SP212 Blurb 10 2010

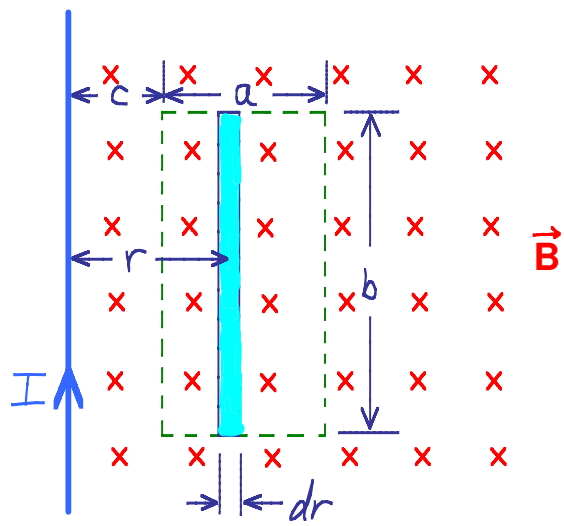
1. Magnetic Flux and Gauss' Law for **B**.
2. Faraday's Law
3. Induced emf
4. Motional emf
5. Eddy Current Demos

Magnetic Flux and Gauss' Law for B.

Since magnetic monopoles do not exist, **B** lines are always complete loops. Thus, any closed surface has as many **B** lines going in as going out. We can say that the net magnetic flux through a closed surface is always zero and write this result as,

$$\oint_S \vec{B}\cdot\hat{n}\,da = 0 \quad \text{(Gauss' Law for } \mathbf{B}\text{)}$$

Soon, though, we'll need to calculate magnetic flux through a non-closed surface. As an example consider the flux through an a x b rectangle near a long straight current carrying wire. Assume the wire is in the plane of the loop and that the long side of the rectangle is parallel to the current carrying wire.



Since thin vertical strips of width dr have a constant distance from the wire, we can calculate the flux $d\Phi_B$ through this strip and integrate to get the total flux.

$$\Phi_B = \int \vec{B}\cdot\hat{n}\,dA = \int B\,dA \quad \text{since } \hat{n} \parallel \vec{B}$$

$$dA = b\,dr \qquad B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r} b\,dr = \frac{\mu_0 I b}{2\pi}\int_c^{a+c} \frac{dr}{r}$$

$$= \frac{\mu_0 I b}{2\pi} \ln r \Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi}\left[\ln(a+c) - \ln c\right]$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

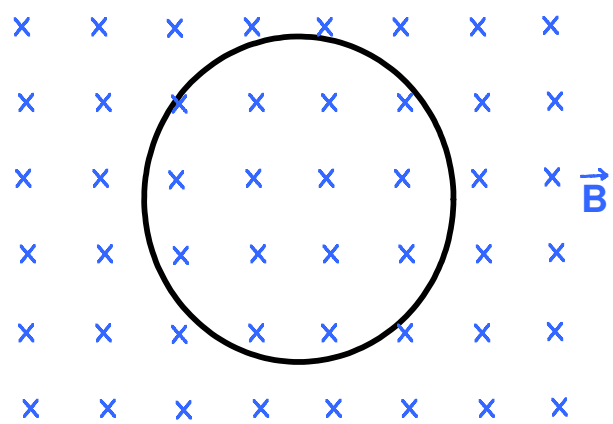
Faraday's Law

Since electric currents produce magnetic fields, it seems reasonable to ask if magnetic fields can somehow produce electric currents. All attempts with static magnetic fields failed. Michael Faraday discovered that if the magnetic flux through a conducting loop was changing, then a current appeared in the loop. Since current in a loop implies emf in the loop, the changing magnetic flux must induce an emf. Faraday's Law is stated as,

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad ; \quad \mathcal{E} = -N\frac{d\Phi_B}{dt}$$ If the loop has N turns.

where ε is the induced emf and Φ_B is the magnetic flux through the loop.

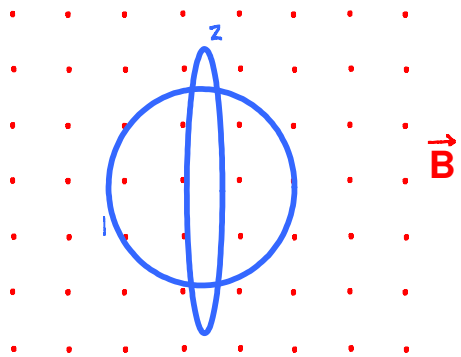
Lenz' Law is, basically, the minus sign in the expression above. It means that the direction of the induced emf is such that the change in flux is opposed.



