

SP212 Blurb 14 2010

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Concave Spherical Mirrors

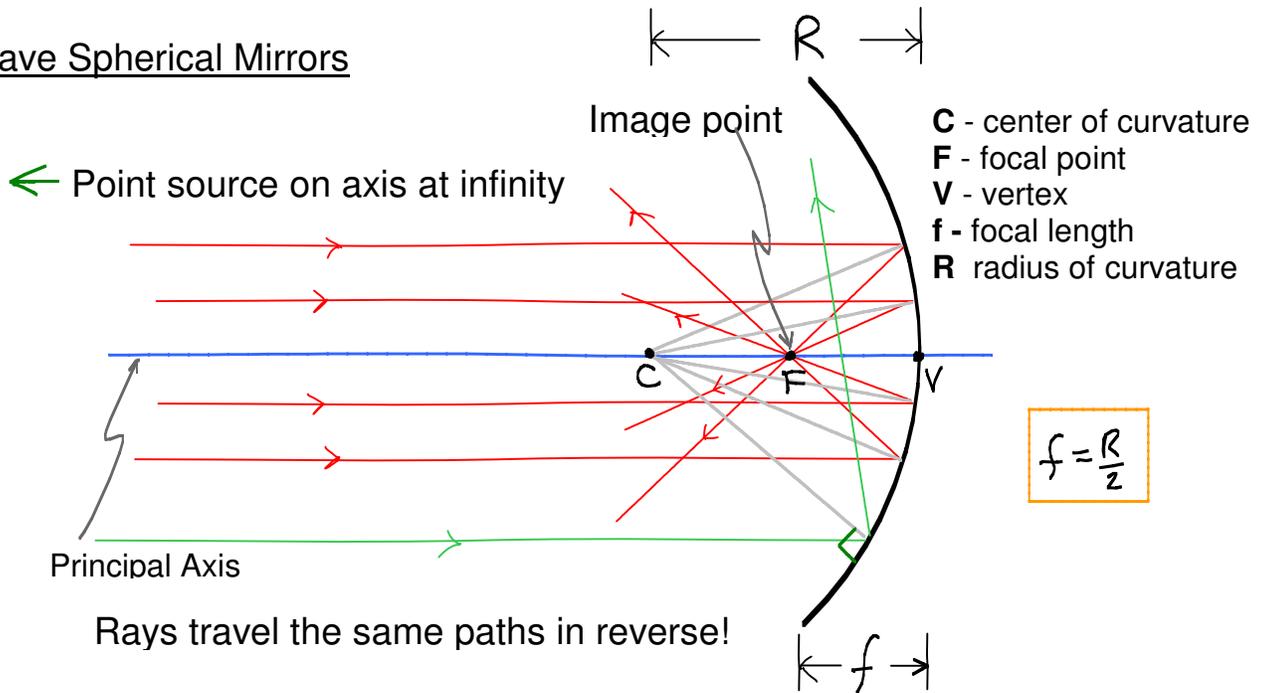
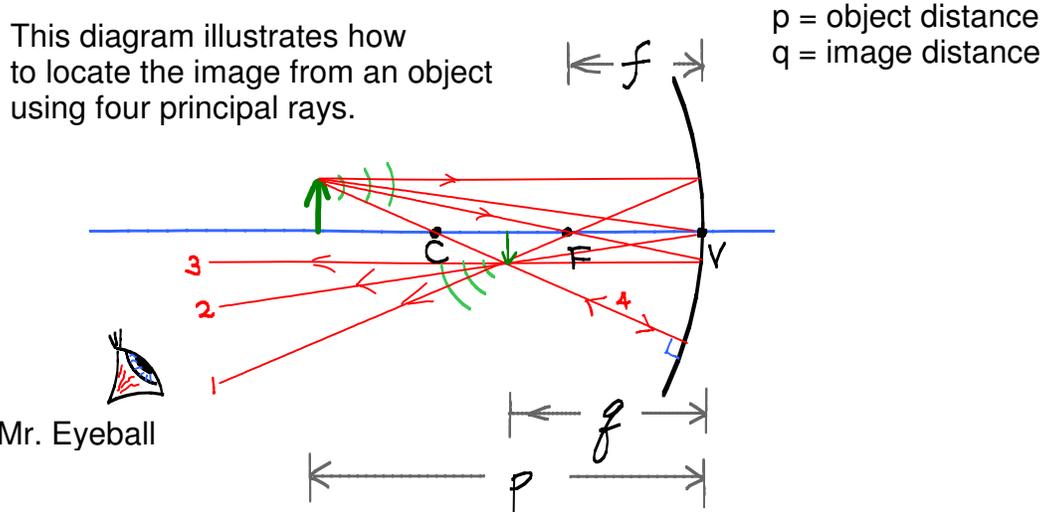
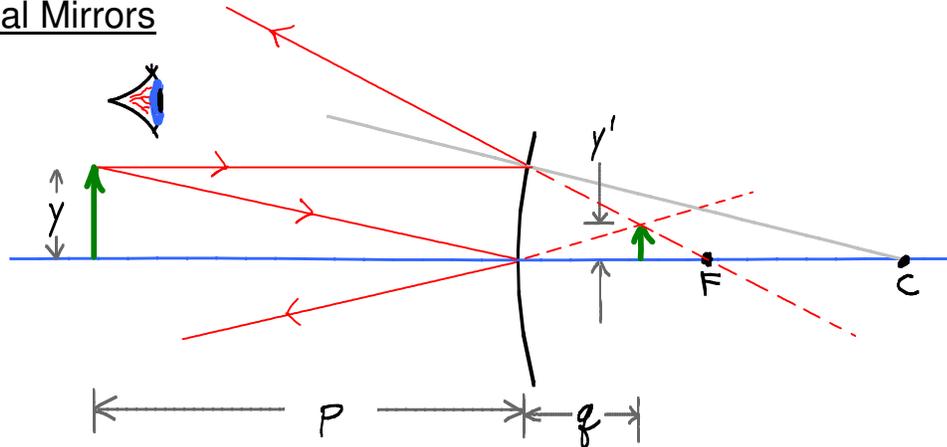


