

## SP212 Blurb 8 2010

1. Magnets (Demo magnetic field lines).
2. Demo magnetic force on a moving charge.
3. Cross product form of the magnetic force law.
4. Application: Cyclotron.
5. Application: Magnetic bottles.
6. Application: The velocity selector.
7. Application: The mass spectrometer.
8. Magnetic force on a current carrying wire.
9. Application: Measuring B with a pan balance.
10. The Hall Effect
11. The magnetic dipole.

Magnets always have North & South poles: No isolated monopoles!

Corollary: **B** lines are always loops!

**B** lines emerge from North poles and enter South poles.

**Magnetic force** on a moving charge.

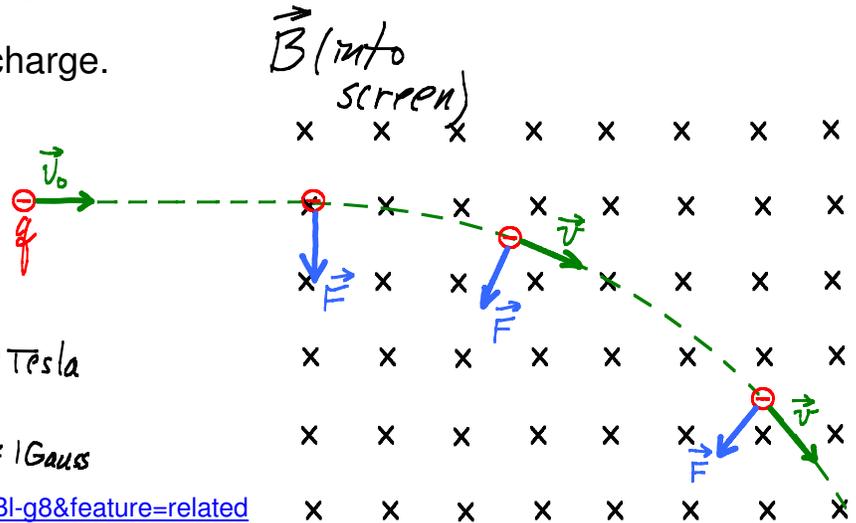
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$|\vec{F}| = qvB \sin\theta$$

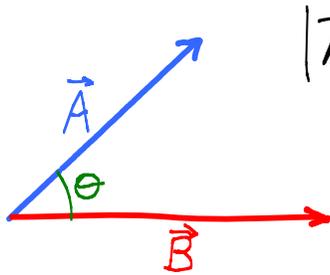
$$B = \frac{|\vec{F}|}{qv \sin\theta} \Rightarrow \frac{N}{C \cdot m/s} = \frac{Ns}{Cm} = 1 \text{ Tesla}$$

1 Tesla = 10,000 Gauss ;  $B_{earth} \approx 1 \text{ Gauss}$

[http://www.youtube.com/watch?v=a2\\_wUDBI-g8&feature=related](http://www.youtube.com/watch?v=a2_wUDBI-g8&feature=related)



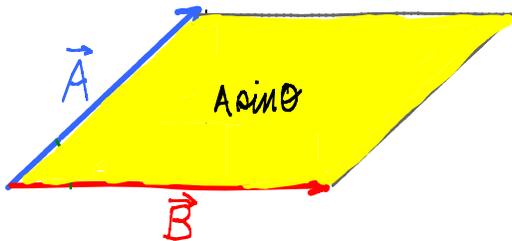
Cross product review



$|\vec{A} \times \vec{B}| = AB \sin\theta$  direction is given by right hand rule.

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} ; \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - B_x A_z) + \hat{k}(A_x B_y - A_y B_x)$$



Geometrically, we can interpret the magnitude of the cross product as the area of the parallelogram bounded by the two vectors.

$$\text{Area} = \text{Base} \cdot \text{height} = BA \sin\theta = |\vec{A} \times \vec{B}|$$

Work done on a moving charge by a static magnetic field.

