dx + \int_{L/3}^{L}\left(\left(\frac{3\,h}{2} - \frac{3\,h\,x}{2\,L}\right)\text{Sin}\left[\frac{n\,\pi\,x}{L}\right]\right)dx\right)\right]$$

$$\frac{12\,h\,\text{Sin}\left[\frac{n\,\pi}{3}\right]^3}{n^2\,\pi^2}$$

VERTICAL MASS LOADED STRING 2.11.2

Note Title 9/30/2010

GIVEN: $M = 0.2\ kg$ $m_s = 0.05 kg$ $L = 1m$

FING: c, f_1, f_2, $\frac{y(\text{ANTINODE})}{y(\text{MASS})}$ IN f_2

$$T = Mg = (.2kg)(9.8\ m/s^2) = 1.96N$$

$$\rho_\ell = \frac{m_s}{L} = \frac{0.05kg}{1m} = .05\ kg/m$$

$$c = \sqrt{\frac{T}{\rho_\ell}} = \sqrt{\frac{1.96N}{.05\ kg/m}} = 6.26\ m/s$$

A. FOR A FIXED MASS LOADED STRING:

$$\cot(kL) = \frac{M}{m_s} kL = \frac{.2kg}{.05kg} kL = 4\ kL$$

$$kL = 0.4801,\ 3.2191$$

$k_1 = 0.4801\ \frac{r}{m}$ $k_2 = 3.2191\ r/m$

$\omega_1 = ck_1 = (6.26\ m/s)(.4801\ \frac{r}{m}) = 3.005\ r/s$

$\omega_2 = ck_2 = (6.26\ \frac{m}{s})(3.2191\ \frac{r}{m}) = 20.15\ r/s$

$f_1 = \frac{3.005\ r/s}{2\pi} = \boxed{.478\ Hz}$ $f_2 = \frac{20.15\ r/s}{2\pi} = \boxed{3.21\ Hz}$

$$\tilde{y} = \tilde{A} \sin kx \, e^{j\omega t}$$

FOR f_2, AT THE <u>ANTINODE</u> $\sin kx = 1$, $y_{max} = A$

AT $\underline{x = L}$, $y_{max} = A \sin kL = A \sin(3.2191)$

$$\frac{A}{|A \sin(3.2191)|} = \boxed{12.9}$$

B. FOR A DRIVEN, MASS LOADED STRING:

WE SHOWED $\tan kL = -\left(\frac{M}{m_s}\right) kL$

$$\tan kL = -\left(\frac{.2\,kg}{.05\,kg}\right) kL = -4\,kL$$

$$kL = 1.7155,\ 4.7648$$

$$k = 1.7155\ r/m,\ 4.7648\ r/m$$

$$\omega = ck = 10.739\ r/s,\ 29.8276$$

$$f = 1.71\ Hz,\ 4.74\ Hz$$

PART (b) IS MUCH HARDER IN THIS CASE BECAUSE THE TRANSVERSE DISPLACEMENT IS MORE COMPLICATED.

$$y = \frac{-F e^{j\omega t}}{\rho_\ell c \omega} \quad \frac{\cos\left[k(L-x)\right] - \frac{m\omega}{\rho_\ell c}\sin\left[k(L-x)\right]}{\frac{m\omega}{\rho_L c}\cos kL + \sin kL}$$

NODES OCCUR WHEN: $0 = \cos\left[k(L-x)\right] - \frac{m\omega}{\rho_\ell c}\sin\left[k(L-x)\right]$

ANTINODE OCCUR WHEN NUMERATOR IS A MAX

LET $u = k(L-x)$

$A = \frac{m\omega}{\rho_\ell c}$

$$\frac{\partial}{\partial u}\left(\cos u - A\sin u\right) = 0$$

$$-\sin u - A\cos u = 0$$

$$-\sin u = A\cos u$$

$$\tan u = -A = \frac{(-.2\,kg)(29.83\,\frac{m}{s})}{(.05\,\frac{kg}{m})(6.26\,m/s)}$$

$$\tan u = -19.06$$

$$u = -1.518,\ 1.623$$

$$\left(\approx \frac{\pi}{2}\right)$$

$$k(L-x) = 1.623$$

$$4.7648 - 4.7648x = 1.623$$

$$x = \frac{4.7648 - 1.623}{4.7648}\ m$$

$$= .659\ m$$

$$\frac{y(\text{ANTINODE})}{y(L)} = \frac{\cos(-1.518) - 19.06\sin(-1.518)}{\cos(kL) - 19.06\sin(kL)}$$

$$= \frac{\cos(-1.518) - 19.06\sin(-1.518)}{\cos(4.7648) - 19.06\sin(4.7648)}$$

$$= \frac{19.08}{19.08} = 1$$

ENERGY IN A FIXED-FIXED STRING 2.12.2

Note Title 11/1/2010

GIVEN:

$L = 31.4\text{ cm} = 10\pi\text{ cm}$

$\rho_L = 0.1\text{ gm/cm}$

$y = 2\sin\frac{x}{5}\cos 3t$ $\qquad y, x - [\text{cm}]$, $t - [\text{s}]$

FIND:

c, f, k

$y\left(\frac{L}{2}\right), u\left(\frac{L}{2}\right) \qquad \mathcal{E}\left(\frac{L}{2}\right), \mathcal{E}\left(\frac{L}{4}\right)$

