SP212 Equation Sheet

Prefixes: 10^3 kilo k, 10^6 mega M, 10^9 giga G, 10^{12} tera T, 10^{15} peta P, 10^{-3} milli m, 10^{-6} micro μ, 10^{-9} nano n, 10^{-12} pico p, 10^{-15} femto f

Constants: $\epsilon_0 = 8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}$ $\quad \frac{1}{4\pi\epsilon_0} = k = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$ $\quad e = 1.602\times10^{-19}\,\mathrm{C}$
$m_e = 9.109\times10^{-31}\,\mathrm{kg}$ $\quad m_p = 1.673\times10^{-27}\,\mathrm{kg}$ $\quad \mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A} \approx 1.26\times10^{-6}\,\mathrm{T\cdot m/A}$
$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3.00\times10^8\,\mathrm{m/s}$ $\quad 1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$

Electric Charge: $i = \frac{dq}{dt}$ $\quad F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$ $\quad q = ne \quad n = \pm1, \pm2, \pm3, \ldots,$

Electric Fields: $\vec{E} = \frac{\vec{F}}{q_0}$ $\quad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ $\quad \vec{p} = q\vec{d}$ $\quad \vec{E}(z) = \frac{1}{2\pi\epsilon_0}\frac{\vec{p}}{z^3}$ $\quad \vec{dE} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$ $\quad dq = \lambda\, ds$
$dq = \sigma\, dA$ $\quad dq = \rho\, dV$ $\quad \vec{F} = q\vec{E}$ $\quad \vec{\tau} = \vec{p}\times\vec{E}$ $\quad U = -\vec{p}\cdot\vec{E}$

Gauss' Law: $\Phi_E = \int \vec{E}\cdot\vec{dA}$ $\quad \epsilon_0\oint \vec{E}\cdot\vec{dA} = q_{\mathrm{enc}}$, $\epsilon_0(EA) = q_{\mathrm{enc}} \rightarrow E = \frac{\sigma}{\epsilon_0}$,
$\epsilon_0 E(2\pi r h) = q_{\mathrm{enc}} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$, $\epsilon_0(EA + EA) = q_{\mathrm{enc}} \rightarrow E = \frac{\sigma}{2\epsilon_0}$, $\epsilon_0 E(4\pi r^2) = q_{\mathrm{enc}} \rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

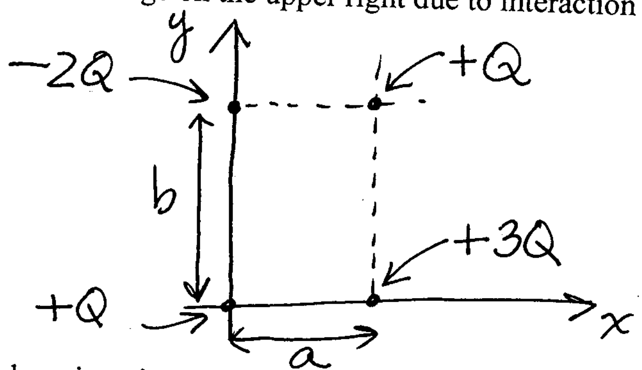
SP212 Practice Exam 1

This practice exam is to give you a feel for the kind of exams I am planning on giving this semester. Provided I stick with the same format, I won't provide practice exams with each exam. Your practice problems come from the WileyPlus suggested problems (minus the one from each set that is the least appropriate for testing).

Do your best to convey a line of thinking and box all of your answers. Good Luck.

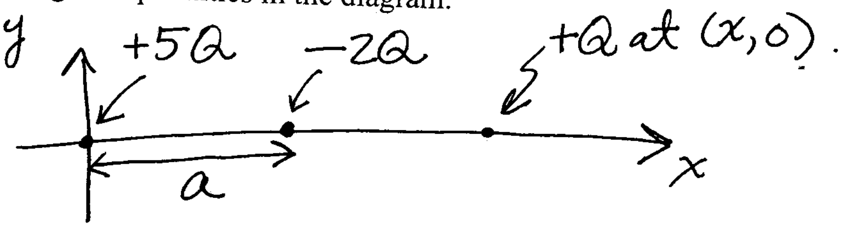
Problem 1) For this problem, imagine Q is some positive fixed amount of charge. The charges in the diagram are positive and negative integer multiples of this charge. Find an expression for the y component of the force on the charge on the upper right due to interaction with the other charges.