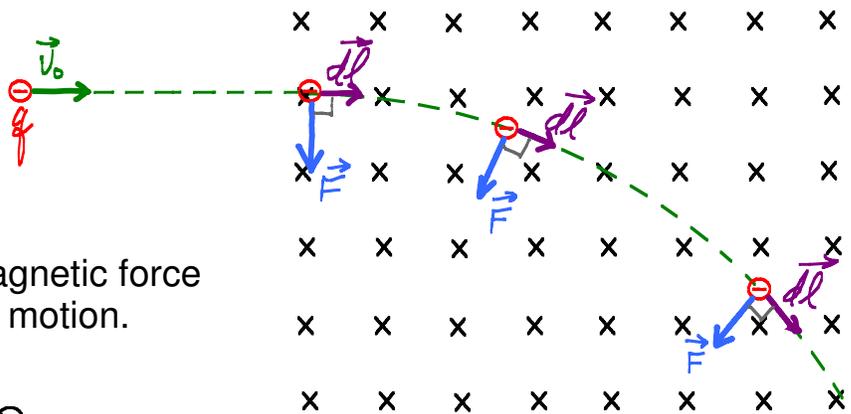
$$W = \int \vec{F} \cdot d\vec{l}$$

As the diagram shows, the magnetic force is always perpendicular to the motion.

$$\Rightarrow \vec{F} \cdot d\vec{l} = 0 \Rightarrow W = 0$$

Static magnetic fields do no work on moving charges.

$\Rightarrow$  Kinetic energy is constant  $\Rightarrow$  speed is constant.



Cyclotron frequency: Consider a charged particle in a constant magnetic field.

v is constant since kinetic energy is constant. Force is perpendicular to v.

$\Rightarrow$  circular motion

$\Rightarrow$  centripetal acceleration

$$\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB$$

*since  $\vec{v} \perp \vec{B}$*

$$F = ma = \frac{mv^2}{r}$$

$$\Rightarrow qvB = \frac{mv^2}{r} \Rightarrow qB = \frac{mv}{r}$$

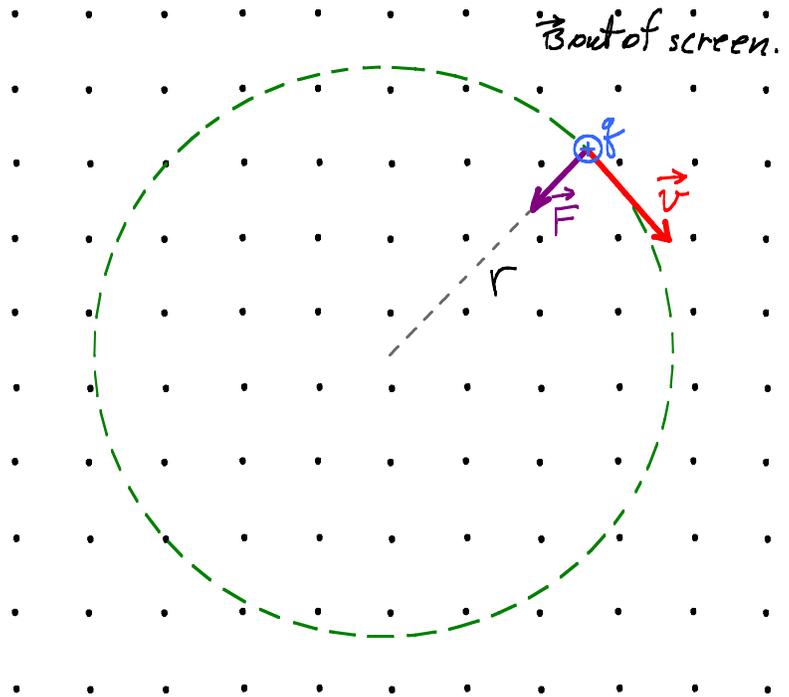
$$\Rightarrow \boxed{v = \frac{qBr}{m}} ; v \propto r$$

