SP212 Blurb 13 2010

1. Ray representation of light waves.

2. Speed o' light

3. Wavelength and frequency ranges of visible light.

4. Law of Reflection

5. Image location with a flat mirror

6. Index of refraction

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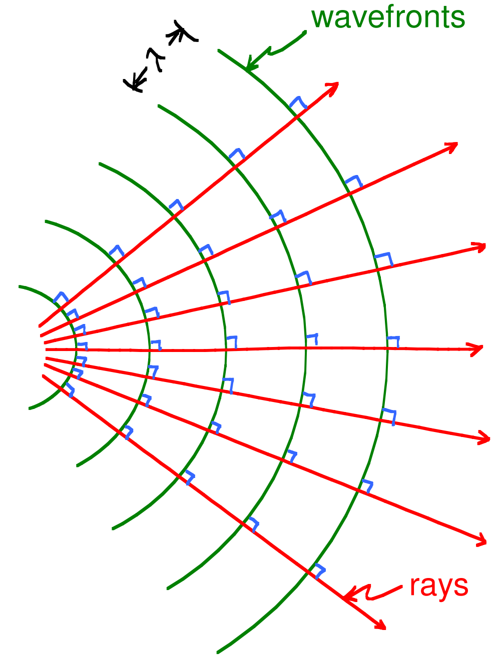
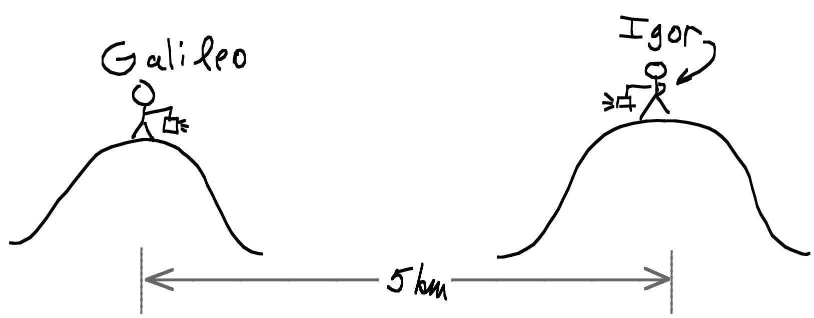
Ray vs wave representation

We can usually use ray representation if the objects the light interacts with are very large compared to a wavelength.



Speed o' Light Measurement

An early attempt by Galileo.



G and I were each on mountain tops with covered lanterns. G uncovers, then I uncovers and G measures time difference between his uncovering and his observation of I's light. Experiment was inconclusive as to whether light traveled at infinite or finite speed. The problem was the small time interval.

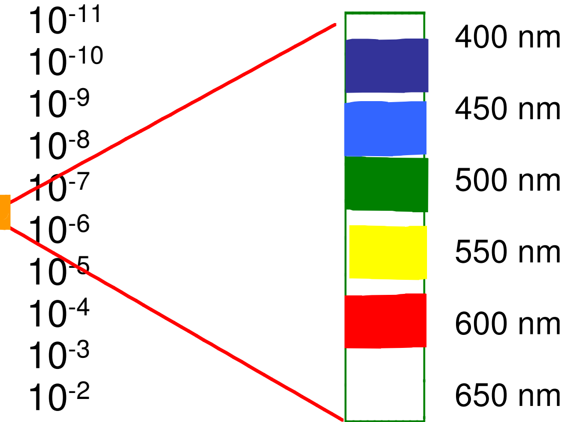
$$\Delta t \cong \frac{2\ell}{c} = \frac{2(5000\,m)}{3\times10^{8}\,m/s} = \frac{10^{4}}{3\times10^{8}}\,s = 0.33\times10^{-4}\,s = 33\,\mu sec$$

G was using his own pulse or a pendulum as a time measuring device!

