$\boxed{\text{HW\#1}}$

(a) <u>VSWUP #2</u>

A set of vectors $\{|v_1\rangle, |v_2\rangle, \ldots\}$ is linearly indep. if the <u>only</u> choice of scalars $\{a_1, a_2, \ldots\}$ that makes

$$a_1|v_1\rangle + a_2|v_2\rangle + \ldots = |0\rangle$$

is $a_1 = a_2 = \ldots = 0$.

① $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{vmatrix} = 1 \quad \therefore \boxed{\text{lin. indep.}}$

② $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -1 & 0 \end{vmatrix} = 0 \quad \therefore \boxed{\text{lin. dep.}}$

③ $\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2 \quad \therefore \boxed{\text{lin. indep.}}$

(The two vectors are NOT parallel!)

④ $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 3 & -3 & 1 \end{vmatrix} = 2 \quad \therefore \boxed{\text{lin. indep.}}$

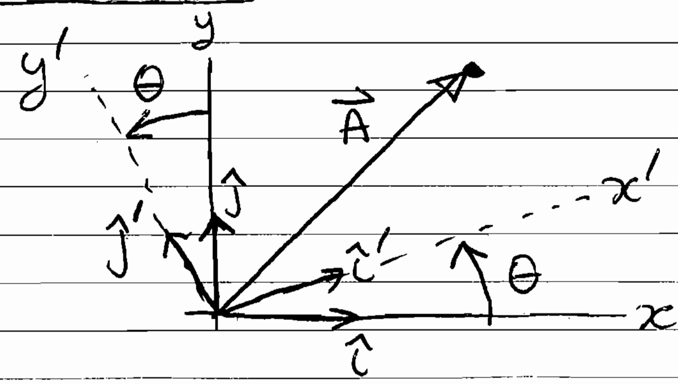
Here I computed the determinant of the coefficients of the unit vectors. Note however the <u>different</u> way I wrote $\hat{i} - \hat{j}$ in case ③ as a 2D vector $(1, -1)$ compared to in case ④ as a 3D vector $(1, -1, 0)$. IOW, I assume that we are checking the N vectors in N-dimensional space. This problem would not arise if you did the problem the other way:

write $a(\hat{i} - \hat{j}) + b(\hat{i} + \hat{j}) = \vec{0}$ in case ③

and $a(\hat{i} - \hat{j}) + b(\hat{i} + \hat{j}) + c(3\hat{i} - 3\hat{j} + \hat{k}) = \vec{0}$ in case ④

provided one understands that $\vec{0} = (0, 0)$ in case ③ and $\vec{0} = (0, 0, 0)$ in case ④.

(b) VSWUP #8



a. $\boxed{\begin{array}{ll} \hat{i}\cdot\hat{i}' = \cos\theta & \hat{i}\cdot\hat{j}' = -\sin\theta \\ \hat{j}\cdot\hat{i}' = \sin\theta & \hat{j}\cdot\hat{j}' = \cos\theta \end{array}}$

b. $\boxed{A_x = \vec{A}\cdot\hat{i}} = (A_x\hat{i} + A_y\hat{j})\cdot\hat{i}$

c. $\boxed{A_y' = \vec{A}\cdot\hat{j}'} = (A_x\hat{i} + A_y\hat{j})\cdot\hat{j}'$

$= \boxed{A_x(\hat{i}\cdot\hat{j}') + A_y(\hat{j}\cdot\hat{j}')}$

$\boxed{A_x' = \vec{A}\cdot\hat{i}'} = (A_x\hat{i} + A_y\hat{j})\cdot\hat{i}'$

$= \boxed{A_x(\hat{i}\cdot\hat{i}') + A_y(\hat{j}\cdot\hat{i}')}$

d.

$$\therefore \boxed{\begin{bmatrix} A_x' \\ A_y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} A_x \\ A_y \end{bmatrix}}$$

This is called the rotation matrix and we'll study it more later in the course. ▨

