SP212 Blurb 9 2010

1. The Biot-Savart Law

2. Applications of Biot-Savart: Loop & line of current

3. Ampere's Law

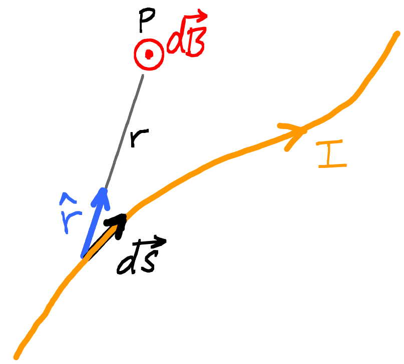
4. Applications of Ampere's law.

5. Force between parallel current carrying wires.

The Biot-Savart Law

The only source of the magnetic field previously mentioned was the permanent magnet, whose magnetism comes from the magnetic dipole moment of electrons. Another source of magnetic field is the electric current. The Biot-Savart law gives a quantitative mathematical description of the magnetic field at a point near an electric current.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$



$d\vec{s} \equiv$ a displacement vector in the direction of current

$\hat{r} \equiv$ a unit vector from the current segment to P

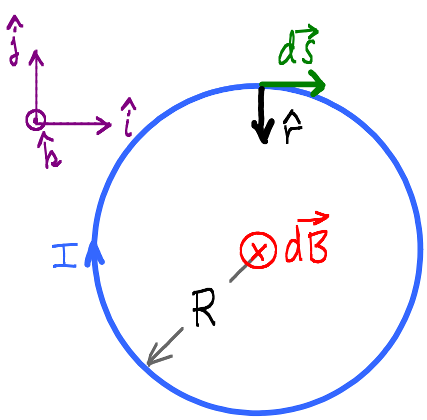
$r \equiv$ the distance from the current segment to P.

$\mu_0 \equiv$ magnetic permeability of free space $= 4\pi \times 10^{-7}\,\frac{Tm}{A}$

To get the total magnetic field at P integrate all the contributions from the entire wire.

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{s}\times\hat{r}}{r^2}}$$

<u>Example 1</u>: **B** at the center of a circular current loop of radius R.



① $\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$; $r = R$

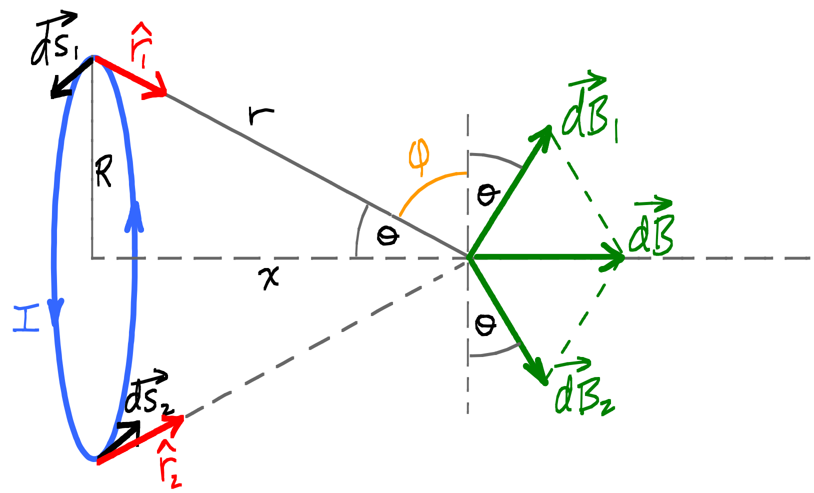
② $d\vec{s}\times\hat{r} = ds(1)(-\hat{k})$

③ $\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{ds(-\hat{k})}{R^2} = \frac{\mu_0 I}{4\pi R^2}(-\hat{k})\int ds$

④ $\vec{B} = \frac{\mu_0 I(-\hat{k})}{4\pi R^2}\,2\pi R$

⑤ $\vec{B} = \frac{\mu_0 I}{2R}(-\hat{k})$

Example 2: **B** for the field on the axis of a loop, a distance x from its center.