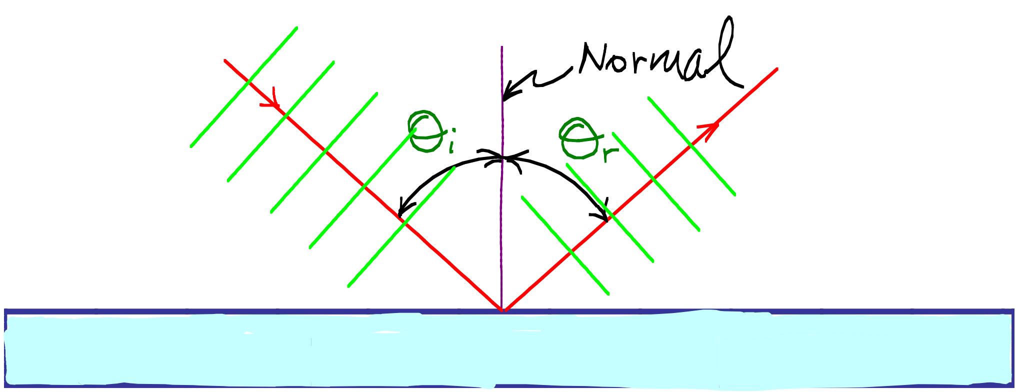
From astronomical observation of Jupiter's largest moon, Io, Romer was able to argue (1676) that the speed o' light is finite. Others calculated a ballpark figure and from that Fizeau was able to design the first (1849) earth based experiment with a rotating toothed wheel and a long path length. (8.6 km) x 2. Michelson did much to zero in on the final measured value. Since 1993 the speed of light has been taken to be 2.99792458×10^{8} m/s = c.

Wavelength & frequency ranges of e-m radiation

Frequency		Wavelength
10^{23}		
10^{22}	Gamma Rays	10^{-14}
10^{21}		10^{-13}
10^{20}		10^{-12}
10^{19}	x-rays	10^{-11}
10^{18}		10^{-10}
10^{17}		10^{-9}
10^{16}	Ultraviolet	10^{-8}
10^{15}	Visible	10^{-7}
10^{14}		10^{-6}
10^{13}	Infrared	10^{-5}
10^{12}		10^{-4}
10^{11}		10^{-3}
10^{10}	Microwaves	10^{-2}
10^{9}	Short Radio	10^{-1}
10^{8}	Waves	10^{0}
10^{7}	TV & FM	10^{1}
10^{6}	Waves	10^{2}
10^{5}		10^{3}
10^{4}	AM Radio	10^{4}
10^{3}	waves	10^{5}
10^{2}		10^{6}
10^{1}		10^{7}



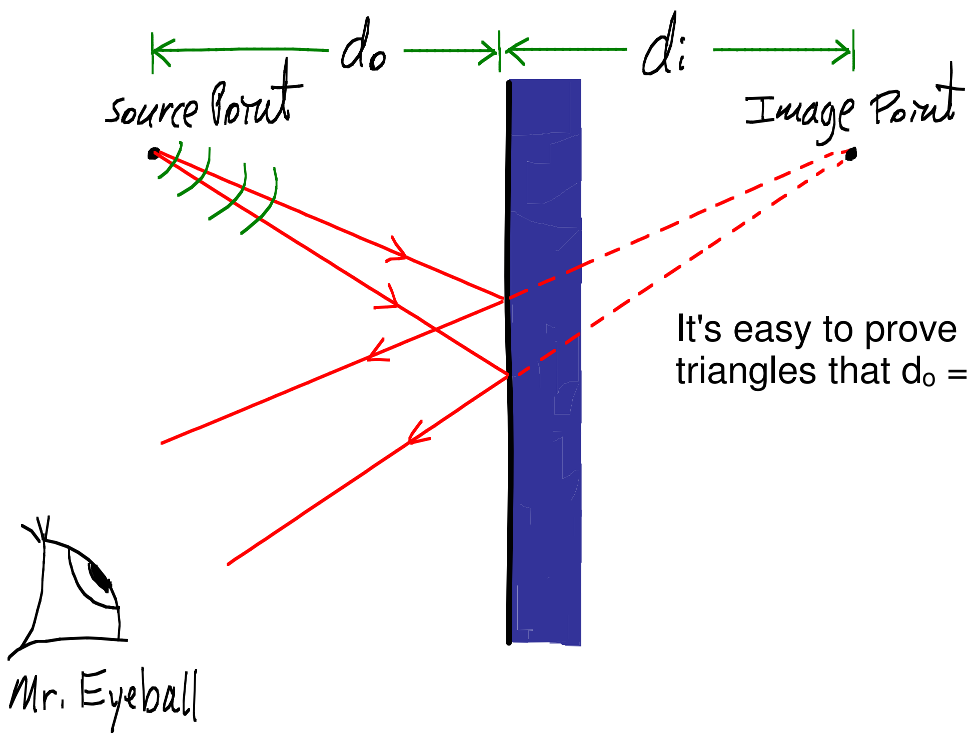
Reflection: Two types are diffuse and specular. An example of diffuse reflection is the reflection from white paint. Specular is the reflection from polished surfaces. Specular reflection obeys the law of reflection.