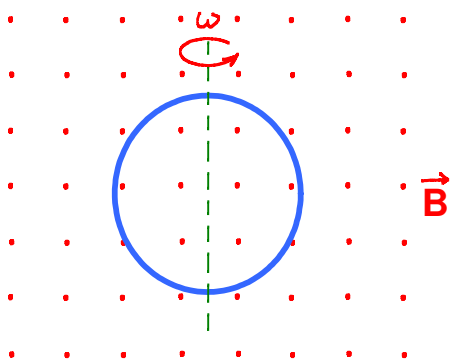
A clockwise current produces a magnetic field within the loop that is into the screen while a ccw current does the opposite. So, if the external **B** shown above is increasing in strength, the loop will produce a field within it that is pointing out of the screen. It does this with a ccw current. On the other hand, if the same external magnetic field is decreasing, the loop will produce a magnetic field within it that is into the screen. This requires a clockwise current and emf,

Applications & Examples:

Suppose we collapse the conducting loop from state 1 to state 2. What is the direction of induced current?



If we rotate a circular conducting loop about a vertical axis, as shown, what is the direction of induced current?



$$\mathcal{E} = -\frac{d\Phi_B}{dt} \; ; \; \Phi_B = BA\cos\omega t$$

$$\Rightarrow |\mathcal{E}| = BA\omega \sin\omega t \; ; \; \mathcal{E}_{max} = BA\omega$$

$$\Rightarrow I = \frac{|\mathcal{E}|}{R} = \frac{BA\omega}{R}\sin\omega t = I_{max}\sin\omega t$$

Find the direction of current induced by moving the bar magnet relative to the conducting loop below.

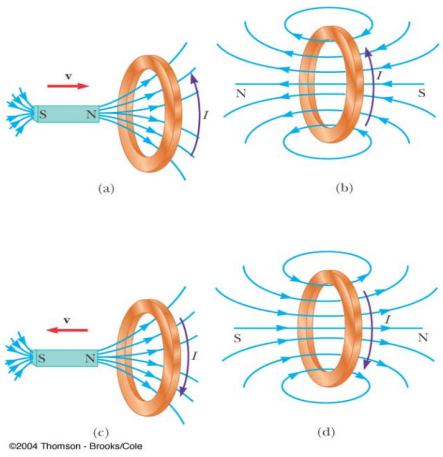


Fig 31-14, p.978

(Farmer story)