① $\vec{B} = \int d\vec{B} = \int |d\vec{B}_1| \sin\theta \, \hat{i}$

since vertical components

② $|d\vec{B}_1| = \dfrac{\mu_0 I}{4\pi} \displaystyle\int \frac{|d\vec{s}_1 \times \hat{r}_1|}{r^2}$

③ $|d\vec{s}_1 \times \hat{r}_1| = ds_1$ since $d\vec{s}_1 \perp \hat{r}_1$

④ $|d\vec{B}_1| = \dfrac{\mu_0 I}{4\pi r^2} \displaystyle\int ds_1$

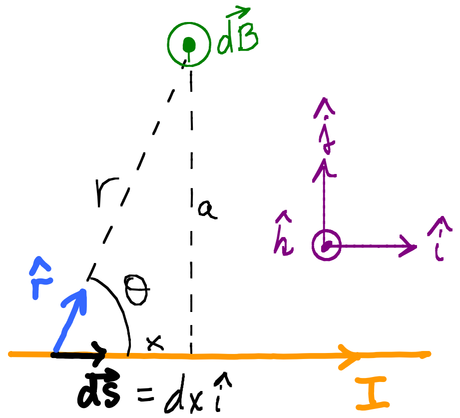
⑤ $\int ds_1 = 2\pi R$ ⑥ $r^2 = R^2 + x^2$

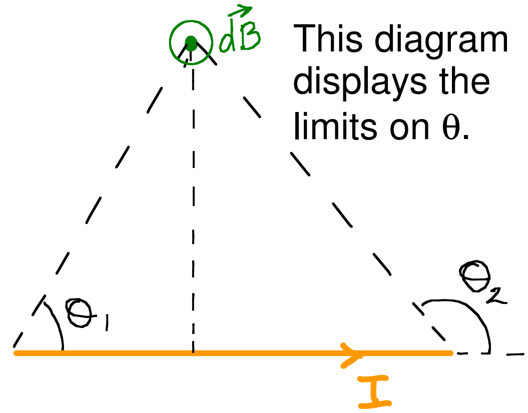
⑦ $|\vec{B}_1| = \dfrac{\mu_0 I \, 2\pi R}{4\pi (R^2+x^2)} = \dfrac{\mu_0 I R}{2(R^2+x^2)}$

⑧ $\vec{B} = \dfrac{\mu_0 I R}{2(R^2+x^2)} \sin\theta \, \hat{i}$ ⑨ $\sin\theta = \dfrac{R}{\sqrt{R^2+x^2}}$

⑩ $\boxed{\vec{B} = \dfrac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \hat{i}}$ If we let $x \to 0$, $\vec{B} \to \dfrac{\mu_0 I R^2}{2R^3}\hat{i} = \dfrac{\mu_0 I}{2R}\hat{i}$ the same result as Example 1.

Example 3: **B** near a line of current using the Biot-Savart Law.





This diagram displays the limits on θ.

① $$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{dx\sin\theta}{r^2}\hat{k}$$

We're going to integrate with respect to θ so get r and dx in terms of θ .

② $$r = \frac{a}{\sin\theta}$$

③ $$x = \frac{a}{\tan\theta}$$

from the left diagram

④ $$\frac{dx}{d\theta} = -\frac{a}{\sin^2\theta} \Rightarrow dx = \frac{-a\,d\theta}{\sin^2\theta}$$

⑤ $$d\vec{B} = \frac{\mu_0 I}{4\pi}\left(\frac{-\cancel{a}\,d\theta}{\cancel{\sin^2\theta}}\right)\frac{\sin^{\cancel{2}}\theta}{a^{\cancel{2}}}\hat{k} = \frac{-\mu_0 I}{4\pi a}\sin\theta\, d\theta\,\hat{k}$$

⑥ $$\vec{B} = -\frac{\mu_0 I}{4\pi a}\hat{k}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = -\frac{\mu_0 I}{4\pi a}\hat{k}\cos\theta\Big|_{\theta_1}^{\theta_2}$$

⑦ $$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi a}\hat{k}(\cos\theta_1 - \cos\theta_2)}$$

In the limit as the wire gets infinitely long, $\theta_1 \rightarrow 0$ and $\theta_2 \rightarrow \pi \Rightarrow (\cos\theta_1 - \cos\theta_2) \rightarrow 2$.