Angles are measured with respect to surface normal. The law of reflection says,

$$\theta_i = \theta_r$$

We can use this relationship and rays to locate an image formed by reflection from a plane mirror.



It's easy to prove using similar triangles that $d_o = d_i$.

Index of Refraction

In transparent media the speed of light slows down and we characterize the media with index n where n is defined as,

$$n \equiv \frac{c}{v}$$

where v is the speed of light in the medium. Some n's that I remember are:

$n_{air} = 1.00027$ $n_{glass} = 1.5$ $n_{water} = 1.33$ $n_{diamond} = 2.4$

Wavelength in Transparent Media

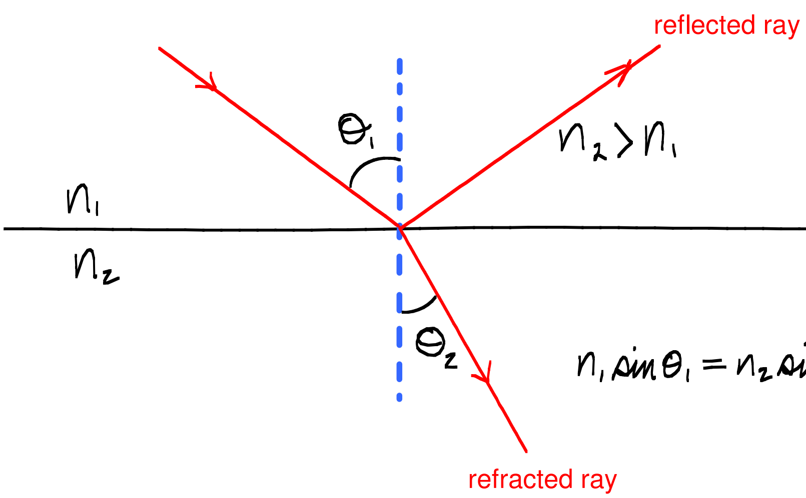
In transparent media the speed of light slows down, the frequency stays fixed, so from the relationship v = λf and we see that λ must get smaller. So,

$$c = \lambda_0 f \text{ and } v = \lambda f$$

$$\Rightarrow \quad \frac{c}{v} = \frac{\lambda_0 \cancel{f}}{\lambda \cancel{f}} = n \quad \Rightarrow \quad \boxed{\lambda = \frac{\lambda_0}{n}}$$

Which tells us that the wavelength in the medium gets smaller by 1/n compared to the vacuum value of the wavelength.