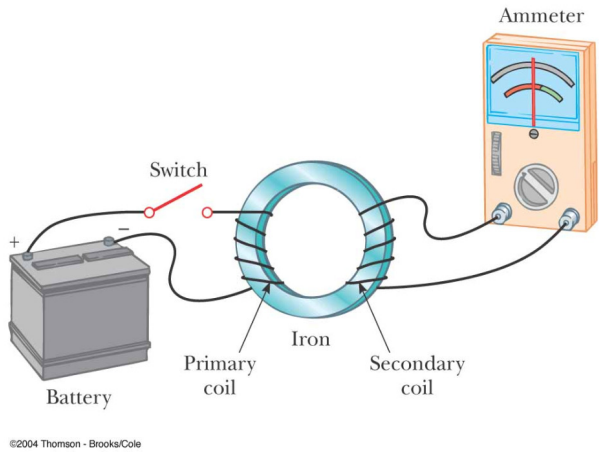


Fig 31-2, p.969

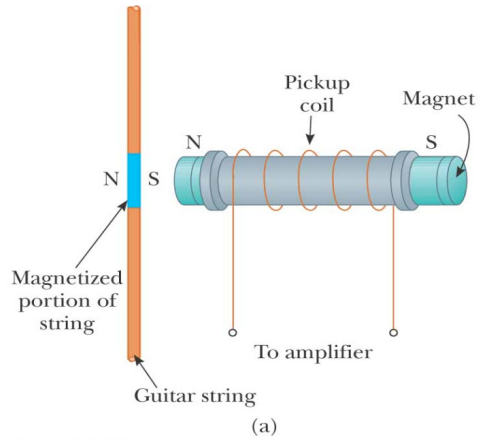


Fig 31-6a, p.972

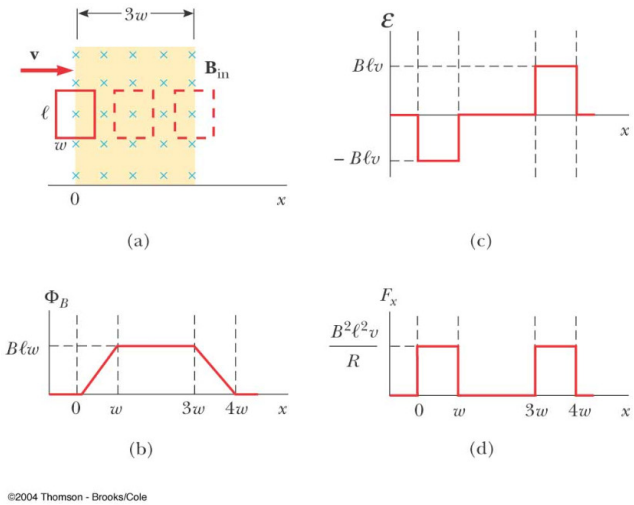
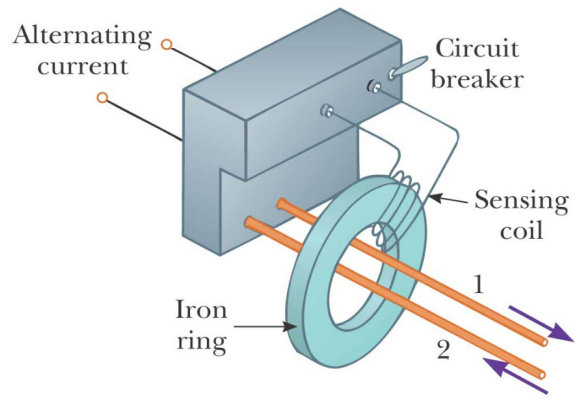


Fig 31-18, p.980



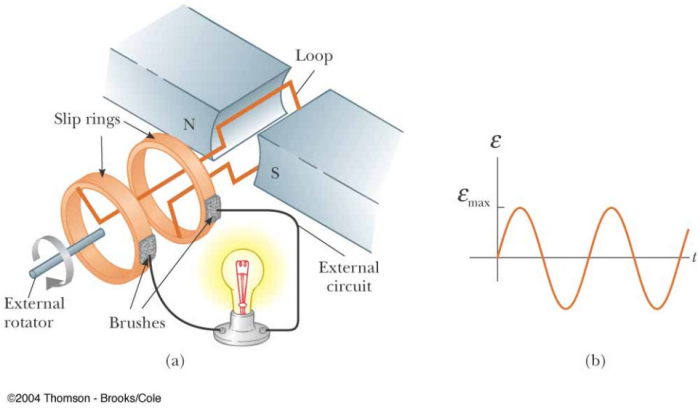


Fig 31-21, p.983

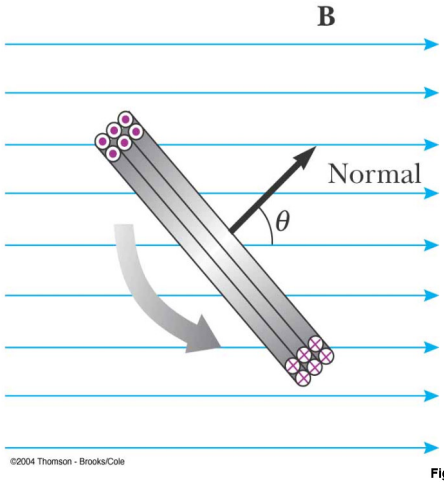
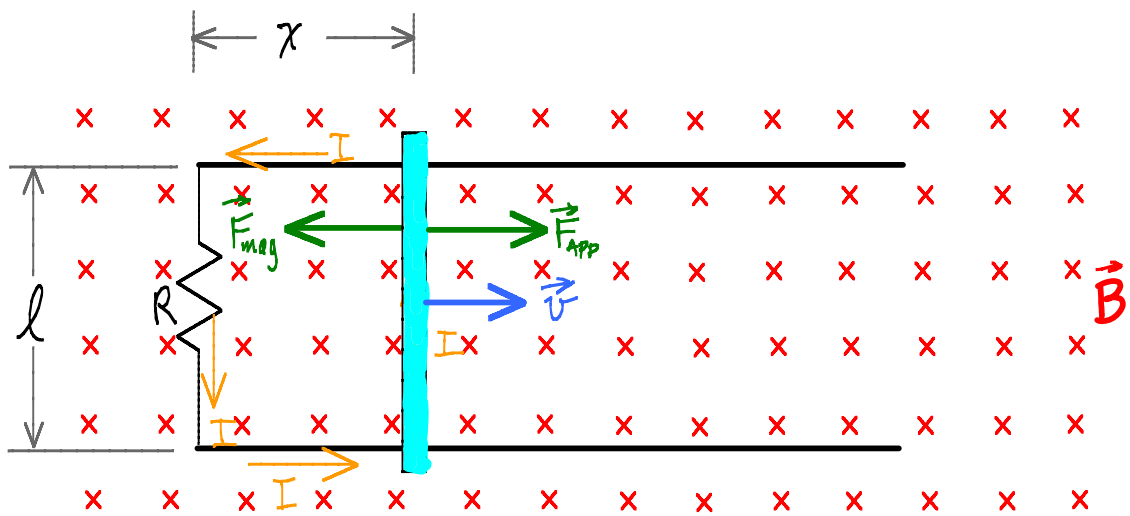