but  $v = \frac{2\pi r}{T}$  ; T  $\equiv$  period

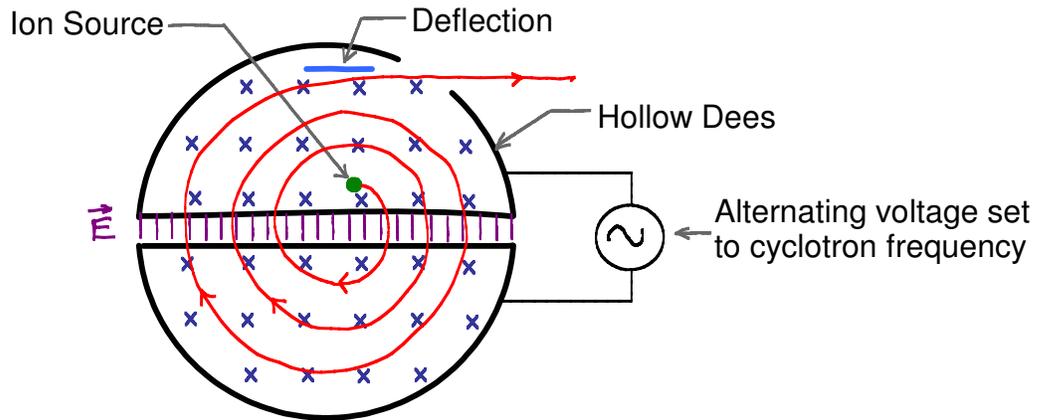
$$\frac{2\pi r}{T} = \frac{qBr}{m} \Rightarrow \frac{2\pi}{T} = \frac{qB}{m} ; \omega_c \equiv \frac{2\pi}{T} = \frac{qB}{m}$$

$$\boxed{\omega_c = \frac{q}{m} B}$$

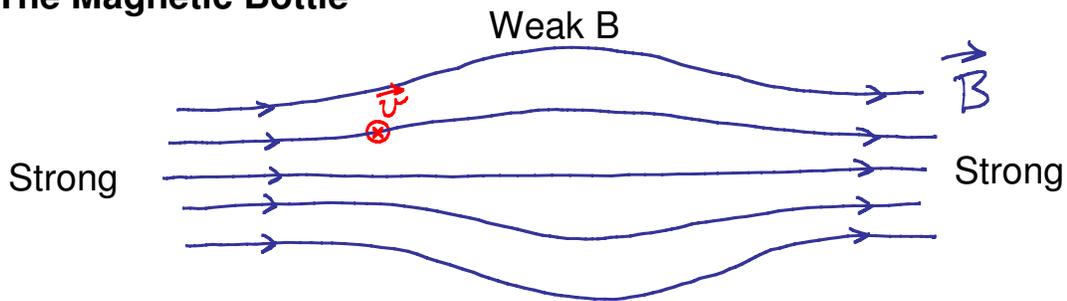
Does not depend on v!