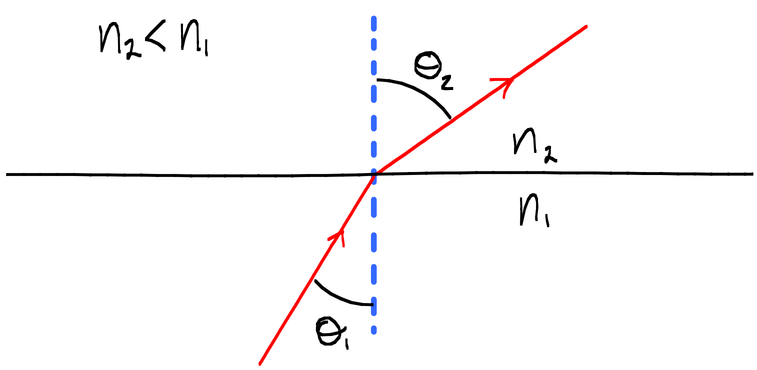
Refraction is observed when light traverses an index change.



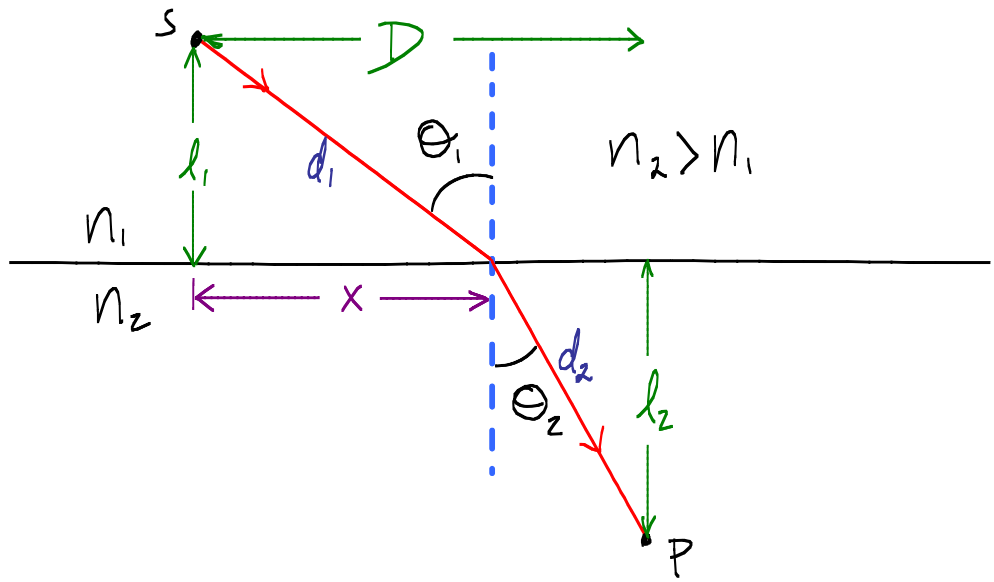
Using Maxwell's equations it is possible to find the relative intensities of the reflected and refracted light, though it's a bit beyond the level of this course.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{(Snell's Law)}$$

So, rays bend toward the normal when light enters a lower speed medium with higher Index. They bend away from the normal when light goes from a high index (slow) medium to a to a low index (fast) medium.



My favorite proof of Snell's law uses <u>Fermat's extremum principle</u>.



Fermat's extremum principle says that of all possible paths from S to P, the one that takes an extremum in time (here this means the least time) is the true path. In the situation depicted this boils down to what is x? To get the time for the SP trip,

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1}{c/n_1} + \frac{d_2}{c/n_2} = \frac{\sqrt{l_1^2 + x^2}}{c/n_1} + \frac{\sqrt{l_2^2 + (D-x)^2}}{c/n_2}$$

