Tabletop demonstration of multiple source interference using ultrasonic transducers

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Superposition of multiple sources, constructive and destructive interference, and array beam forming are common topics demonstrated in undergraduate acoustics. Student learning experience is enhanced by actually building a system with speakers and measuring the resulting amplitude at various angles with a microphone. This experiment is often performed in an anechoic chamber for best agreement between theory and experiment. A simple tabletop apparatus using inexpensive ultrasonic transducers yields comparable results.

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I. INTRODUCTION

Many acoustics classes have included a rotating speaker array and microphone system described by Meiners to demonstrate the interference pattern.[1] This method provides a qualitative observation of constructive and destructive interference directions. Challenges with this demonstration include the annoying loud tone in the room and modal interference from objects in the room. To obtain results agreeing quantitatively with theoretical predictions, the experiment is usually performed in an anechoic chamber. From the student's perspective the ability to visualize the experiment is diminished.

The use of ultrasonic waves to demonstrate acoustic interference was proposed by Correll in the mid 1960s.[2] Today hobbyists, security systems, robotics, and other applications have developed a market for inexpensive (~ \$5/pair) airborne ultrasonic transducers shown in Fig. 1. These piezoelectric devices have wavelengths of roughly a centimeter favoring a tabletop experiment more compact than its counterpart in the audible range. Additionally, digital oscilloscopes have made the acquisition of 20 sec of data simple and convenient. Rotating tables driven by stepper motors are common positioning systems available in many laboratories. These factors suggest a need to revisit and modify Meiners's original experiment.

The demonstration described here is simple and easy to perform on the tabletop of any physics laboratory or demonstration room. The apparatus scales to the ultrasonic wavelength much like commercially available microwave kits scale to demonstrate electromagnetic wave properties.[3] The ultrasonic transducers are simple and inexpensive, yet robust enough to handle student misuse.

II. MULTIPLE SOURCE INTERFERENCE

The interference pattern from two or more omni-directional acoustic sources in the far field is developed in standard acoustic texts.[4] The phase difference between two sources separated by a distance, d, at some distant point an angle, θ, from the line bisecting the sources is

$$\phi \approx kd \sin\theta \,. \tag{1}$$

where the path difference traveled by the sound from the two sources is approximately $d\sin\theta$ and $k=2\pi/\lambda$, is the magnitude of the wave vector.

The time averaged acoustic intensity (W/m^2) resulting from these two sources at this distant point is given by

$$\langle I \rangle = \frac{\left\langle \left(p_1 + p_2 \right)^2 \right\rangle}{\rho c} \tag{2}$$

where p_1 and p_2 are traveling pressure plane waves from the two sources in the far field, ρ is the density of the medium and c is the sound speed for the medium. This allows the definition of a directional factor or a beam pattern function that describes the normalized pressure field at any angle, θ, around the two sources,

$$H(\theta) \equiv \sqrt{\frac{\langle I(\theta) \rangle}{\langle I(\theta = 0) \rangle}} \,. \tag{3}$$

The directional factor for two sources is

$$H_2(\theta) = \left| \cos\left(\frac{\pi d \sin\theta}{\lambda} \right) \right|. \tag{4}$$

Similarly, the directional factor for an array of N identical sources is

$$H_N(\theta) = \left| \frac{1}{N} \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda} \right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda} \right)} \right|. \tag{5}$$

Solving Eq. (4), the angular locations of maxima in the beam pattern corresponding to constructive interference are given by

$$d \sin\theta = m\lambda \tag{6}$$

and minima corresponding to destructive interference are located at

$$d \sin\theta = (m + \frac{1}{2})\lambda \tag{7}$$

where m is any integer. Eqs. (6) and (7) are studied when examining the interference of two sources in a ripple tank or two slit optical interference.[5]

Eqs. (4) and (5) were developed as the beam pattern function around transmitting sources. The equations are equally valid to describe the beam pattern function around two passive omni-directional receivers. The apparatus described in the next section can be modified to demonstrate either case.

III. APPARATUS

The essential elements of this demonstration are small inexpensive ultrasonic ceramic transducers shown exploded in Fig. 1. These transducers are a piezoelectric ceramic disc bender design at a resonant frequency of either 25 or 40 kHz. The piezoelectric bender is bonded with a metal cone and mounted in a small can behind a protective screen with a diameter of 1.3 cm. The diameter of the 25 kHz transducer cone is 1.2 cm while the 40 kHz transducer cone has a diameter of 0.7 cm. Nominal minimum sound pressure level for a transmitter is 101 dB re 20 μPa at 1 m with minimum receiver sensitivity of -65 dB re 1 V/0.1 μPa,[6] both over a very narrow bandwidth around the resonant frequency. The transmitters are capable of dissipating over 10 Vrms. In practice, the receivers deliver over 100 mV when placed in the far field at distances 1 m from two or more transmitters. These piezoelectric transducers are typically sold in pairs, one optimized as a transmitter and the other optimized as a receiver. They are commercially available from many suppliers.[7,8]

The ultrasonic transducers are installed in the configuration shown in Fig. 2 with two or more transmitters on the rotary table excited by a signal generator. The motor controller turns the rotary table at a known speed with a rotational period of about 30 sec. The rotary table is controlled with a joystick or from a computer using manufacturer supplied software or experiment controlling software such as LabView®.

