

## SP212 Blurb 6 2010

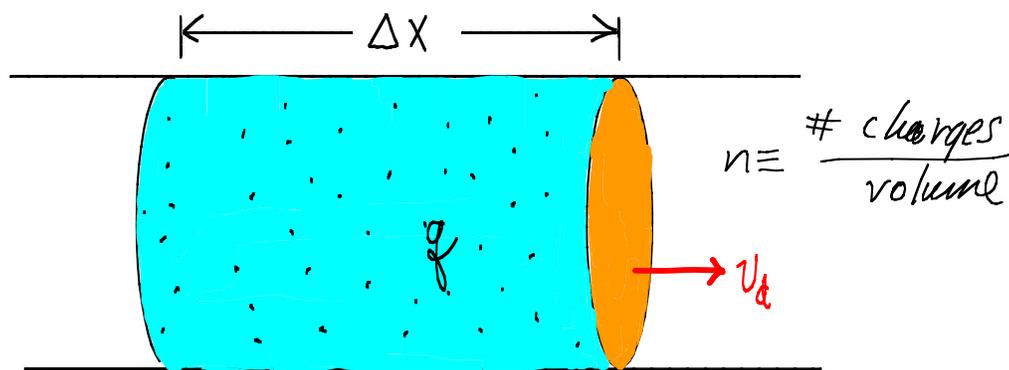
1. Electric current & charge carriers
2. Current density & conductivity (Microscopic model)
3. Resistance, current density + conductivity
4. Drude model of conductor
5. Temperature dependence of resistance
6. Electric power
7. Battery with internal resistance.
8. Load matching.

## Electric Current

$$I \equiv \frac{\Delta Q}{\Delta t} \xrightarrow{\Delta t \rightarrow 0} \frac{dQ}{dt} \quad \text{units } \frac{C}{s} \equiv \text{Ampere (A)}$$

It was early assumed that positive charges in conductors were mobile. That usually turns out not to be the case.

## Microscopic Model



$$\Delta Q = n A \Delta x q \Rightarrow I = \frac{\Delta Q}{\Delta t} = \frac{n A \Delta x q}{\Delta t}$$

$$\frac{\Delta x}{\Delta t} = v_d \quad \text{the drift velocity of the charges}$$

$$\Rightarrow I = n q v_d A$$

**Example 27.1** from text: A 12 gauge copper wire carries 10 Amps of current. Find the drift velocity of electrons in the wire.

$$M_{Cu} = 63.5 \text{ g/mol}$$

$$\Rightarrow A = 3.31 \times 10^{-6} \text{ m}^2$$

$$\Rightarrow V = \frac{m}{\rho} = \frac{63.5 \text{ g}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3 \quad \text{is the volume of one mole}$$

Assuming one conduction electron per atom,

$$\Rightarrow n = \frac{N_A}{V} = \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3 \times 1 \text{ m}^3 / 10^6 \text{ cm}^3} = 8.49 \times 10^{28} \frac{\text{electrons}}{\text{m}^3}$$

$$\Rightarrow v_d = \frac{I}{nqA} = \frac{10 \text{ A}}{(8.49 \times 10^{28} \frac{\text{elec}}{\text{m}^3})(1.6 \times 10^{-19} \text{ C})(3.31 \times 10^{-6} \text{ m}^2)}$$

$$= 2.22 \times 10^{-4} \text{ m/s} = 0.22 \text{ mm/s} !$$

## Resistance

Current per unit area  $\equiv$  current density,  $J$

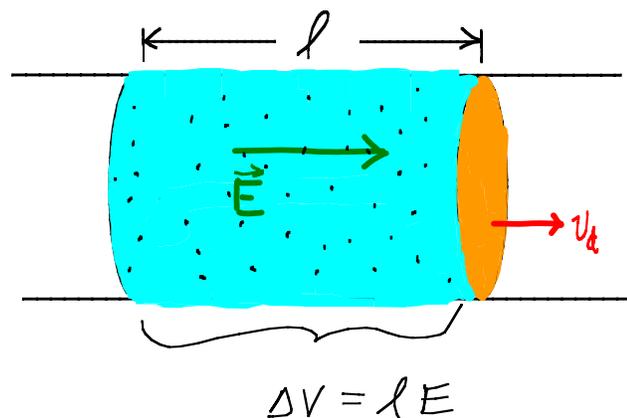
$$J = \frac{I}{A} = nq v_d \text{ from earlier result.}$$

$J \propto E$  in conductor  $J = \sigma E$   $\sigma \equiv$  conductivity and is material dependent

see Table 27.1 on p.837 of text.