## The Cyclotron

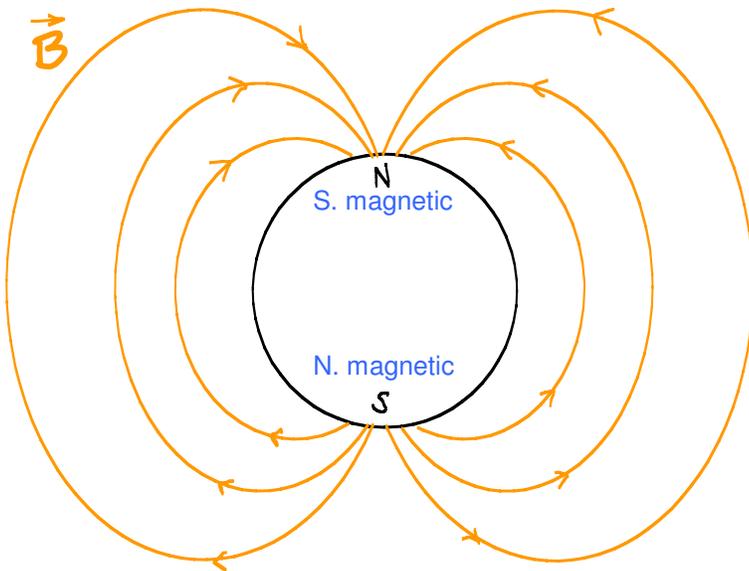


## The Magnetic Bottle



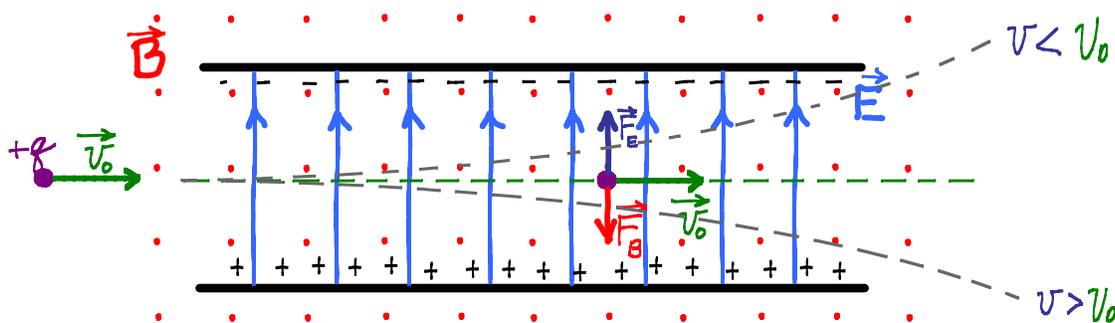
## Earth's Magnetic Bottle

<http://www.youtube.com/watch?v=6CpNOu4l1dM&feature=related>



## The Velocity Selector

Atoms can be stripped of electrons to make charged ions that can then be manipulated by electric and magnetic fields. One useful ion manipulator is the velocity selector, depicted below. It is basically a capacitor with a magnetic field perpendicular to the electric field between the plates. Charged ions moving perpendicular to both fields experience opposing electric and magnetic forces, as shown in the diagram. For one velocity the forces are equal and opposite, resulting in no deflection. Ions with larger or smaller speeds are deflected up or down, depending on the relative strength of the electric and magnetic forces.

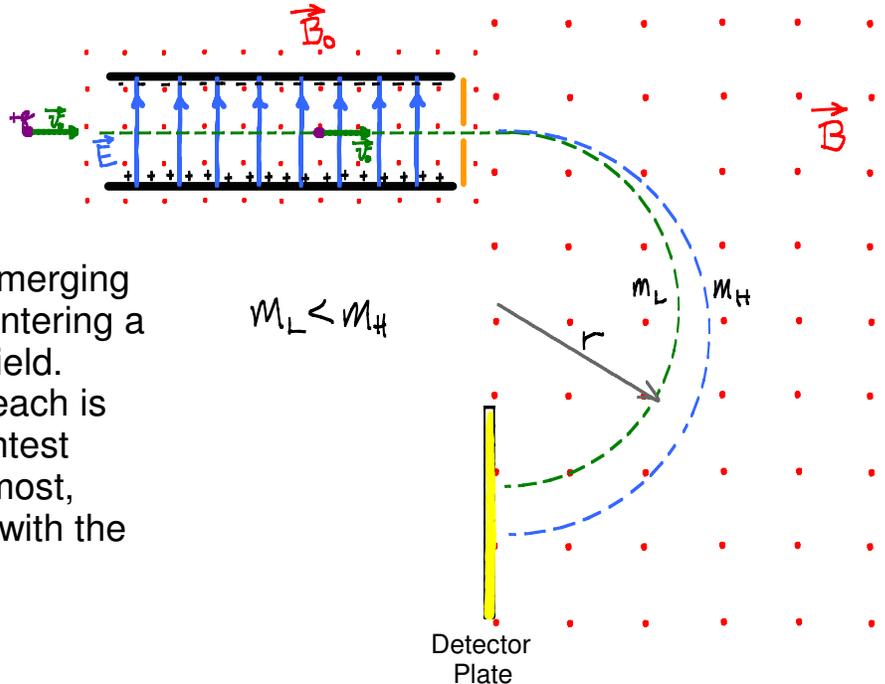


For the selected velocity,  $v_0$ , the electric and magnetic forces have the same magnitude. So,

$$F_B = F_E \Rightarrow qv_0B = qE \Rightarrow v_0 = \frac{E}{B}$$

## The Mass Spectrometer

It is possible to detect the atomic makeup of an unknown sample using a device called a mass spectrometer, depicted below. First the sample is heated in an oven to break all chemical bonds, then atoms emerging from the oven are ionized to give them charge. Since the ions will have a large range of speeds when they emerge from a hot oven, they are passed through a velocity selector before entering a region of constant magnetic field. The radius of trajectory in the constant field depends on the ion's mass, causing ions of differing mass to strike different points on a detector. The strength of signal on the detector can be calibrated to reveal the amount of each atom in the sample.