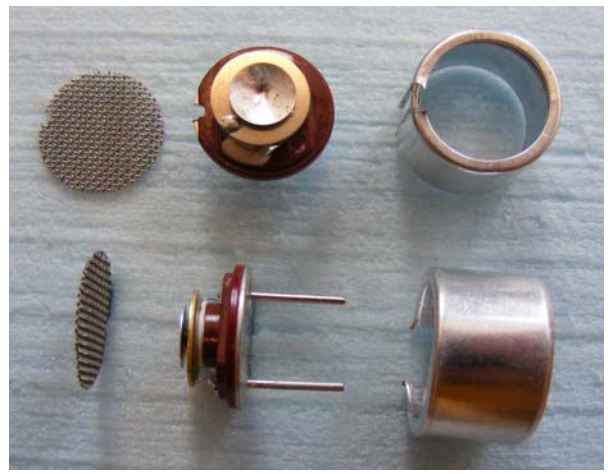

Fig. 1. Cutaway view of a piezoelectric ultrasonic transducer.

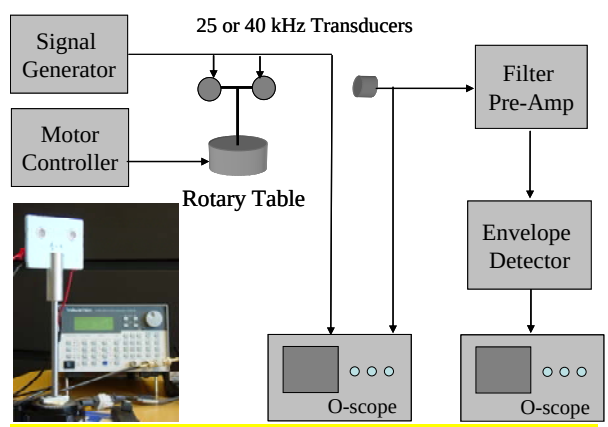

Fig. 2. A typical ultrasonic beamforming apparatus is shown here. In this configuration the transmitting transducers are mounted on the rotary table and driven in parallel from the same function generator to deliver equal output acoustic level. The receiving transducer is mounted on a lab stand in the in the far field creating a signal for the receiving electronics. Two digital oscilloscopes allow monitoring the receiver amplitude on a short time scale (20 μsec/div) while simultaneously viewing the envelope on a long time scale (2 sec/div) as the rotary table turns.

As the transmitting array turns, one oscilloscope is set to display the unprocessed waveform from the

receiving transducer. The student observes constructive and destructive interference as relative maxima and minima received from the receiving transducer on this oscilloscope. This oscilloscope is not essential, but observing the changing waveform amplitude is valuable for replacing the response that would be sensed at audible frequencies. An angular scale on the rotary table allows Eqs. (6) and (7) to be verified.

To obtain the directional factor around the rotating array, the output of the receiving transducer is sent to a filter preamplifier. The output impedance of the receiving transducer can be as high as 10000 Ω near resonance and the input impedance of the Stanford Research Systems SR560 preamplifier is 100 MΩ. After appropriate band pass filtering (10 kHz to 100 kHz on the SR560), the signal leaves the low impedance (50 Ω) output of the preamplifier. An envelope detector shown in Fig. 3 then removes the 25 or 40 kHz carrier and displays only the slowly varying peak signal. To function properly, the envelope detector requires an input of at least 1 V obtained with a gain of 20 on the SR560 preamplifier. While many oscilloscopes have a built in peak detection function, this circuit completely eliminates the high frequency carrier and displays only the upper envelope. This signal is sent to the second oscilloscope set on a long time scale (~2 sec/division) to acquire approximately 20 sec of data as the rotary table turns. The resulting trace is then sent to a computer for analysis and comparison to theoretical prediction. It is convenient to collect the envelope over a range of 90 deg either side of the central maximum. Additionally, stopping the oscilloscope acquisition when the central maximum is at the position assigned to zero seconds simplifies the conversion of time to angle, θ, using the rotating table's angular speed.