Adding directionality:  $\vec{J} = \sigma \vec{E}$

This last expression is closely related to Ohms Law,



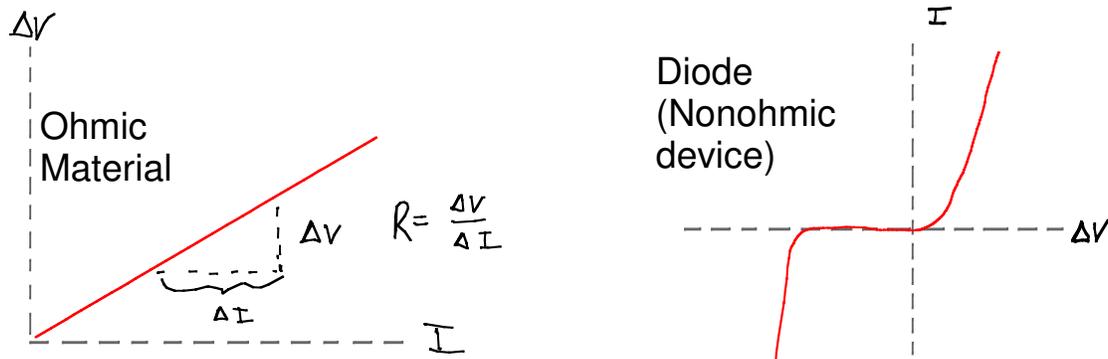
$$J = \sigma E \Rightarrow \frac{I}{A} = \sigma \frac{\Delta V}{l}$$

$$\Rightarrow \Delta V = I \left( \frac{l}{\sigma A} \right) \text{ Compare to } \Delta V = IR \Rightarrow R = \frac{l}{\sigma A}$$

Alternative to conductivity is resistivity  $\rho \equiv \frac{1}{\sigma}$

$$\Rightarrow R = \rho \frac{l}{A} \quad \text{for uniform cross section}$$

materials that have  $I \propto \Delta V$  are called ohmic.



**Example:** What is the diameter of a 1 m gold wire with a resistance of 1 ohm?

$$R = \rho \frac{l}{A} = \frac{\rho l}{\pi r^2} \Rightarrow r^2 = \frac{\rho l}{\pi R} \Rightarrow r = \sqrt{\frac{\rho l}{\pi R}}$$

$$\Rightarrow r = \sqrt{\frac{(2.44 \times 10^{-8} \Omega \cdot \text{m})(1 \text{ m})}{\pi (1 \Omega)}} = 88 \times 10^{-6} \text{ m}$$

$$\Rightarrow \text{diam} = 167 \mu\text{m} = 0.167 \text{ mm}$$

"Gas model" of electrical conduction developed by Drude ~ 1900

Thermal speed of electrons @ room temp ~  $10^6$  m/s we've seen drift speeds are ~  $10^{-4}$  m/s

Consider that electrons loose all their energy with each atomic collision. If there is an  $\mathbf{E}$  field present in the conductor then between collisions,

$$\vec{v}_f = \vec{v}_i + \vec{a}t \quad ; \quad \vec{F}_e = m_e \vec{a} \quad ; \quad \vec{F}_e = q \vec{E}$$

$$\Rightarrow \vec{v}_f = \vec{v}_i + \frac{q \vec{E}}{m_e} t$$

Now let's average over many collisions.

$$\overline{\vec{v}_i} = 0 \quad ; \quad \vec{v}_f = \vec{v}_d \quad ; \quad \overline{t} = \tau$$

$\tau \equiv$  mean time between collisions

$$\Rightarrow \vec{v}_d = f \frac{\vec{E} \tau}{m_e} \xrightarrow[\text{terms}]{\text{scalar}} v_d = f \frac{E \tau}{m_e} \star$$

Now recall  $J = \sigma E$  and  $J = nq v_d$

$$\Rightarrow nq v_d = \frac{nq^2 E \tau}{m_e} \text{ from } \star$$

$$\Rightarrow J = \frac{nq^2 \tau}{m_e} E = \sigma E \Rightarrow \sigma = \frac{nq^2 \tau}{m_e}$$