Shown at right are two ions emerging from a velocity selector and entering a region of constant magnetic field. Since the magnetic force on each is the same, the ion with the lightest mass  $m_L$ , is accelerated the most, corresponding to a trajectory with the smallest radius.

①  $F = ma$

②  $a = \frac{v^2}{r}$  centripetal acceleration

③  $F = qvB$  magnetic force with  $v \perp B$ .

④  $\frac{mv^2}{r} = qvB$

⑤  $v = \frac{E}{B_0}$  from the velocity selector

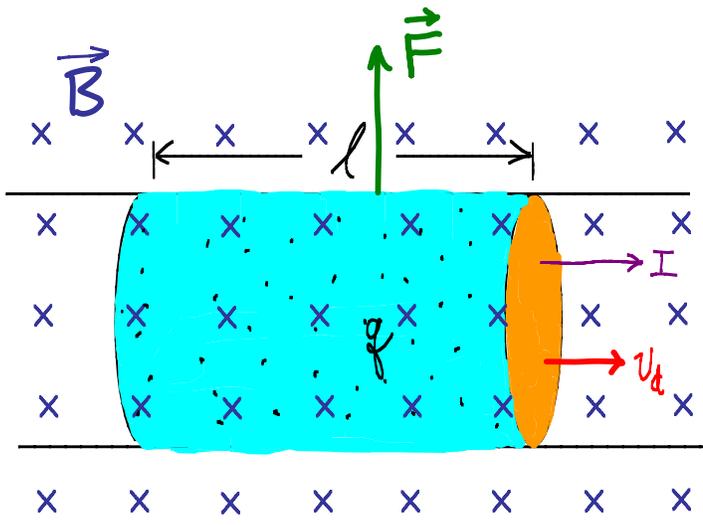
⑥  $\frac{m}{r} \frac{E}{B_0} = qB$

⑦  $\frac{q}{m} = \frac{E}{rB B_0}$

So, knowing the magnitudes of the electric and magnetic fields and measuring  $r$  with the detection plate allows one to identify the charged particles going into the mass spectrometer.

### Magnetic Force on a Current Segment

We know the magnetic force on a single moving charge is  $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ . Since electric current in a wire is many small charges traveling at the drift velocity in response to an electric field in the wire, you might correctly suspect that a magnetic field will exert force on a current carrying wire.



$$\vec{F} = N q \vec{v}_d \times \vec{B}; \quad N = \# \text{ of moving charges}$$

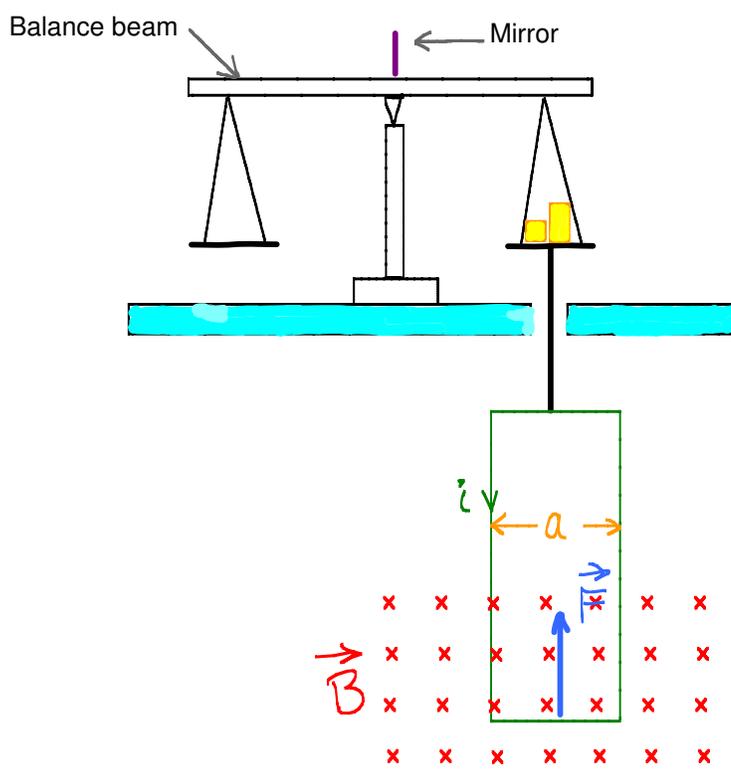
$$N = n A l; \quad n \equiv \# \text{ of charges/volume}$$