Image Location with Ray Diagrams



From Mr. Eyeball's perspective the reflected light appears to originate at the tip of the image arrow. In fact, the light energy really goes through the image so we call it a **real image**.

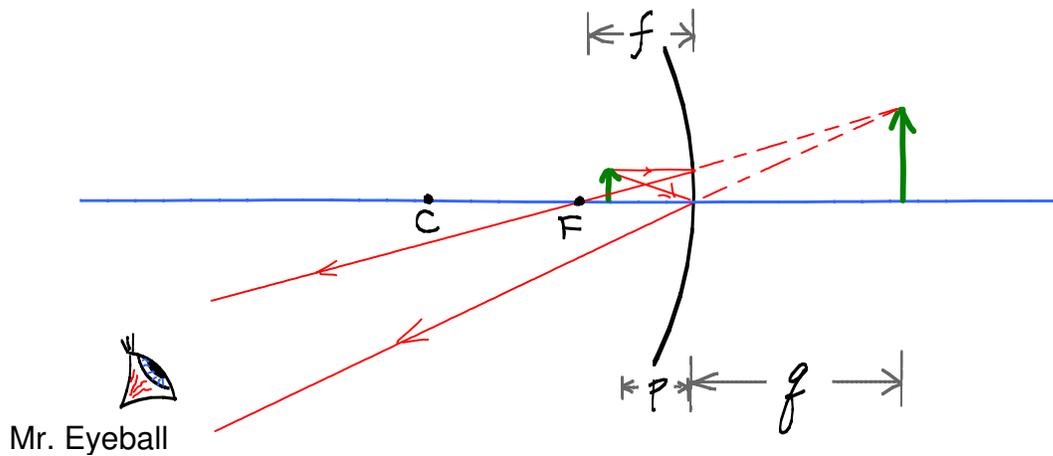
Convex Spherical Mirrors



Now from Mr. Eyeball's perspective the reflected light appears to originate at the tip of the image arrow. In fact, the light energy really doesn't go through the image so we call it a **virtual image**.

Real images are inverted, virtual images are not.

In the next diagram we'll place an object inside the focal point.



Once again the energy of the image rays do not originate at the image points, so this image is also virtual. Note also that the image is upright and magnified.