Fig 31-22, p.983

Motional Emf

Most of the examples we've seen of emf generated by changing magnetic flux involved changing field strength. Another option is to change the geometry of a loop. Consider the system below. Two long conducting rails are connected by a resistor. A moving segment of rail lies atop the fixed rails, completing the circuit. If the moving rail is pulled right at constant speed v, find the emf generated, the current, $\mathbf{F}_{mag}$, and the power delivered by the pulling agent.



As the rail moves right in response to $\mathbf{F}_{app}$, the number of B lines through the complete circuit increases. To counteract this change an emf is generated in the circuit to produce a B field opposite the direction of the static B field. By the right hand rule this requires a ccw current around the circuit.

$$\Phi_B = Blx \quad ; \text{Now use Faraday's law,} \quad \mathcal{E} = \frac{-d\Phi_B}{dt}$$

$$\Rightarrow \mathcal{E} = \frac{-d}{dt}(Blx) = -Bl\frac{dx}{dt}$$

$$\Rightarrow \mathcal{E} = -Blv \quad \Rightarrow I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

Now we can get the power dissipated by the resistor from

$$P_R = I^2R \Rightarrow P_R = \frac{B^2l^2v^2}{R^2}R \Rightarrow P_R = \frac{B^2l^2v^2}{R}$$

Since there is an upward current in the moving rail there must be a magnetic force $\mathbf{F}_{mag}$ that is opposite $\mathbf{F}_{app}$. It is the balance of these forces that allows the rail to move at constant speed.

$$\text{from } \vec{F} = I\vec{\ell} \times \vec{B} \Rightarrow \vec{F}_{mag} = I\ell B(-\hat{i}) = \vec{F}_{mag} = \frac{B\ell v}{R}(\ell B)(-\hat{i})$$

$$\Rightarrow \vec{F}_{mag} = \frac{B^2\ell^2 v}{R}(-\hat{i})$$

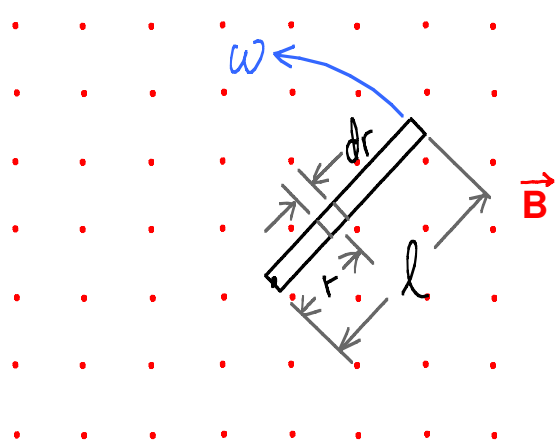
For the rail to be moving at constant v the net force must be zero. Thus, $\mathbf{F}_{mag} = \mathbf{F}_{app}$. If we look at the power expended by the applied force,

$$P_{App} = \vec{F}_{App} \cdot \vec{v} = F_{App}\, v = F_{mag}\, v$$

$$\Rightarrow P_{App} = \frac{B^2\ell^2 v^2}{R} \Rightarrow P_R = P_{App}, \text{ as conservation of energy demands.}$$

Speaking of conservation of energy, consider what would happen if Faraday's law didn't have the minus sign. Current would go clockwise in our example above and $\mathbf{F}_{mag}$ would point in the same direction as $\mathbf{F}_{app}$. Thus, a net force would lead to acceleration that would grow without bound. A small energy investment would give the rail infinite energy, violating energy conservation. So, the minus sign in Faraday's law is vital.