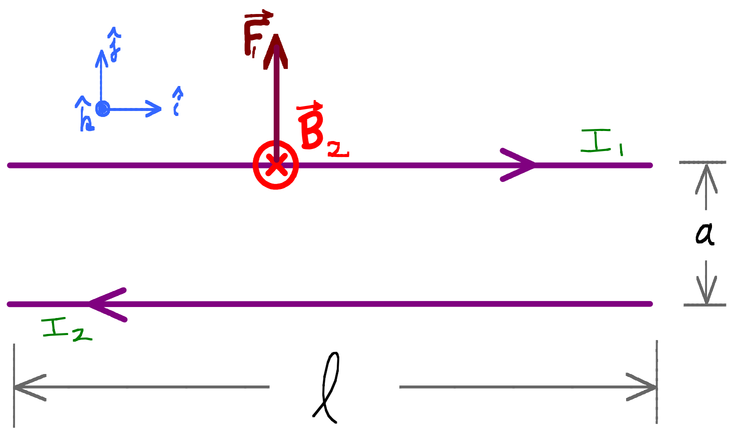
⑧ $$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{k}}$$

This is a surprisingly important result!

Forces between parallel current carrying wires.

We're now able to find the strength of **B** near a long straight wire and we know that a current carrying segment of wire in an external magnetic field experiences a force described by $I\ell$ x **B**. We will now put these two facts together to get quantitative info about the force between current carrying wires.

The case for oppositely directed currents will be worked out below.



① $\vec{F}_1 = I_1 \vec{\ell} \times \vec{B}_2$ where $\mathbf{F}_1$ is the force on current 1 and $\mathbf{B}_2$ is the magnetic field at the location of current 1 caused by current 2.

② $\vec{F}_1 = I_1 \ell B_2 \hat{j}$

③ $B_2 = \dfrac{\mu_0 I_2}{2\pi a}$ from our earlier result for the magnetic field near a long straight wire.

④ $\vec{F}_1 = \dfrac{I_1 \ell \mu_0 I_2}{2\pi a} \hat{j}$

⑤ $\dfrac{F_1}{\ell} = \dfrac{\mu_0 I_1 I_2}{2\pi a}$ We can easily see that this repulsive force becomes attractive if the currents are in the same direction.

Ampere's Law says the line integral of the magnetic field around a closed curve is proportional to the net current intersecting the area bounded by the curve.

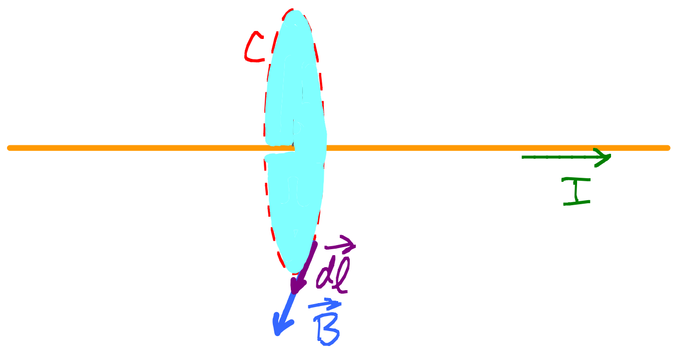
$$\oint_C \vec{B} \cdot \vec{d\ell} = \mu_0 I$$

Example: **B** field near a long straight wire carrying current I.

First, try to find a symmetric "Amperian Loop" that best fits the problem.

Use the right hand rule to get the direction of **B** on C.

Choose a direction of integration along the path. Currents in a direction consistent with the right hand rule are positive, otherwise they're negative.



For the problem at hand the best curve is a circular loop centered on the wire. Along this curve the magnitude of **B** is constant and the direction is tangent to the curve. At every point on C, **B** and **d**ℓ are parallel.