Sign Conventions:

1. If the object is on the same side of the reflector (refractor) as the incident light the object distance is positive. Otherwise the object distance is negative.
2. If the image is on the same side of the reflector (refractor) as the reflected (refracted) light the image distance is positive. Otherwise the image distance is negative.
3. If the center of curvature is on the same side of the reflector (refractor) as the reflected (refracted) light the radius is positive. Otherwise the radius is negative.
4. Inverted images have negative height. Upright images have positive height.

Ray diagrams are good for finding qualitative results. For quantitative results use the sign conventions and the relationship,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Example: In the diagram above let $p = 3.0$ cm and $R = 8.0$ cm to find the image distance, q .

$f = R/2$ and $R > 0$ since C is on the reflected side, $\Rightarrow f = 4.0$ cm

$p > 0$ since the object is on the incident side

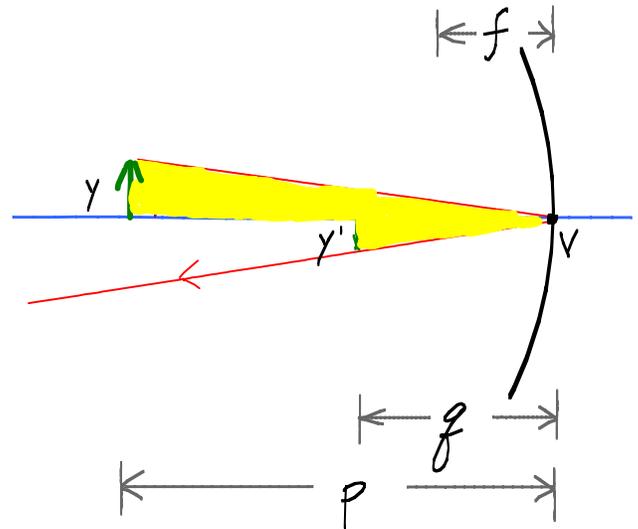
$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{4.0} - \frac{1}{3.0} = \frac{3}{12.0} - \frac{4}{12.0} = \frac{-1}{12.0} \Rightarrow q = -12.0 \text{ cm}$$

Negative image distance means the image is not on the reflected side of the mirror, in qualitative agreement with the diagram above. Also notice that the image is larger than the object. We define magnification as image height divided by object height, i.e. $m = y'/y$ where y' and y are image and object height, respectively. We will soon show that $m = -q/p$. Calculating m in the example above gives,

$$m = \frac{y'}{y} = \frac{-q}{p} = \frac{-(-12.0)}{3.0} = +4.0$$

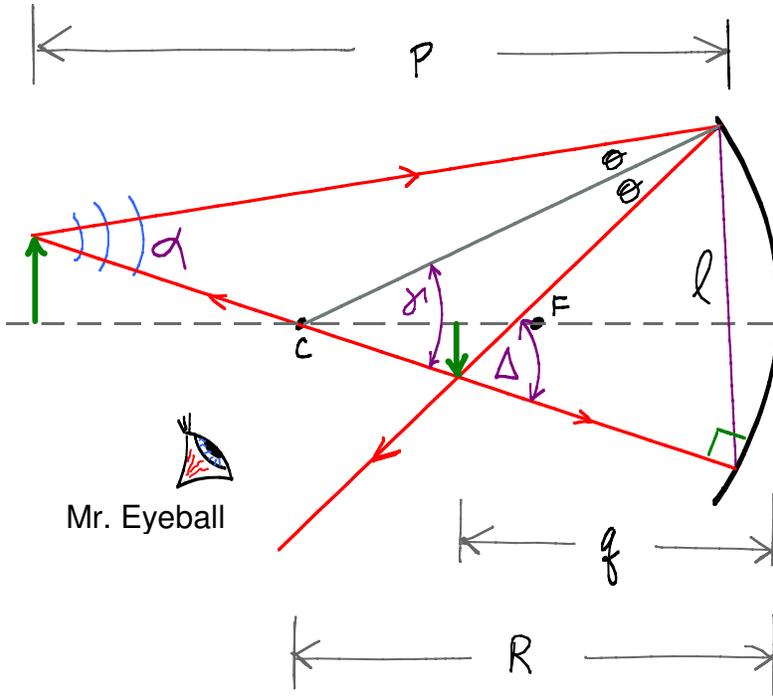
The image height is 4 times the object height. The positive sign indicates the image is upright, matching the predictions of the ray diagram pretty well.