Numbers (optional):
$Q = 2.00\text{e-}6$ C
$a = 0.300$ m
$b = 0.400$ m



Problem 2) For this problem, imagine Q is some positive fixed amount of charge. A charge $+Q$ is placed at position x is observed to experience no net force due to its interaction with the other charges. Solve for x in terms of the other given quantities in the diagram.

Numbers (optional):
$Q = 2.00\text{e-}6$ C
$a = 0.300$ m

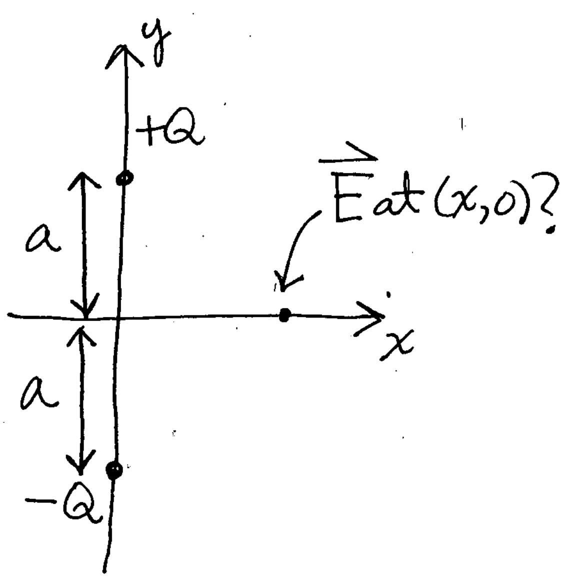


Problem 4) Explain with words and pictures what would happen to a charge $+Q$ if released from rest somewhere on the negative x-axis..

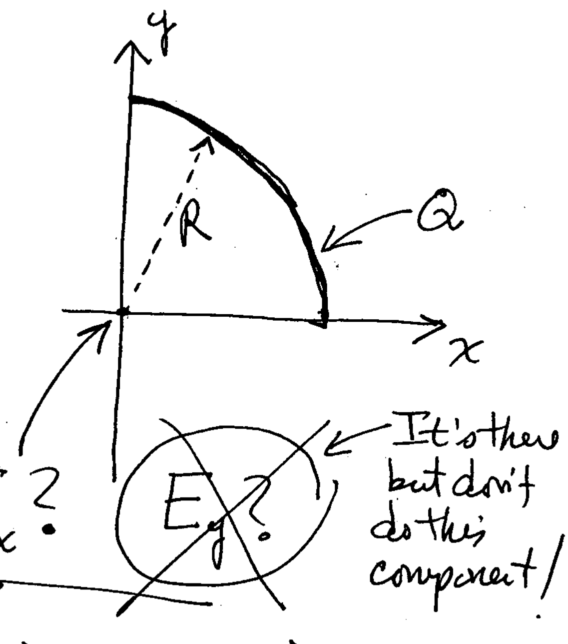
Problem 5) The diagram below shows an electric dipole, but let's just think of it as two charges at the given coordinates (no approximations or dipole moments here). Find an exact expression for the electric field at the point $(x, 0)$ where x is a coordinate on the positive x-axis.

YOUR ANSWER SHOULD CONVEY DIRECTION SOMEHOW!

Numbers (optional):
$Q = 2.00\text{e-}6$ C
$a = 0.300$ m
$x = 0.400$ m



Problem 6) The diagram shows a quarter circle line of charge. The charge Q is spread uniformly over the length. The electric field at the point designated due to this line of charge certainly both x and y components. **Let's just focus on the x-component.**

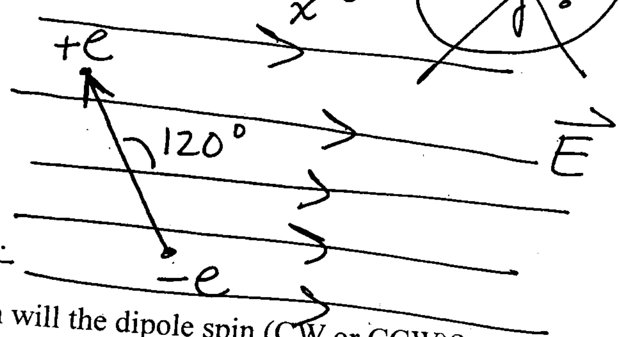


Develop an integral expression for the x-component of the electric field at the designated point due to the line of charge.