$y\left(\frac{L}{4}\right), u\left(\frac{L}{4}\right) \qquad E_{TOT}$

$$c = \frac{\omega}{k} = \frac{3\text{ r/s}}{\frac{1}{5}\text{ r/cm}} = 15\text{ cm/s}$$

$$k = \frac{1}{5}\text{ r/cm}$$

$$f = \frac{\omega}{2\pi} = \frac{3\text{ r/s}}{2\pi\text{ rad}} = .477\text{ Hz}$$

$$y\left(\frac{L}{2}\right) = (2\text{cm})\sin\left(\frac{1}{5}\frac{\text{r}}{\text{cm}}\frac{10\pi\text{ cm}}{2}\right)\cos 3t = 0$$

$$y\left(\frac{L}{4}\right) = (2\text{cm})\sin\underbrace{\left(\frac{1}{5}\frac{\text{r}}{\text{cm}}\frac{10\pi}{4}\text{cm}\right)}_{\pi/2}\cos 3t = 2\text{cm}\cos 3t$$

$$u = \frac{\partial y}{\partial t} = -(2\text{cm})\left(3\frac{\text{r}}{\text{s}}\right)\sin\frac{x}{5}\sin 3t$$

$$= -6\frac{\text{cm}}{\text{s}}\sin\frac{x}{5}\sin 3t$$

$$u\left(\frac{L}{2}\right) = -6\frac{cm}{s}\sin\left(\frac{1}{\cancel{5}\,cm}\frac{r}{}\frac{\cancel{10}\pi\,\cancel{cm}}{\cancel{2}}\right)\sin 3t = 0$$

$$u\left(\frac{L}{4}\right) = -6\frac{cm}{s}\sin\left(\underbrace{\frac{1}{5}\frac{r}{\cancel{cm}}\frac{10\pi}{4}\cancel{cm}}_{\hookrightarrow \pi/2}\right)\sin 3t$$

$$= -6\frac{cm}{s}\sin 3t$$

$$\mathcal{E} = \frac{\partial E}{\partial x} = \frac{1}{2}\rho_L c^2\left[\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{1}{c}\frac{\partial y}{\partial t}\right)^2\right]$$

$$\frac{\partial y}{\partial x} = 2cm\left(\frac{1}{5}\frac{r}{cm}\right)\cos\frac{x}{5}\cos 3t = \frac{2}{5}\cos\frac{x}{5}\cos 3t$$

$$\mathcal{E} = \frac{1}{2}\rho_L c^2\left[\frac{4}{25}\cos^2\frac{x}{5}\cos^2 3t + \frac{1}{c^2}\left(36\frac{cm^2}{s^2}\right)\sin^2\frac{x}{5}\sin^2 3t\right]$$

$$\mathcal{E}\left(\frac{L}{2}\right) = \frac{1}{2}\rho_L c^2\left[\frac{4}{25}\cos^2\cancelto{1}{\left(\frac{1}{5}\frac{10\pi}{2}\right)}\cos^2 3t + \frac{1}{c^2}\left(36\frac{cm^2}{s^2}\right)\cancelto{0}{\sin^2\left(\frac{1}{5}\frac{10\pi}{2}\right)}\sin^2 3t\right]$$

$$= \frac{1}{2}\left(.1\frac{g}{cm}\right)\left(15\frac{cm}{s}\right)^2\frac{4}{25}\cos^2 3t$$

$$\left(\text{OR } \frac{ERG}{cm}\right) \quad = (1.8\ \text{DYNE})\cos^2 3t$$

$$\mathcal{E}\left(\frac{L}{4}\right) = \frac{1}{2}\rho_L c^2\left[\frac{4}{25}\cos^2\cancelto{0}{\left(\frac{1}{5}\frac{10\pi}{4}\right)}\cos^2 3t + \frac{1}{c^2}\left(36\frac{cm^2}{s^2}\right)\cancelto{1}{\sin^2\left(\frac{1}{5}\frac{10\pi}{4}\right)}\sin^2 3t\right]$$

$$= \frac{1}{2}\rho_L \cancel{c^2}\left[\frac{1}{\cancel{c^2}}36\frac{cm^2}{s^2}\sin^2 3t\right] = \frac{1}{2}\left(.1\frac{g}{cm}\right)\left(36\frac{cm^2}{s^2}\right)\sin^2 3t$$

$$= (1.8\ \text{DYNE})\sin^2 3t$$

THE TIME AVE $\langle \sin^2(3t) \rangle = \langle \cos^2(3t) \rangle = \frac{1}{2}$

SO ON AVERAGE, $\frac{1}{2}$ ENERGY IS POTENTIAL AN $\frac{1}{2}$ IS KINETIC. TO SIMPLIFY PICK A TIME $t=0$ WHEN ALL POTENTIAL

$$\mathcal{E} = \underbrace{\frac{1}{2}\rho_L c^2 \frac{4}{25}}\cos^2 \frac{1}{5}x$$

$$E_{TOT} = 1.8\,\text{DYNE} \int_0^{10\pi} \cos^2 \frac{x}{5}\,dx = 1.8\,\text{DYNE} \int_0^{10\pi} \left(\frac{1}{2} + \cancel{\cos\frac{2x}{5}}^{\,0}\right) dx$$

$$= 1.8\,\text{DYNE}\left(\frac{1}{2}\right) 10\pi\ \text{cm}$$

$$= 28.3\ \text{ERGS}\left(\frac{1\,\text{J}}{10^7\,\text{ERG}}\right) = \boxed{2.83\ \mu\text{J}}$$