To prove $y'/y = -q/p$ examine the yellow right triangles highlighted in the diagram. Since the law of reflection insures they have same angle at V the triangles are similar, so the ratio y'/y must equal q/p . The negative sign is simply added to accomodate our sign convention that inverted images have negative height.



Now that we've seen them in action, we'll prove the expressions above by using the paraxial ray approximations.

Reflection by a spherical mirror. (Assume paraxial rays)



Paraxial rays make small angles with respect to the principal axis and are not far off axis compared to R .

$$\Delta = \alpha + 2\theta \quad \text{eliminate } \theta$$

$$r = d + \theta \quad \Rightarrow \quad \Delta = \alpha + 2(r - d) \Rightarrow \Delta + \alpha = 2r$$

$$\text{but } \alpha \approx \frac{l}{p} ; r = \frac{l}{R} ; \Delta \approx \frac{l}{f}$$

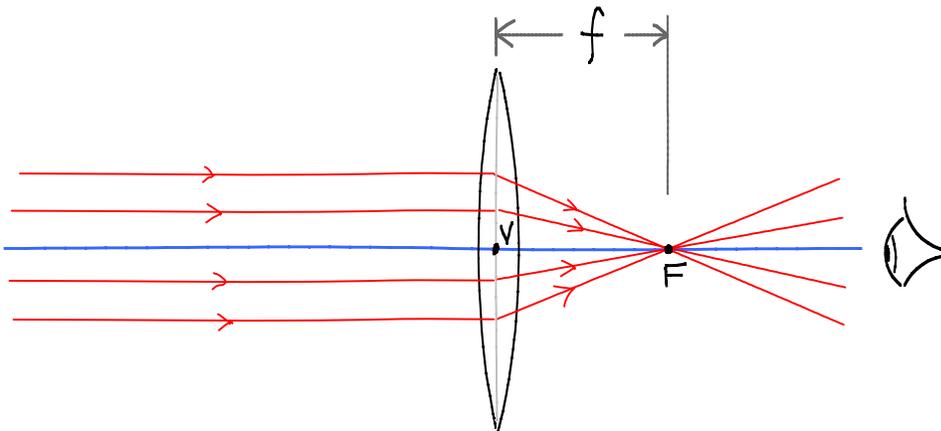
$$\Rightarrow \frac{l}{f} + \frac{l}{p} = \frac{2l}{R} \Rightarrow \frac{1}{p} + \frac{1}{f} = \frac{1}{R/2} = \frac{1}{f}$$

where f is obtained when $p \rightarrow \infty$

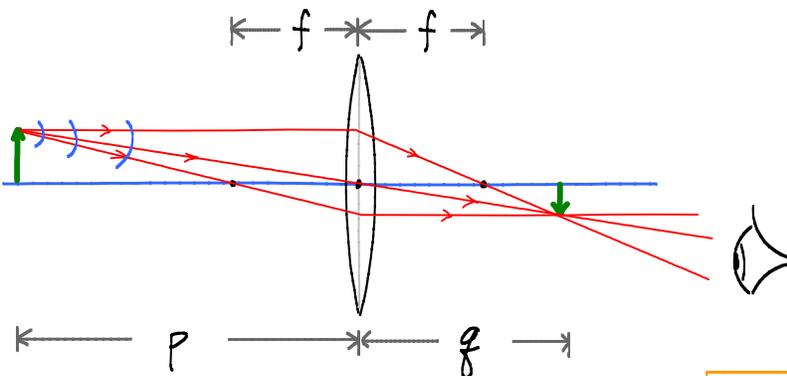
Thin Lenses

Since the procedures and conventions for lenses are so similar to the above, we'll treat them next. Later, we'll make the connection to the basic rule of refraction, Snell's law, and develop the basic rule of thin lens design. Here too we will operate within the confines of the paraxial ray approximation.

Rays are refracted at the front and back surfaces of the lens, but we're going to assume that the lens is so thin we can imagine all the refraction takes place at one plane surface inside the lens.