You do not need to evaluate the integral, but do get it into the form that is ready to integrate mathematically.

Numbers (optional):
$Q = 2.00\text{e-}6$ C
$R = 0.300$ m

Problem 7) EVERYONE CRANK THE NUMBERS ON THIS ONE. An electric dipole consists of charges $+e$ and $-e$ separated by 0.700 nm at an angle of 120 degrees relative to the direction of the electric field which is of strength 4.00e6 N/C.

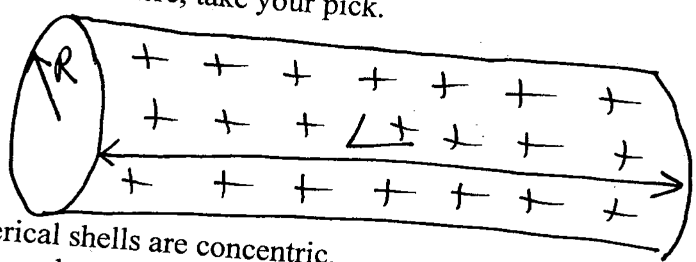


- If released from rest, in what direction will the dipole spin (CW or CCW)?
- What is the potential energy of the dipole in this orientation?
- What are the max and min possible values of the potential energy over all possible orientations?

Problem 8) A charged conductor in the shape of a sheet wrapped into the shape of a cylinder has total charge Q on its surface that is distributed uniformly over its surface (the cylinder is of radius R and length L). What is the strength of the electric field just off the surface of the conducting sheet?

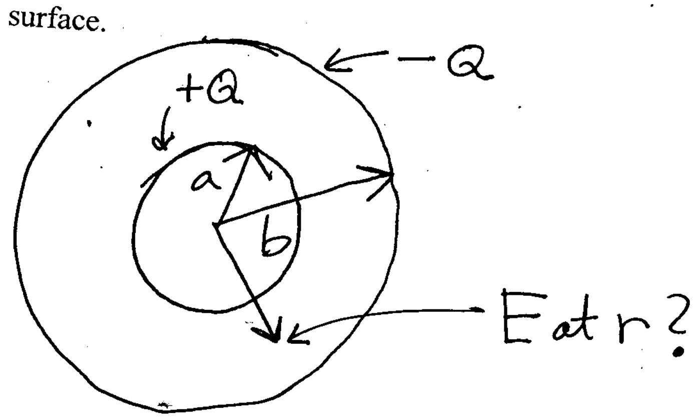
NOTE: There are a few valid approaches here, take your pick.

Numbers (optional):
$Q = 7.00\text{e-}6$ C
$R = 0.200$ m
$L = 0.900$ m



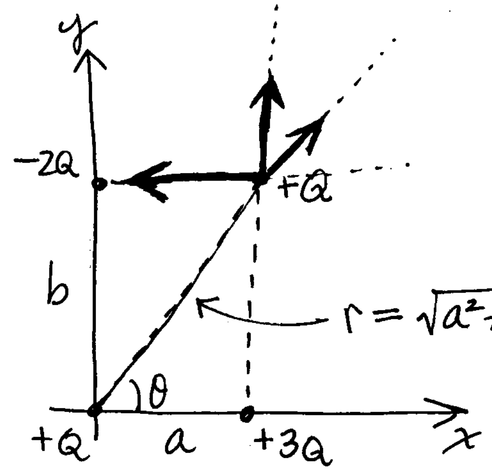
Problem 9) Two conducting spherical shells are concentric. The inner shell is of radius a and has charge $+Q$ on its surface. The outer shell is of radius b and has charge $-Q$ on its surface. Find an expression for the E field in the region where $a < r < b$.