① $\oint_C \vec{B} \cdot \vec{d\ell} = \mu_0 I$

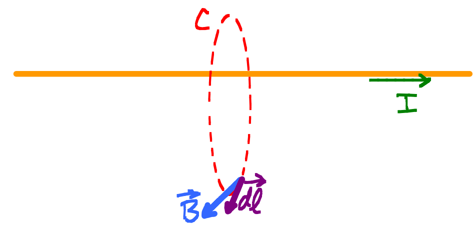
② $\oint_C B\, d\ell = \mu_0 I$ since B II dℓ

③ $B \oint_C d\ell = \mu_0 I$ since magnitude of **B** is constant on C

④ $B\, 2\pi r = \mu_0 I$

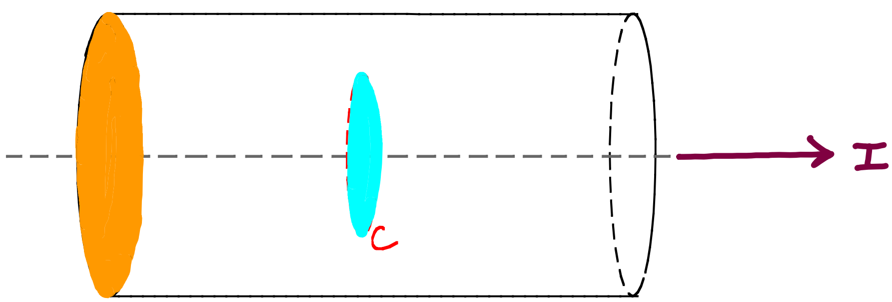
⑤ $\boxed{B = \dfrac{\mu_0 I}{2\pi r}}$ The same result we got earlier with the Biot-Savart law. (... and a lot more trouble!)

Note: There are only a few problems that can be solved with Ampere's Law. For the off-center curve shown below, **B** is not parallel to **d**ℓ on most segments of the curve so, **B•d**ℓ does not reduce to Bdℓ as it did in step 2 above.

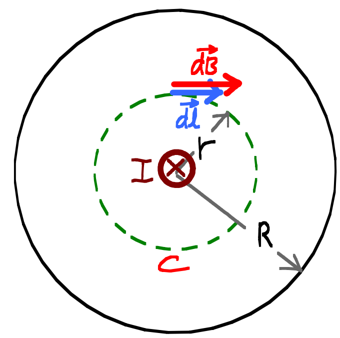


B inside a long current carrying wire.

Ampere's law easily allows the determination of the magnetic field inside the wire. To do this with the Biot-Savart law would be much more difficult.