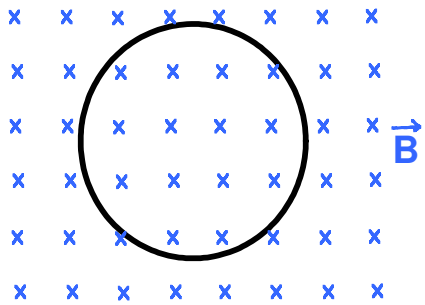
Example: Consider a conducting bar rotating in a plane perpendicular to a magnetic field. Find the emf between the ends of the bar in terms of B, the bar's length l, and ω, the angular rotation rate of the bar.



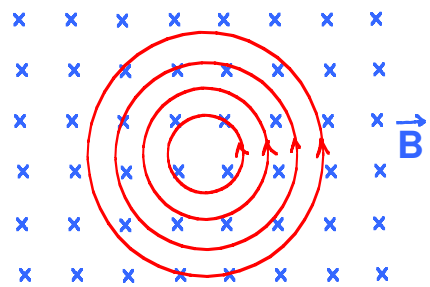
Each small segment of length dr generates a small emf given by dε = Bvdr. The emf in the entire bar is obtained by integrating over r. Since v is different for each segment, use v = ωr in order to integrate

$$\mathcal{E} = \int Bv\,dr = \int_0^{\ell} B\omega r\,dr = \boxed{\frac{B\omega\ell^2}{2} = \mathcal{E}}$$

Induced emf and electric fields



Recall that changing the magnetic flux through the wire loop produced a clockwise or ccw current. Current in a wire is caused by charges moving in response to an electric field. So, there must be a cw or ccw E field induced in the wire that is produced whether a wire is present or not!



Is **B** increasing or decreasing in strength at left?

The E field shown above is different from the electrostatic E field. We see that it does not originate or terminate on charge. In an electrostatic field there is no net work done on a charge by the field if it moves in a complete loop. In the field depicted above net work would be done on a charge of qE2πr (force*distance) for every lap around a loop of radius r. This makes the induced E field above nonconservative and means there is no potential function that can be associated with it. From the work done on a charge going around the circular loop we write,

$$q\mathcal{E} = qE2\pi r \Rightarrow \mathcal{E} = E2\pi r$$

so the emf of the nonconsecutive E is electric field times path length. We can generalize this in the following way,

$$\mathcal{E} = \int \vec{E}\cdot d\vec{s}$$

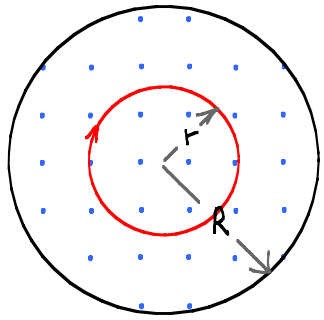
This allows us to rewrite Faraday's Law as

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \rightarrow \boxed{\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}}$$

<u>Summarizing</u>: A changing magnetic flux produces a nonconservative electric field!

Example: Suppose the current in a solenoid varies. Find the E field induced in the solenoid.Recall $B=\mu_o ni$ inside a solenoid.

B increasing implies clockwise **E**.



End view of solenoid

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$$\Rightarrow E\,2\cancel{\pi}\cancel{r} = -\frac{d}{dt} B\cancel{\pi} r^{\cancel{2}}$$

$$\Rightarrow E = -\frac{r}{2}\frac{dB}{dt} = -\frac{r}{2}\mu_o n \frac{dI}{dt}$$

So, we get the direction of **E** from Lenz' Law and magnitude of **E** is,

$$E = \frac{\mu_o n}{2}\frac{dI}{dt} r$$

If $I = I_o \sin\omega t$ then $\frac{dI}{dt} = I_o\omega\cos\omega t$

$$E = \frac{\mu_o n I_o \omega r \cos\omega t}{2}$$

Is an electric field induced outside the solenoid?