Numbers (optional):
$Q = 2.00\text{e-}6$ C
$a = 0.300$ m
$r = 0.400$ m
$b = 0.500$ m



Solutions

Problem 1)



Use diagram. For each vector,

1) magnitude (ignore + −).
2) sign
3) trig

$$\sin\theta = \frac{b}{\sqrt{a^2+b^2}} \quad *$$

$$F_y = +\frac{k(3Q)(Q)}{b^2} + \frac{k(Q)(Q)}{(a^2+b^2)}\sin\theta$$

More Practice: What is F_x?

from diagram

just magnitude

(1) $$F_x = +\frac{k(Q)(Q)}{(a^2+b^2)}\cos\theta - \frac{k(2Q)(Q)}{a^2}$$

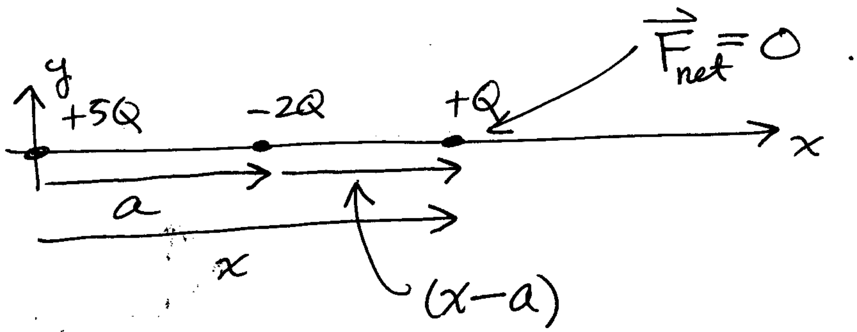
(2) $$\cos\theta = \frac{a}{\sqrt{a^2+b^2}} \quad *$$

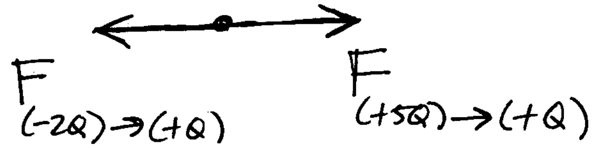
Use of θ is OK because we can get angle from (2).

* Any single trig statement that fixes θ is fine. For instance,

$\tan\theta = \frac{b}{a}$ would be O.K.

Problem 2)





$$\frac{k(2Q)(Q)}{(x-a)^2} = \frac{k(5Q)(Q)}{x^2}$$

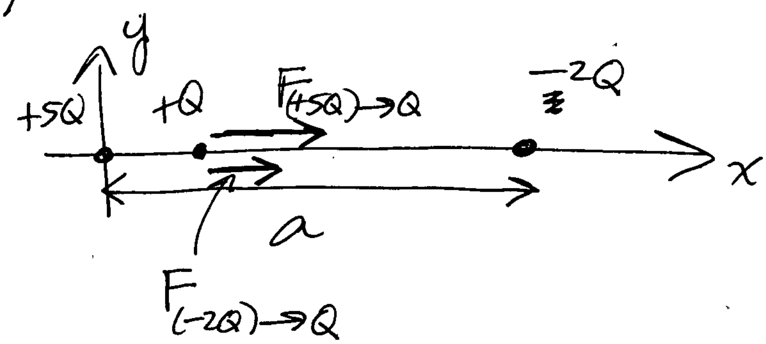
$$\boxed{2x^2 = 5(x-a)^2}$$

numerically solve for x. ← ACCEPTABLE ANSWER. with this statement

After initial most basic simplifications you can do this (this would be ~~and~~ easy to use with nsolve calculator function).

→ For more trivial ~~calculat~~ manipulations please **DO** grind to the end. If unsure, just ask.