Each term make sense : bigger  $n \Rightarrow$  bigger  $\sigma$

"  $f \Rightarrow$  " "

"  $\tau \Rightarrow$  " "

"  $m_e \Rightarrow$  smaller  $\sigma$

Since we calculated  $v_d = 0.22$  mm/s earlier in a copper wire let's find  $\tau$  in the same copper wire under the same conditions.

$$\sigma = \frac{1}{\rho} = \frac{nq^2 \tau}{m_e} \quad ; \quad \tau = \frac{m_e}{nq^2 \rho}$$

$$\Rightarrow \tau = \frac{9.11 \times 10^{-31} \text{ kg}}{\left(8.49 \times 10^{28} \frac{\text{elec}}{\text{m}^3}\right) (1.6 \times 10^{-19} \text{ C})^2 (1.7 \times 10^{-8} \Omega \text{ m})}$$

$$\tau \approx 2.5 \times 10^{-14} \text{ sec}$$

Assuming an average electron speed of  $1.6 \times 10^6$  m/s, how far does an electron move between collisions?

$$l = v \tau = (1.6 \times 10^6 \text{ m/s})(2.5 \times 10^{-14} \text{ s})$$

$$= 4 \times 10^{-8} \text{ m} = 40 \text{ nm}$$

A small distance, but hundreds of inter atomic spacings.

**Temperature dependence of resistance:** R for a sample is temperature dependent. We define a temperature coefficient of resistivity ( $\alpha$ ) by,

$$\alpha = \frac{\Delta \rho / \rho_0}{\Delta T}; \quad \frac{\Delta \rho}{\rho_0} \text{ is fractional change in } \rho$$

$$\Rightarrow \alpha = \frac{(\rho - \rho_0) / \rho_0}{\Delta T} \Rightarrow \rho_0 \alpha \Delta T = \rho - \rho_0$$

$$\Rightarrow \rho = \rho_0 [1 + \alpha (T - T_0)] \quad 27.19 \text{ in text}$$

multiply previous by,  $l/A$

$$\Rightarrow R = R_0 [1 + \alpha (T - T_0)] \quad R_0 = \text{room temp. } R$$

The expression you used to get the temperature of a hot light bulb filament in lab.

## Electric Power

If a charge Q travels through a region with potential difference  $\Delta V$ , then the potential energy change  $\Delta U = Q\Delta V$ . If we're dealing with very small amount of charge,  $dU = QdV$ . If this happens over a small time interval then,

$$\frac{dU}{dt} = \Delta V \frac{dQ}{dt} \Rightarrow P = \Delta V I$$

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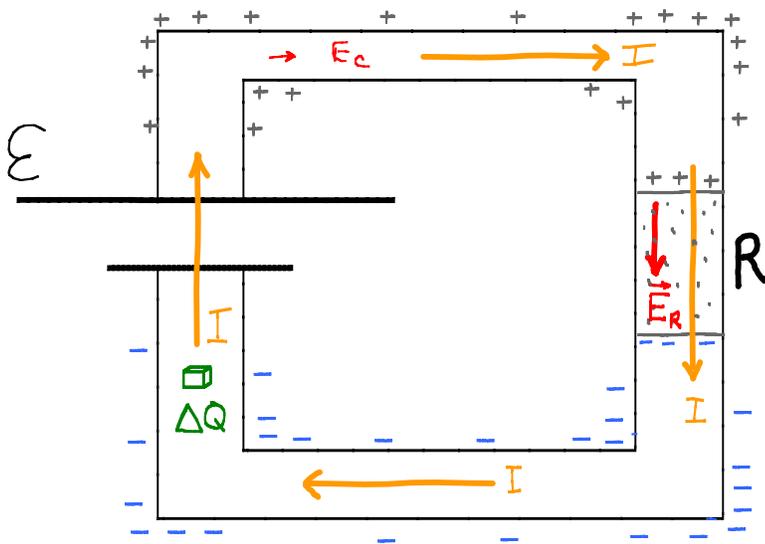
So, the power lost by charges going through a resistor is,

$$P_{\text{lost}} = (IR)I = I^2R$$

while power gained by charges going through a battery with emf  $\epsilon$  is,

$$P_{\text{gained}} = \epsilon I$$

### Conceptual Picture



If we follow a small charge,  $\Delta Q$  around the circuit at left, one complete lap has to result in no energy change to conserve energy. The energy boost through the emf of the battery is  $\epsilon \Delta Q$  while the energy change through the resistor is  $-\Delta Q \Delta V$ , where  $\Delta V$  is the magnitude of the potential difference across the resistor. The energy drops through the resistor because the charge travels downstream in the field,  $E_R$ .