Parallel rays enter a lens and refract to the focal point F with a focal length f that extends from V to F . Since reversing the rays sends them along the same path, there is a focal point on each side of the lens. We'll use this ray behavior to locate the image for an object a finite distance from the lens.



Once again p , q , and f are related by,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

The magnification relationships are similar as well,

$$m = \frac{y'}{y} = -\frac{q}{p}$$

For an analytical solution of the situation in the diagram above let $f = 5.0$ cm and $p = 13.0$ cm to find image distance and magnification.

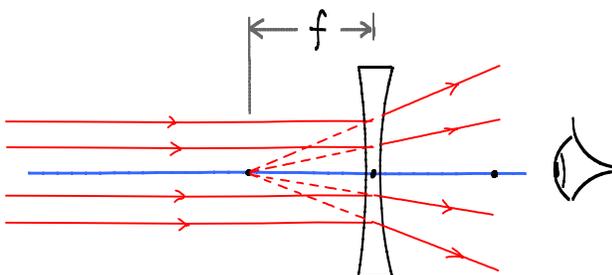
$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{5.0} - \frac{1}{13} \Rightarrow \frac{1}{q} = \frac{13.0 - 5.0}{13.0 \times 5.0} = \frac{8.0}{65.0} \Rightarrow q = \frac{65.0}{8.0} = 8.1 \text{ cm}$$

$$m = \frac{-q}{p} = \frac{-8.1}{13.0} = -0.62$$

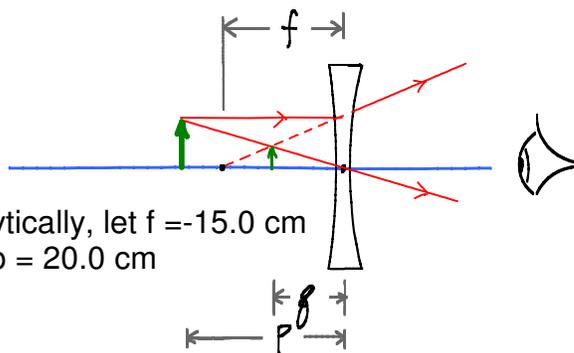
So the final image is 62% as tall as the object and is inverted since m is negative.

Center Ray: One of the rays in the diagram above goes undeviated through the center of the lens. The surfaces of the lens are nearly parallel at the lens center, causing the emerging central ray to be parallel to the incident central ray.

Negative Lenses are thinner in the middle than on the edges, causing the rays to bend away from the principal axis.



Parallel rays into the lens diverge as if they originated at the focal point indicated on the incident side of the lens. f is negative in this case. Again, we will use this ray behavior to locate the image for an object a finite distance from the lens.



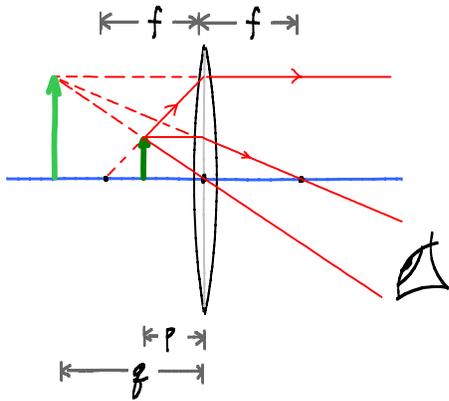
Analytically, let $f = -15.0$ cm and $p = 20.0$ cm

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{-15.0} - \frac{1}{+20.0} = \frac{-4}{60.0} - \frac{3}{60.0} = \frac{-7}{60.0}$$

$$\Rightarrow q = \frac{-60.0}{7} = -8.6 \text{ cm}$$

$$m = \frac{-q}{p} = \frac{-(-8.6 \text{ cm})}{20.0 \text{ cm}} = 0.43$$

The negative image distance indicates the image is virtual. Positive magnification less than one means the image is upright and smaller than the



The diagram at left illustrates a virtual image formed by a positive biconvex lens when the object is placed inside the focal point. This is a description of a magnifying glass.