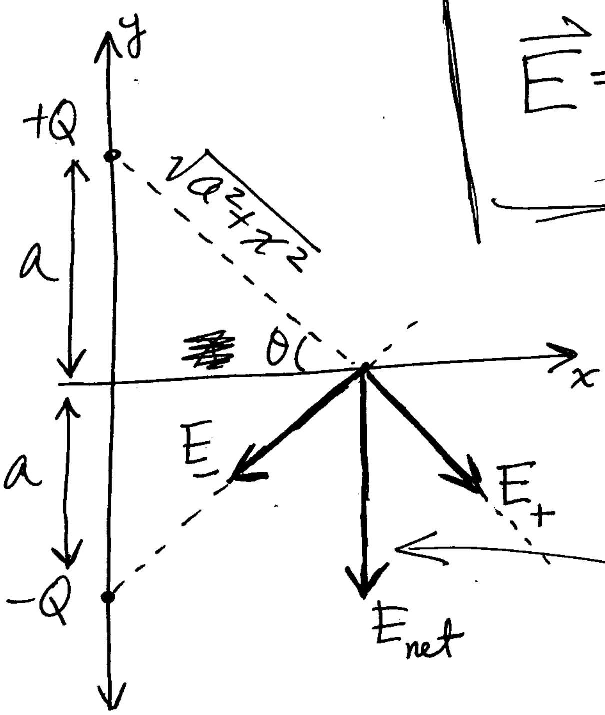
Problem 4)



- +5Q pushes
- −2Q pulls

→ cooperative! ⟹ (+Q will accelerate in +x direction)

Problem 5)



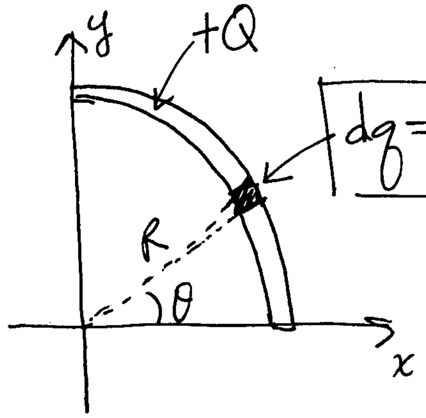
$$\sin\theta = \frac{a}{\sqrt{a^2+x^2}}$$

$$\vec{E} = 2\frac{kQ}{(a^2+x^2)}\sin\theta\,(-\hat{j})$$

two ways direction is conveyed (one is fine both are nice).

θ not given so you need to show how to get θ.

Problem 6)



$$\lambda = \frac{Q}{\frac{1}{4}(2\pi R)}$$

$$dq = \lambda\, ds$$

$$ds = R\, d\theta$$

$$E_x = \int dE_x = \int_0^{\pi/2} -\frac{k\,dq}{R^2}\cos\theta$$

$$E_x = \int_0^{\pi/2} \frac{-k\left[\frac{Q}{\frac{1}{4}(2\pi R)} R\, d\theta\right]\cos\theta}{R^2}$$

$$E_x = -\frac{kQ}{R^2}\left(\frac{2}{\pi}\right)\int_0^{\pi/2}\cos\theta\, d\theta$$

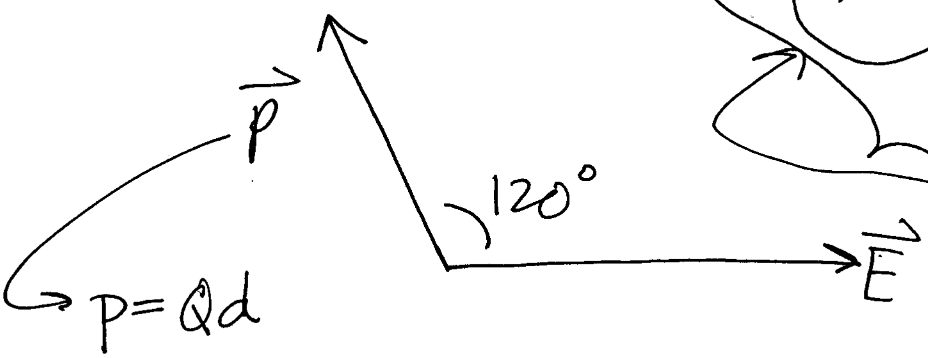
Not required here, but $= \sin\theta\Big|_0^{\pi/2} = 1$