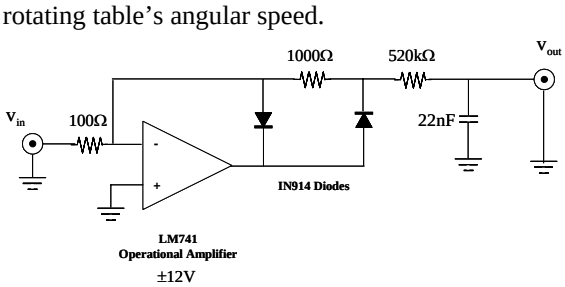

Fig. 3. Envelope detector circuit

To map the beam pattern of a receiving array, simply swap the transmitting elements on the rotary table with receiving elements. Place a transmitter in the far field where the receiver was previously and swap the transmitting and receiving electronics. For more advanced demonstrations, the polarity of the leads for one of the transmitting elements can be swapped, placing the transmitters out of phase in a dipole fashion.[9] This allows the demonstration of the beam pattern similar to Lloyd's mirror interference from a pressure release surface.[10]

Attention must be paid when mounting the transducers to prevent interaction between the elements. In this apparatus, the transducers were mounted in Styrofoam plates with holes cut to the outer diameter of the transducer can (1.6 cm). Double sided foam tape secured the transducers in the plate. For calculation convenience, holes were cut at separations, *d*, near to integer multiples of a wavelength. Temperature affects the sound speed in air and requires the separation to wavelength ratio to be reevaluated. In practice the ratio is used as a fit parameter.

In this apparatus, a single signal generator output was split and sent to both transducers. Occasionally the transducer transmit voltage response varies from the nominal value causing a distortion in the interference pattern. The effect is much like array shading where the amplitude of array elements is independently controlled.[11] If multiple signal generators are available and can be synchronized in phase, each can be adjusted separately to obtain a balanced output. For advanced students, this offers the ability to demonstrate array shading and its effect on the beam pattern function.

IV. RESULTS

The transducers described have directional beam patterns and corrections to the omni-directional source model motivating Eqs. (4) and (5) must be considered. The design of these transducers results in a complicated geometry for the source of the ultrasonic wave. The bender component is vibrating in a radial mode with a conical piston attached. Both components are placed in a cylindrical can with a slightly larger radius. Despite this complex construction, a single transducer can be approximated as a baffled circular piston with a directional factor,

$$H_{piston}(\theta) = \left| \frac{2\,J_1(ka\sin\theta)}{ka\sin\theta} \right|. \tag{8}$$

Here, a is the radius of the vibrating piston and J_1 is the first order Bessel function. Fig. 4 shows measured directional factors for a single transducer compared to those predicted using the baffled piston approximation of Eq. (8) with radius 0.6 cm. The good agreement allows the use of the product theorem to predict the interference beam pattern for two or more sources. The resulting direction factor is the product of Eq. (8) and either Eq. (4) or (5).

Typical results obtainable by students on a laboratory table top are shown in Fig. 5 through Fig. 7. Voltages obtained from the oscilloscope via the envelope detector are normalized and compared with

the result predicted from the directional factor product theorem, shown as a dashed curve. While the side lobe amplitudes may vary as much as 20% from predicted, the minima and maxima occur within 2 deg of their predicted angular direction. In addition to transducer response variation already discussed, reflections from adjacent surfaces resulting in modal interference and alignment of transducers account for these differences. These results satisfactorily demonstrate multiple source interference.

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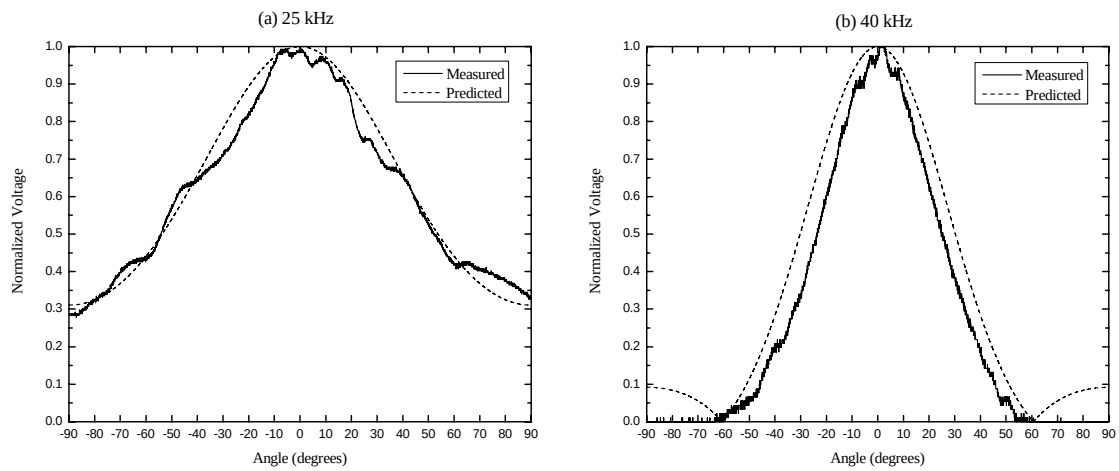

Fig. 4. Normalized directional factor for a single transducer transmitting at (a) 25 kHz and (b) 40 kHz. A radius of 0.6 cm is used for the predicted approximation of a baffled piston.