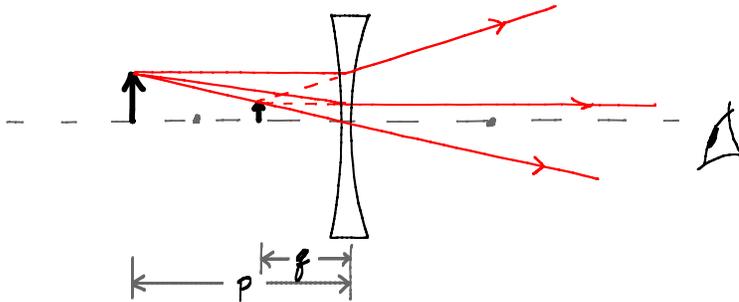
Analytically, if $f = 8.0$ cm and $p = 5.0$ cm then,

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{8.0} - \frac{1}{5.0} = \frac{5-8}{40} = -\frac{3}{40}$$

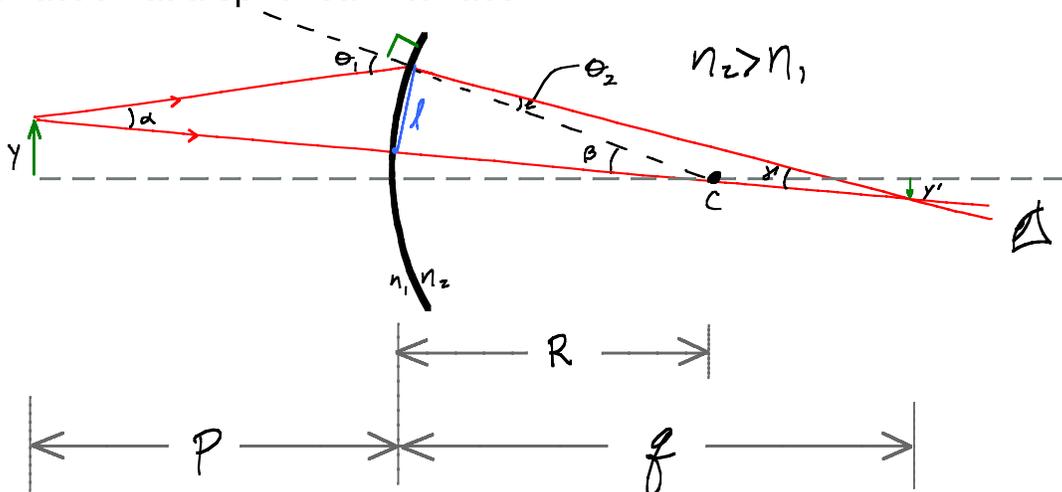
$$\Rightarrow q = -\frac{40}{3} = -13.3 \text{ cm} \Rightarrow m = \frac{-(-13.3)}{5} = 2.7$$

\Rightarrow an upright, magnified image.

Negative f , Negative q



Refraction at a spherical interface



$$\beta = \theta_2 + \delta \quad \theta_1 = \alpha + \beta \quad n_1 \theta_1 \approx n_2 \theta_2$$

Eliminate θ_1 and θ_2 ; $\frac{n_1}{n_2} \theta_1 = \beta - \delta \Rightarrow \frac{n_1}{n_2} (\alpha + \beta) = \beta - \delta$

$$\Rightarrow n_1 (\alpha + \beta) = n_2 (\beta - \delta)$$

$$\Rightarrow (n_1 - n_2) \beta = -n_1 \alpha - n_2 \delta$$

$$\Rightarrow (n_2 - n_1) \frac{l}{R} = n_1 \frac{l}{P} + n_2 \frac{l}{f}$$

$$\therefore \boxed{\frac{n_1}{P} + \frac{n_2}{f} = \frac{n_2 - n_1}{R}}$$

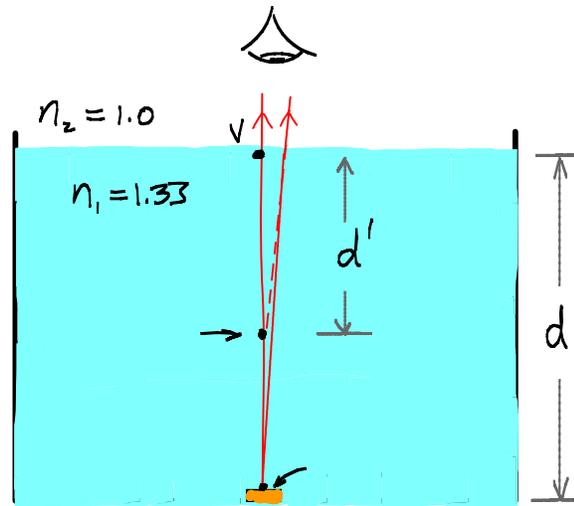
An easy application is to find the apparent depth of a pool of water. Imagine a coin at the bottom of the pool and an observer looking down from above. Find the image distance to find the apparent depth.