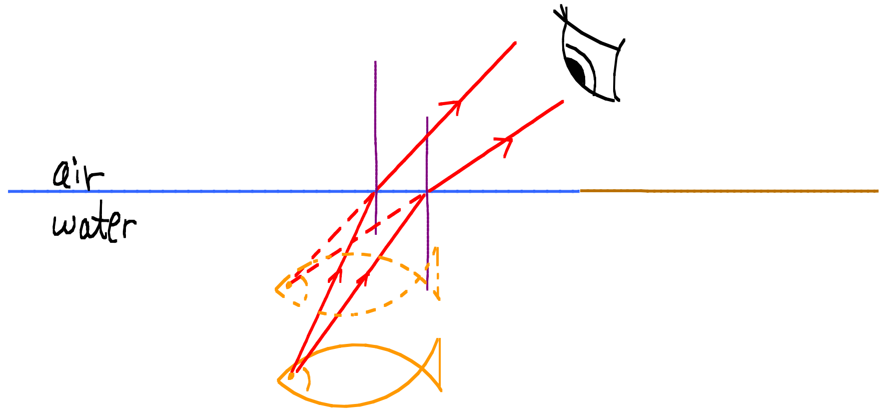
Now we have t as a function of x and can minimize t(x).

$$\frac{dt}{dx} = \frac{n_1}{c}\frac{1}{2}\left(l_1^2 + x^2\right)^{-1/2} 2x + \frac{n_2}{c}\frac{1}{2}\left(l_2^2 + (D-x)^2\right)^{-1/2} 2(D-x)(-1) = 0$$

$$\Rightarrow \quad \frac{n_1\, x}{\sqrt{l_1^2 + x^2}} = \frac{n_2\,(D-x)}{\sqrt{l_2^2 + (D-x)^2}} \quad \Rightarrow \quad n_1 \frac{x}{d_1} = \frac{n_2\,(D-x)}{d_2} \quad \Rightarrow$$

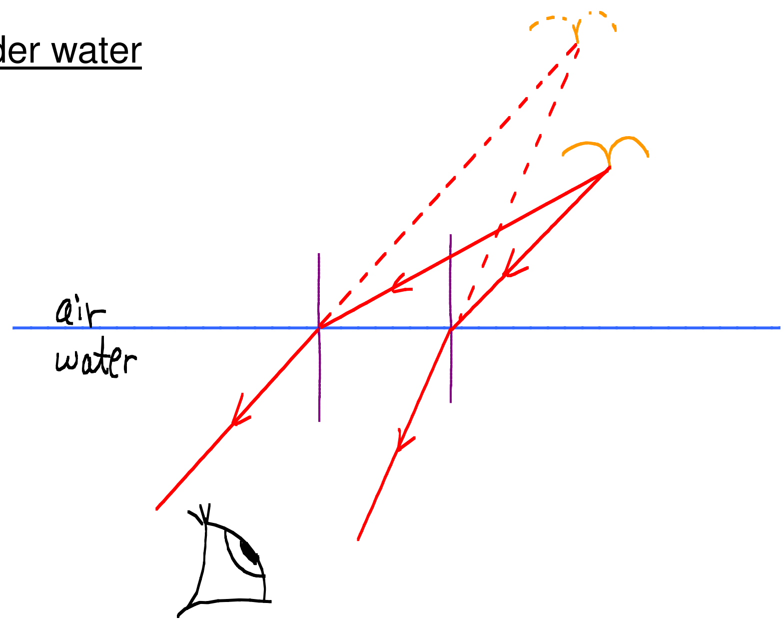
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

A similar proof can be developed for the law of reflection, Shooting fish from shore



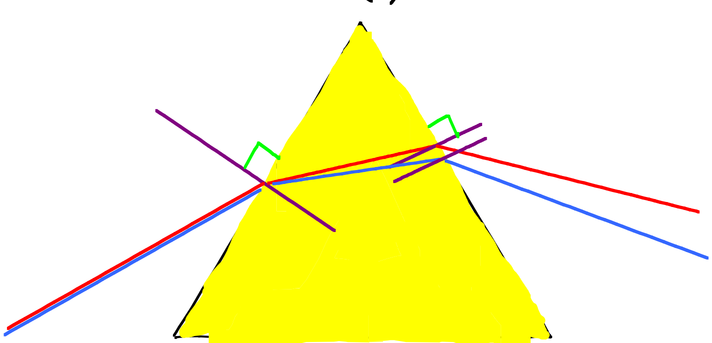
So, the hunter aims below the image of the fish to hit the actual fish.