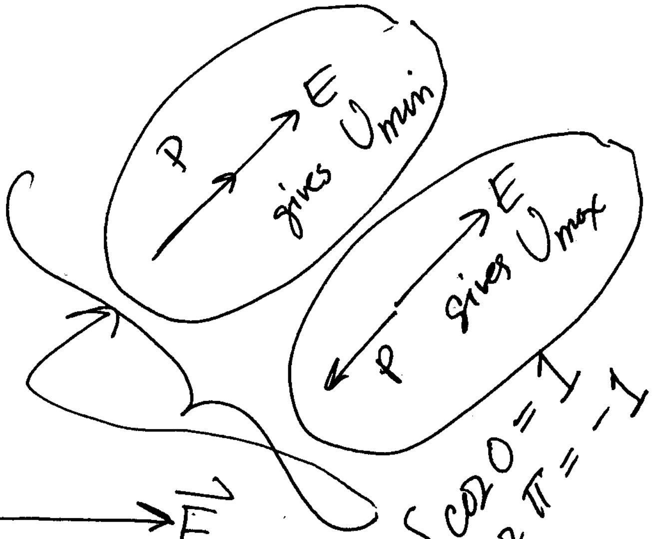
$\frac{kQ}{R^2}$: ✓ DA.

$\frac{2}{\pi}$: <1 ✓ (Why is this expected?)

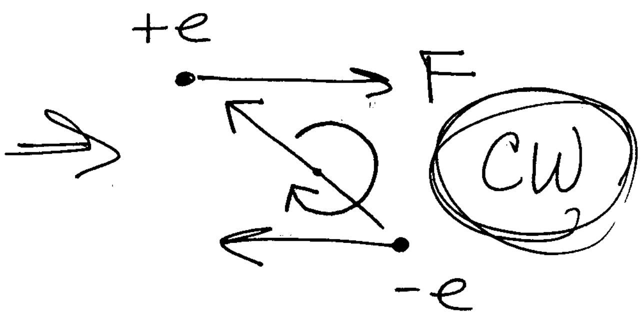
$$E_x = -\frac{2}{\pi}\frac{kQ}{R^2}$$

Problem 7)





$\cos 0 = 1$
$\cos \pi = -1$



$$\Rightarrow U_{\substack{max\\min}} = \pm e d E$$

$$U_{\substack{max\\min}} = \pm 4.49\text{e-}22\ \text{J}$$

$$\Rightarrow U = -pE\cos\theta$$

$$U = -edE\cos\theta$$

$$U = -(1.602\text{e-}19\ \text{C})(0.7\text{e-}9\ \text{m})\left(4\text{e}6\ \frac{\text{N}}{\text{C}}\right)(\cos 120°)$$

$$U = +2.24\text{e-}22\ \text{J}$$

Problem 8)

method 1

$$E = \frac{\sigma}{\epsilon_0} = \boxed{\frac{\left(Q/2\pi R L\right)}{\epsilon_0}}$$

$$\hookrightarrow E = \frac{(Q/L)}{2\pi\epsilon_0 R}.$$

method 2

Cylindrical symmetry with Gaussian Surface at $r=R$.

$$\epsilon_0 E(2\pi R L) = Q$$

$$\hookrightarrow E = \frac{(Q/L)}{2\pi\epsilon_0 R}$$ ✓ same answer

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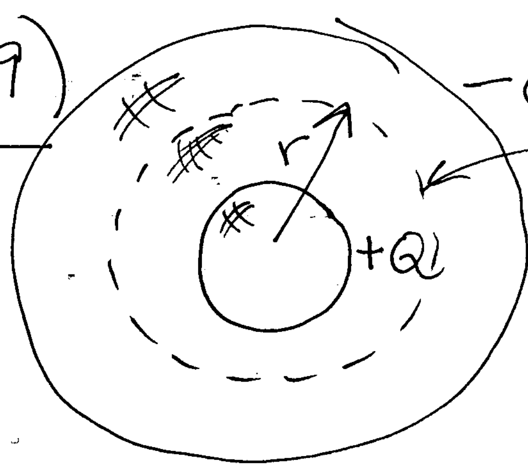
Problem 9)



−Q

spherical symmetry.

$$\epsilon_0 E 4\pi r^2 = \frac{q_{enc}}{\epsilon_0}$$ (this is just +Q)

$$\hookrightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2} \text{ (outward)}}$$