End View



① $\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_c$

② $\oint_c B\,dl = \mu_0 I_c$

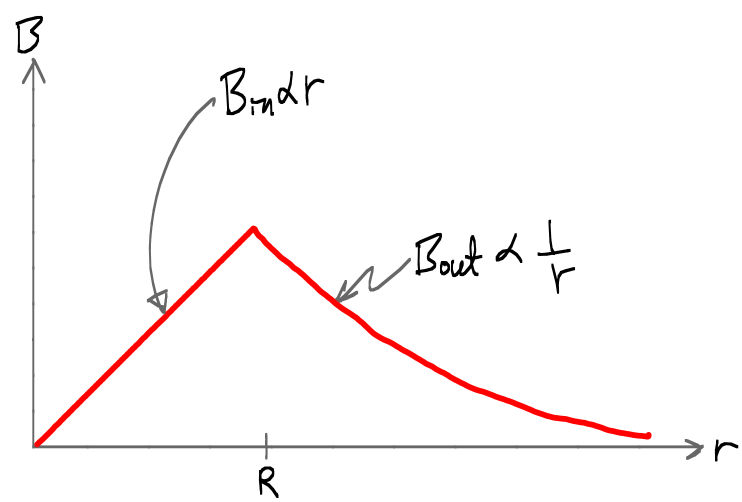
③ $B\oint_c dl = \mu_0 I_c$

④ $B\,2\pi r = \mu_0 I_c$

⑤ $I_c = \frac{I}{\pi R^2}\pi r^2 = I\frac{r^2}{R^2}$

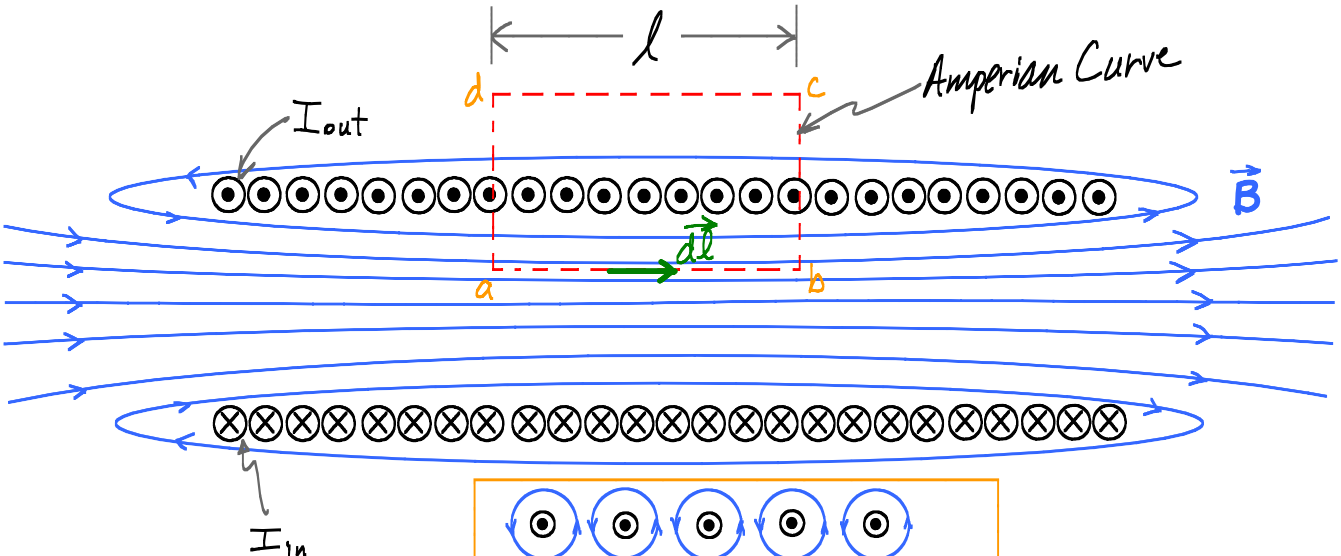
⑥ $B\,2\pi \cancel{r} = \mu_0 I \frac{r^{\cancel{2}}}{R^2}$

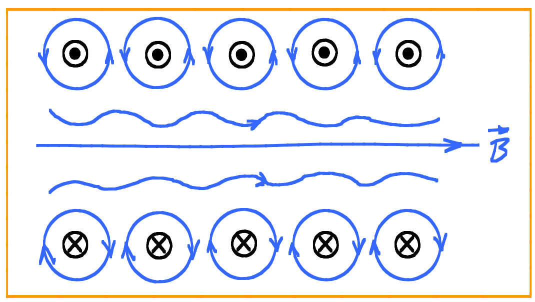
⑦ $\boxed{B = \frac{\mu_0 I}{2\pi R^2} r}$



The Solenoid is a very important electromagnetic device. It is simply a long wire wound into a helix so that the magnetic field contributions from each turn add to produce an intense, constant B field inside the coil and very weak B outside.

We'll use Ampere's Law to relate the field inside the solenoid to the number of coils per meter, n, and the current in the wire, I.





Expanded view details contributions from individual loops.

① $\oint_C \vec{B}\cdot\vec{d\ell} = \mu_0 I$

② $\oint_C \vec{B}\cdot\vec{d\ell} = \int_a^b \vec{B}\cdot\vec{d\ell} + \int_b^c \vec{B}\cdot\vec{d\ell} + \int_c^d \vec{B}\cdot\vec{d\ell} + \int_d^a \vec{B}\cdot\vec{d\ell}$

③ $\int_a^b \vec{B}\cdot\vec{d\ell} = \int_a^b B\,d\ell$ Since $\vec{B} \parallel \vec{d\ell}$ on the a to b segment.

④ $\int_a^b \vec{B}\cdot\vec{d\ell} = B\int_a^b d\ell$ Since B is constant on the a to b segment

⑤ $\int_a^b \vec{B}\cdot\vec{d\ell} = B\ell$

⑥ $\int_b^c \vec{B}\cdot d\vec{l} = 0$ Since $\vec{B}=0$ outside the solenoid and $\vec{B}\perp d\vec{l}$ inside

⑦ $\int_c^d \vec{B}\cdot d\vec{l} = 0$ and $\int_d^a \vec{B}\cdot d\vec{l} = 0$ (See previous)