Shooting gulls from under water



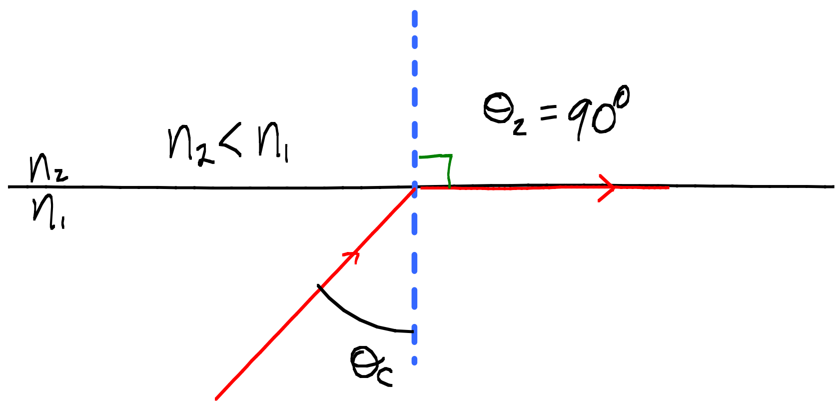
Dispersion: For most materials n is not the same value for all wavelengths.

$n = n(\lambda)$



$n_{blue} > n_{red}$ in most cases. Hence, the color separation depicted above because blue light bends more.

Critical Angle and Total Internal Reflection

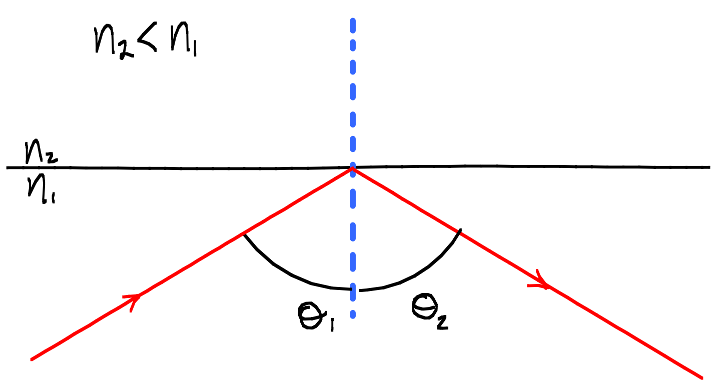


At the critical angle it is apparent that $\theta_1 = \theta_c$ and $\theta_2 = 90°$.

$$n_1 \sin\theta_c = n_2 \sin 90^\circ \Rightarrow \boxed{\theta_c = \sin^{-1}(n_2/n_1)}$$

When the incident angle is larger than the critical angle the light is totally internally reflected. Note there is no critical angle or total internal reflection when the incident medium has the smaller index.

Total Internal Reflection



The direction of the reflected ray is determined by the law of reflection. The light is nearly perfectly reflected. Aluminum and silver mirrors are only about 92% reflecting and degrade over time. In some situations it is better to use prisms in TIR rather than mirrors. Binoculars are an example. Another is guiding high power laser beams that can vaporize the metal off a metal mirror. This is also the principle that makes fiber optics possible.

Normal incidence reflection at an interface between transparent materials.

The difference in index at the interface between transparent materials determines the fraction of light reflected and transmitted. This ratio is also dependent on angles, but we only state the result for normal incidence. Namely,

$$\frac{I_r}{I_0} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

This result can be proved using Maxwell's equations. A very similar result applies to sonar waves bouncing off an interface between two media.

Application: Index matching epoxy to repair glass fibers, wet fabrics.....