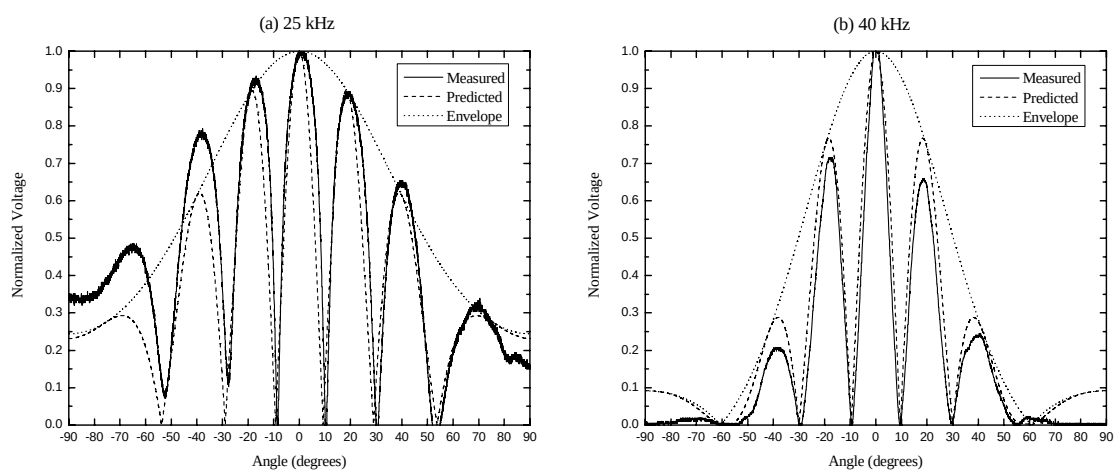

Fig. 5. Far Field beam pattern functions for (a) two 25 kHz transmitters (d/λ = 3.1) and (b) two 40 kHz transmitters (d/λ = 3.0). Each array pattern is modulated by the single source pattern.

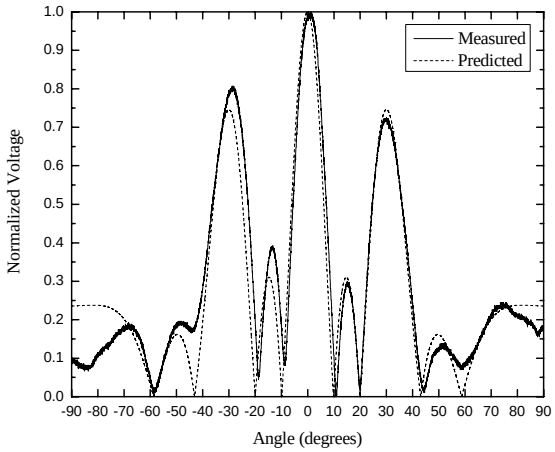

Fig. 6. Far field beam pattern function for three 25 kHz transmitters ($d/\lambda = 2.0$). As expected for three sources, one subsidiary maximum is clearly observed between adjacent major maxima..

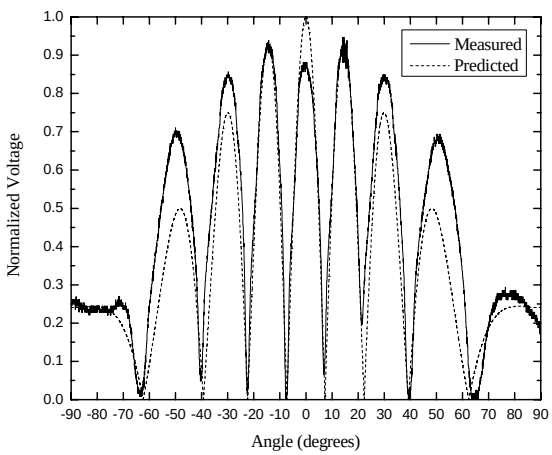

Fig. 7. Beam pattern function for two 25 kHz receiving transducers ($d/\lambda = 4.0$). A single transmitting transducer is placed in the far field. This is one possible modification to the apparatus as described.

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