$$\Rightarrow \epsilon \Delta Q - \Delta V \Delta Q = 0$$

$$\Rightarrow \frac{\epsilon \Delta Q}{\Delta t} - \frac{\Delta V \Delta Q}{\Delta t} = 0 \quad \text{divide by } \Delta t, \text{ the time for the charge to move through each device.}$$

$$\Rightarrow \epsilon I - \Delta V I = 0$$

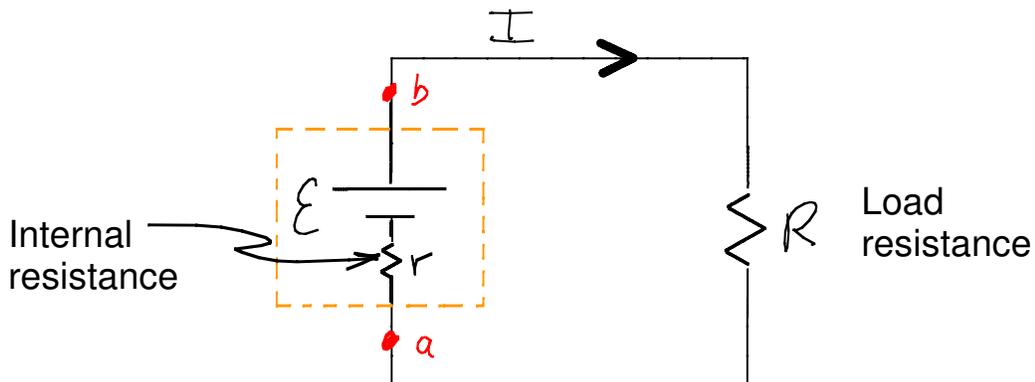
So, the power from the battery equals the power to the resistor.

$$\Rightarrow \epsilon - \Delta V = 0 \quad \text{Now use Ohm's law to write,}$$

$$\Rightarrow \epsilon - IR = 0 \quad \text{Conservation of energy results in the voltage drops around the loop adding to zero. (Kichhoff's Loop rule)}$$

## Battery with internal resistance

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Keeping track of the voltage changes around the circuit we write,

$$\mathcal{E} - IR - Ir = 0 \quad \star$$

The voltage across the battery terminals is not simply  $\mathcal{E}$  but is reduced by the voltage drop across the internal resistance, i.e.,

$$\mathcal{E} - Ir = V_{ab} = V_b - V_a$$

$$\star \Rightarrow I = \frac{\mathcal{E}}{R+r} \quad \text{which tells us the current is also limited by the battery's internal resistance.}$$

Also,

$$\star \Rightarrow \mathcal{E}I = I^2 r + I^2 R \Rightarrow \text{Power delivered by the emf is dissipated by the resistors. Energy conservation, once again.}$$

**Load matching:** Suppose the load resistor is variable and we start with huge resistance that we gradually lower to zero. At what value of load resistance is the battery delivering maximum power to the load? At infinite load resistance there is no power delivered to the load because  $I = 0$  thus,  $I^2 R = 0$ . Also, at zero resistance  $I^2 R = 0$  because  $R = 0$ . So, somewhere in between there must be a

$$P(R) = I^2 R = \left(\frac{\mathcal{E}}{r+R}\right)^2 R \quad \text{To maximize } P(R) \text{ set } dP/dR = 0 \text{ and solve for } R.$$

$$\frac{dP}{dR} = \mathcal{E}^2 \left[ \frac{(r+R)^2 - 2(r+R)R}{(r+R)^4} \right] = 0 \Rightarrow (r+R)^2 = 2(r+R)R \Rightarrow r+R = 2R$$

$$r = R$$

Which means we deliver the maximum power to the load when the load resistance matches the internal resistance of the battery. This tells us that we need the resistance of our automobile starter to be pretty close to the internal resistance of the battery to deliver max power to the starter. For a new battery the internal resistance is small and this works well. As the battery ages its internal resistance goes up and more power is dissipated as heat in the battery, delivering less to the starter. Eventually, the internal resistance gets so large the car won't start.

Load matching is also an important consideration when matching speakers to amplifiers.