$$\frac{n_1}{d} + \frac{n_2}{d'} = \frac{n_2 - n_1}{R} ; R = \infty$$

$$\frac{1.33}{d} + \frac{1.0}{d'} = 0$$

$$\Rightarrow \boxed{d' = \frac{-d}{1.33}}$$

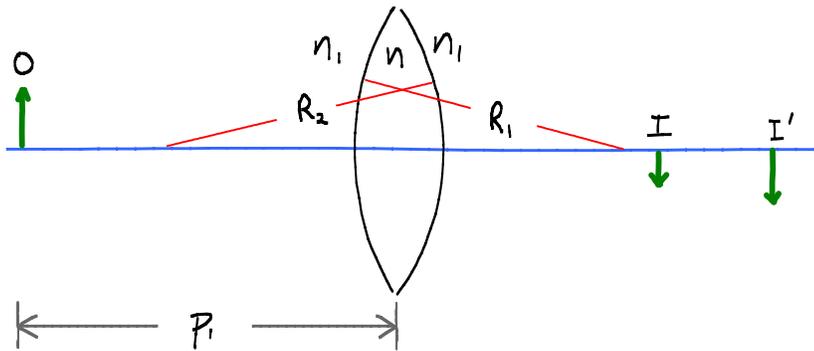
So the image is non-inverted



$$m = \frac{-d'}{d} = \frac{-(-d/1.33)}{d} \Rightarrow = \frac{1}{1.33} = \boxed{0.75}$$

The most significant application of $\boxed{\frac{n_1}{P} + \frac{n_2}{f} = \frac{n_2 - n_1}{R}}$ is to lenses

with spherical surfaces. The surfaces refract rays according to Snell's Law. We'll find the image I' from the first surface by completely ignoring the second interface and then use the image I' as the object for the second interface to find the final image I.



$$\frac{n_1}{p_1} + \frac{n}{f_1} = \frac{n-n_1}{R_1} \quad \text{where } q_1 \text{ is the distance from surface 1 to image I'}$$

$$\frac{n}{p_2} + \frac{n_1}{f_2} = \frac{n_1-n}{R_2} \quad \text{applies to the second surface where } q_2 \text{ is the final image}$$

By ignoring the thickness of the lens and applying the sign convention for object distances it is apparent that $q_1 = -p_2$. So, the last equation becomes,

$$-\frac{n}{f_1} + \frac{n_1}{f_2} = \frac{n_1-n}{R_2}$$

Adding this equation to the equation for surface one above gives,

$$\begin{aligned} \frac{n_1}{p_1} + \frac{n_1}{f_2} &= \frac{n-n_1}{R_1} + \frac{n_1-n}{R_2} \\ \Rightarrow \frac{1}{p_1} + \frac{1}{f_2} &= \frac{n-n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \end{aligned}$$

Now we can drop the subscripts and write,

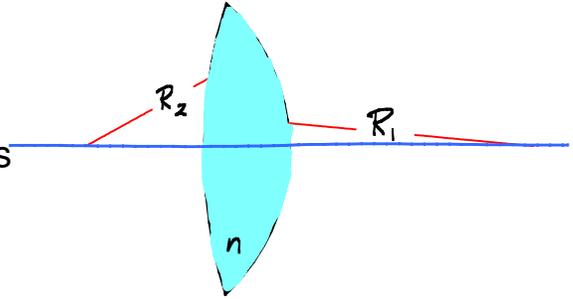
$$\frac{1}{p} + \frac{1}{f} = \frac{n-n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Moving the object to infinity results in an image distance of f , where f is the focal length of the lens. Thus,

$$\boxed{\frac{1}{f} = \frac{n-n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{Lensmaker's Formula}}$$

which is a prescription for designing a lens of a specific focal length f .