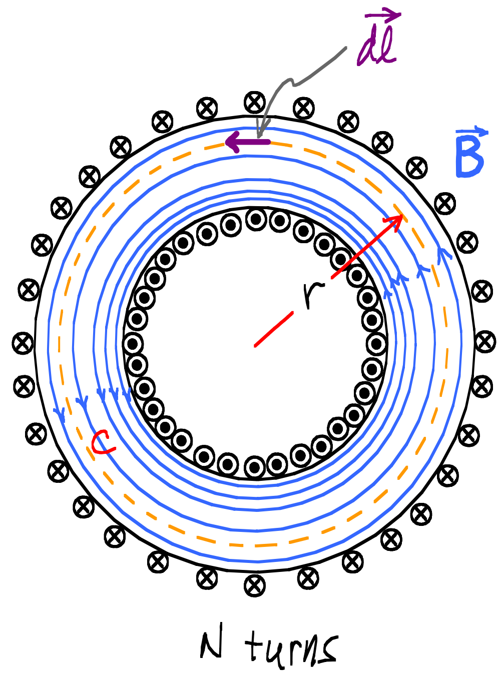
⑧ $I = Ni$ where $N \equiv$ # of loops within C.

⑨ $Bl = \mu_0 N i$ ⑩ $N = nl$ where $n \equiv$ # of turns per unit length

⑪ $B\cancel{l} = \mu_0 n \cancel{l} i$ ⑫ $\boxed{B = \mu_0 n i}$

This simple result allows us to design a solenoid with a specific magnetic field. Applications are almost innumerable but include, automobile starters, MRI machines, electrical relays and actuators....

The Toroid



① $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$

② $\oint_C B\,dl = \mu_0 I$ since **B** ll **dl**

③ $B\oint_C dl = \mu_0 I$ since magnitude of **B** is constant on C

④ $B\,2\pi r = \mu_0 I$

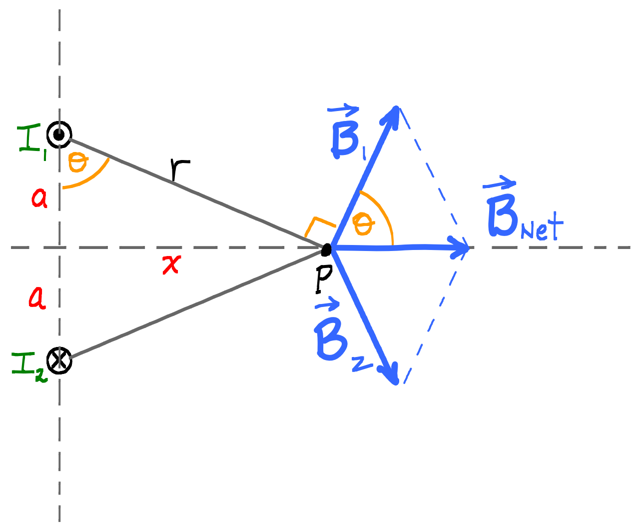
⑤ $I = Ni$

⑥ $B\,2\pi r = \mu_0 N i$

⑦ $\boxed{B = \dfrac{\mu_0 N i}{2\pi r}}$ We see B α 1/r inside the toroid

Example: The top wire in the diagram carries current I_1 out of the page while the bottom wire carries equal current I_2 into the page. Find the magnetic field for a point on the horizontal axis a distance x from the origin.

Use the right hand rule to find the direction of each magnetic field contribution at point P. Add vector components to find the net field.



① $\vec{B}_{NET} = \vec{B}_1 + \vec{B}_2$

② $\vec{B}_1 = B_1 \cos\theta\,\hat{i} + B_1 \sin\theta\,\hat{j}$

③ $\vec{B}_2 = B_2 \cos\theta\,\hat{i} - B_2 \sin\theta\,\hat{j}$

④ $B_1 = B_2$

⑤ $\vec{B}_{NET} = 2B_1 \cos\theta\,\hat{i}$

⑥ $B_1 = \dfrac{\mu_0 I_1}{2\pi r}$; $r = \sqrt{a^2+x^2}$

⑦ $\vec{B}_{NET} = 2\left(\dfrac{\mu_0 I_1}{2\pi\sqrt{a^2+x^2}}\right)\dfrac{a}{\sqrt{a^2+x^2}}\,\hat{i}$

⑧ $\boxed{\vec{B}_{NET} = \dfrac{\mu_0 I_1 a}{\pi(a^2+x^2)}\,\hat{i}}$