$$\Rightarrow \vec{F} = n A l q \vec{v}_d \times \vec{B}$$

Recall  $\frac{I}{A} = n q v_d$

$$\Rightarrow \boxed{\vec{F} = I \vec{l} \times \vec{B}}$$

Application: Magnetic Field Measurement



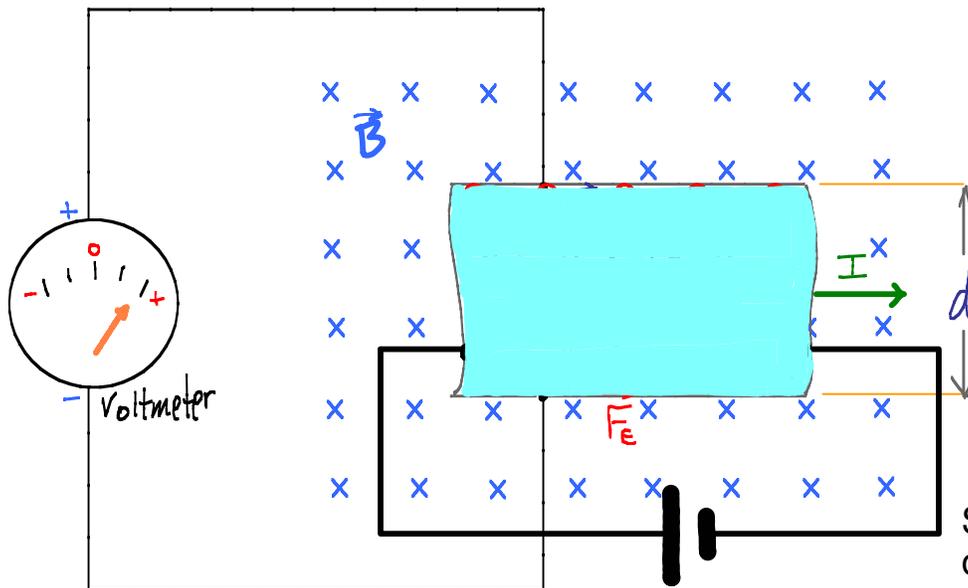
The modified pan balance shown at left can be used to measure the magnetic field. Masses,  $m$ , added to the right pan cause that side of the balance to drop. Current  $i$ , in the loop is increased until the balance moves back to equilibrium. Knowing  $m$ ,  $i$ , and loop width,  $a$ , allows one to calculate the magnetic field strength  $B$ .

- ①  $\vec{F} = i \vec{a} \times \vec{B} \Rightarrow F = i a B$
- ②  $F = mg$  at equilibrium
- ③  $mg = i a B$
- ④  $B = \frac{mg}{ia}$

You will do a variation on this experiment in lab!

**The Hall Effect:** Suppose we have a ribbon of conductor of width  $d$  carrying current in the presence of a magnetic field perpendicular to the plane of the ribbon. Assume current is traveling right. The current can be carried by positive charges moving right or negative charges moving left. Which is it? Electrostatics alone doesn't tell us, but introducing the magnetic field can.