Example: Find the focal length of lens in the diagram if the magnitudes of R_1 and R_2 are 8.0 cm and 16.0 cm, respectively. The lens is made of flint glass with index 1.7 and is to be used in air.



$$\frac{1}{f} = \frac{n - n_i}{n_i} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

If the light is incident from the left, the center of curvature of surface one is on the refracted side while the center of curvature of surface 2 is not. So surface one has positive radius, while surface 2 does not.

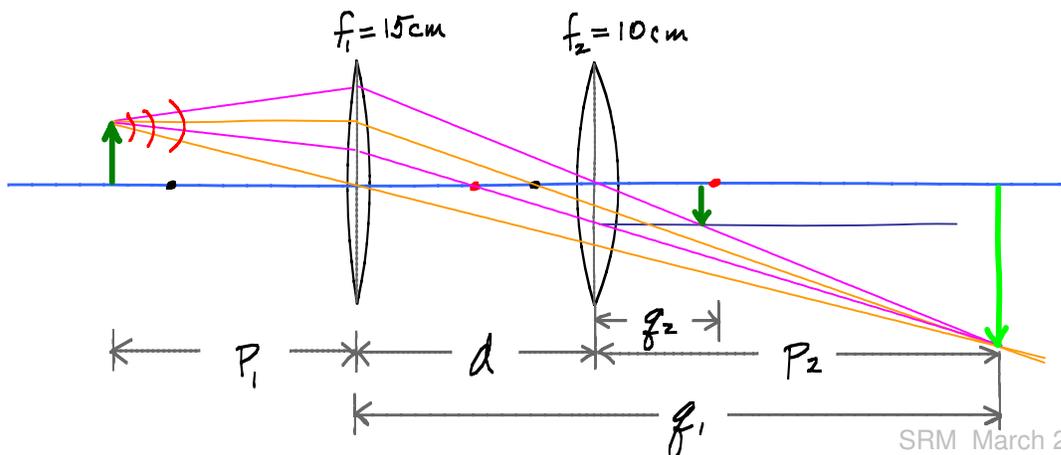
$$\frac{1}{f} = \frac{1.7 - 1.0}{1.0} \left(\frac{1}{16} - \frac{1}{(-8)} \right) = 0.7 * \frac{3}{16} = \frac{2.1}{16} \Rightarrow f = \frac{16}{2.1} = \boxed{7.6 \text{ cm}}$$

If we put the same lens in water there is less index difference between the two media so the focal length gets longer.

$$\frac{1}{f_w} = \frac{1.7 - 1.33}{1.33} \left(\frac{1}{16} - \frac{1}{(-8)} \right) \Rightarrow \boxed{f_w = 19.2 \text{ cm}}$$

Two Lens Systems:

To locate the final image when light enters a two lens system, find the image from the first lens while ignoring the second. Then treat the image from the first lens as the object for the second. This is the same procedure used earlier to analyze the thin lens as two spherical surface refractions. The next example



Analytically, let $p_1 = 20.0 \text{ cm}$, $d = 20.0 \text{ cm}$

$$\Rightarrow \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \Rightarrow \frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{15} - \frac{1}{20} = \frac{4}{60} - \frac{3}{60} = \frac{1}{60}$$

$$\Rightarrow q_1 = 60 \text{ cm} \Rightarrow |p_2| = 60.0 \text{ cm} - 20.0 \text{ cm} = 40.0 \text{ cm}$$

Now treat the second lens. $p_2 = -40 \text{ cm}$ since the object for lens 2 is not on the incident side.

$$\Rightarrow \frac{1}{f_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{10} - \frac{1}{-40} = \frac{1}{10} + \frac{1}{40} = \frac{4}{40} + \frac{1}{40} = \frac{5}{40}$$

$$\Rightarrow f_2 = 8.0 \text{ cm} \quad \text{which agrees well with the ray diagram above.}$$

Overall magnification $m = m_1 * m_2 = \frac{-q_1}{p_1} * \frac{-q_2}{p_2}$

$$\Rightarrow m = \frac{60}{20} * \frac{8}{-40} = -\frac{3}{5} = -0.60$$

which also agrees well with the ray diagram.