Assuming positive charge carriers:



At equilibrium,

$$F_E = F_B \Rightarrow \int E_H = \int v_d B$$

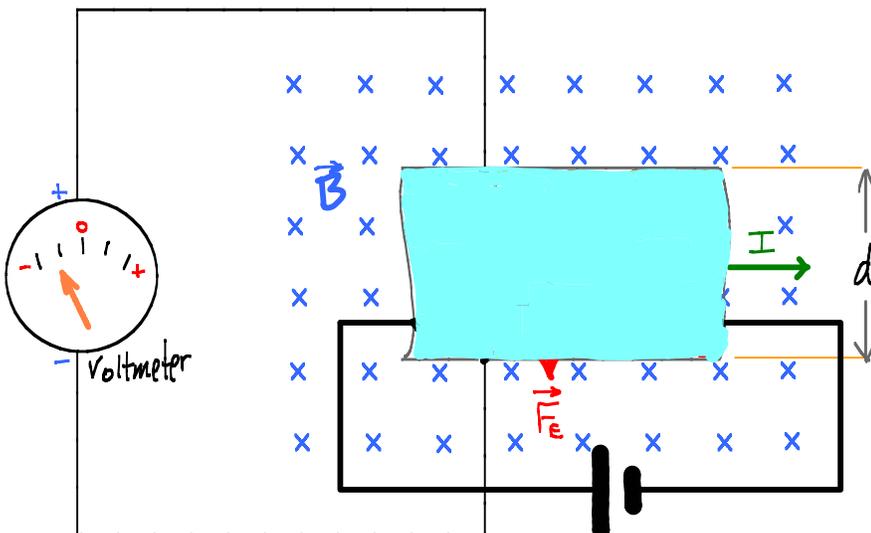
$$\text{but } \Delta V_H = E_H d$$

$$\Rightarrow \frac{\Delta V_H}{d} = v_d B$$

$$\Rightarrow v_d = \frac{\Delta V_H}{Bd}$$

So, not only does the sign of the Hall voltage reveal the sign of the mobile charge carrier, the simple relationship above allows experimental measurement of the drift velocity.

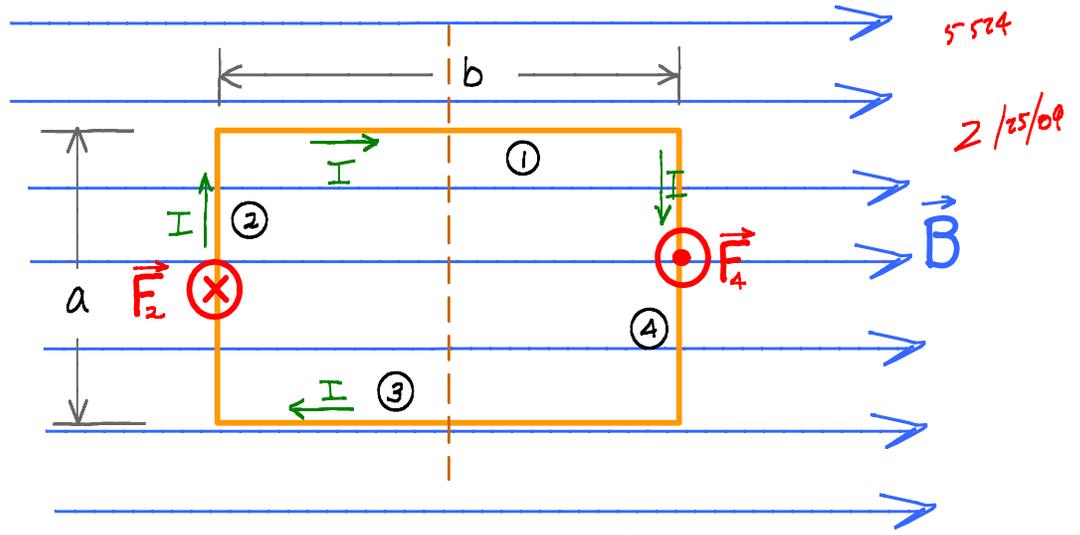
Assuming negative charge carriers:



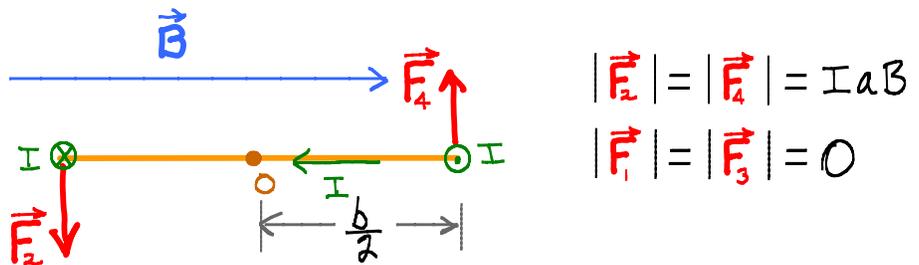
Applications of the Hall effect include: measuring the sign and density of charge carriers, proximity switches, measuring the flow of ionic fluids in a pipe (blood flow in an artery), magnetic field meters.....

# Torque on a current loop

Initially, assume  $\mathbf{B}$  in the plane of the loop.



Side view shows that the loop will rotate due to the torque about

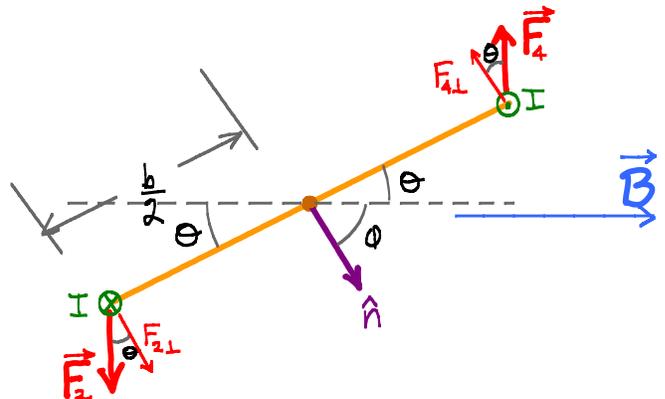


Recall:  $\tau = r F_{\perp} \Rightarrow \tau = \frac{b}{2} F_2 + \frac{b}{2} F_4 = ba I B = A I B$

magnetic dipole moment magnitude,  $\mu \equiv IA$ , where A is the area of the loop

magnetic dipole moment vector  $\vec{\mu} = IA \hat{n}$  where  $\hat{n}$  is given by right hand rule

Now let the loop rotate so that  $\mathbf{B}$  does not lie in the plane of the loop.



$\tau = (F_{2\perp} \frac{b}{2}) 2 = F_{2\perp} b \cos \theta$   
 but  $\cos \theta = \sin \phi \Rightarrow \tau = F_2 b \sin \phi$   
 So,  $\tau = I a b B \sin \phi = I A B \sin \phi$

$\Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$  using the magnetic dipole moment definition.

We can increase the torque by increasing the number of turns in the loop. If the number of turns is  $N$ , we generalize the magnetic dipole moment definition to,

$$\vec{\mu} = NIA\hat{n} \quad \text{units are A m}^2$$

While we developed the above for a rectangular loop, the results apply to any shape. Applications: Electric motors, D'Arsonval galvanometers, satellite torques...

### Potential Energy of a magnetic dipole

$$U = -\vec{\mu} \cdot \vec{B}$$

To see that there is potential energy in the configuration above, imagine doing the work to rotate the loop from the original "flat" orientation to the second orientation where  $\phi$  is no longer  $90^\circ$ . The work invested can be regained by releasing the loop and collecting the kinetic energy when  $\phi$  goes to  $90^\circ$ .

$$\text{Proof: } \Delta U = -\int \tau d\phi \Rightarrow U(\phi) - U(90^\circ) = -\int_{90^\circ}^{\phi} \mu B \sin\phi d\phi$$

$$\Rightarrow U(\phi) - U(90^\circ) = -\mu B \cos\phi \Big|_{90^\circ}^{\phi} = -\mu B \cos\phi$$

$$\text{Set } U(90^\circ) = 0 \Rightarrow U = -\mu B \cos\phi = -\vec{\mu} \cdot \vec{B}$$

## Speakers

For the direction of current shown there is a net downward force on the current loop. If the current reverses the force develops an upward component. The current coil is connected to a cone that moves with the current loop, compressing and rarifying the air next to it. The resulting variations in air pressure propagate as sound.