(c) VS #16

a. $\langle 2|B\rangle = \hat{j}\cdot\vec{B} = \boxed{B_y}$

$\langle 1|A\rangle = \boxed{A_x}$

$\langle 1|2\rangle = \boxed{0}$

$\langle 3|3\rangle = \boxed{1}$

in general $\langle m|n\rangle = \boxed{\delta_{mn}}$

b. $\sum_{m=1}^{3} \hat{e}_m (\hat{e}_m \cdot \vec{A}) = \sum_{m=1}^{3} \hat{e}_m A_m = \boxed{\vec{A}}$

$\sum_{m=1}^{3} \hat{e}_m (\hat{e}_m \cdot \vec{B}) = \boxed{\vec{B}}$ likewise

c. $\sum_{m=1}^{3} \hat{e}_m (\hat{e}_m \cdot \vec{A}) = \boxed{\sum_{m=1}^{3} |m\rangle\langle m|A\rangle}$ just notation!

$\sum_{m=1}^{3} \hat{e}_m (\hat{e}_m \cdot \vec{B}) = \boxed{\sum_{m=1}^{3} |m\rangle\langle m|B\rangle}$ likewise

d. in general $\sum_{m=1}^{3} |m\rangle\langle m|V\rangle = \boxed{|V\rangle}$

e. $1|V\rangle = \boxed{|V\rangle}$ equal!

f. $\therefore \sum_{n=1}^{3} |n\rangle\langle n| \sum_{m=1}^{3} |m\rangle\langle m|V\rangle = \boxed{|V\rangle}$

g. $\therefore \langle A|B\rangle = \boxed{\langle A| \sum_{n=1}^{3} |n\rangle\langle n| \sum_{m=1}^{3} |m\rangle\langle m|B\rangle}$

h. $\langle n|m\rangle = \boxed{\delta_{nm}}$

$\Rightarrow \sum_{n=1}^{3} |n\rangle\langle n|m\rangle = \sum_{n=1}^{3} |n\rangle \delta_{nm} = \boxed{|m\rangle}$

$\Rightarrow \sum_{n,m=1}^{3} |n\rangle\langle n|m\rangle\langle m| = \sum_{m=1}^{3} |m\rangle\langle m| = \boxed{1}$

i. $\langle A| \sum_{n=1}^{3} |n\rangle\langle n| \sum_{m=1}^{3} |m\rangle\langle m|B\rangle = \boxed{\langle A|n\rangle\langle n|m\rangle\langle m|B\rangle}$

j. $\sum_{m=1}^{3} |m\rangle\langle m| = \boxed{1}$ whereas $\sum_{m=1}^{2} |m\rangle\langle m|$

$\boxed{\text{lacks the } 3\,(z) \text{ component}}$ ☒

(d) VS #18

a. $\boxed{|0\rangle}$ can never appear in a set of lin indep vectors

b. Now suppose $\{|A\rangle, |B\rangle, \ldots\}$ are mutually orthogonal and none are zero. Is it possible to find scalars $\{a, b, \ldots\}$ not all zero such that:

$$a|A\rangle + b|B\rangle + \cdots = |0\rangle \qquad (*)$$

To find out, take the I.P. of $|A\rangle$ with both sides to get:

$$a\langle A|A\rangle + b\langle A|B\rangle + \cdots = \langle A|0\rangle$$

But $\langle A|B\rangle = \cdots = 0$ since they're orthogonal, and $\langle A|0\rangle$ since any vector dotted with the zero vector is zero.

$$\therefore \; a\langle A|A\rangle = 0$$

But $\langle A|A\rangle \neq 0$ since the I.P. is normalizable and $|A\rangle$ isn't the zero vector.

$$\therefore \; a = 0$$

Similarly take the I.P. of $|B\rangle, \ldots$ with $(*)$ to deduce that $b = \cdots = 0$.

$\therefore$ The only possible values of $\{a, b, \ldots\}$ that satisfy $(*)$ are $a = b = \cdots = 0$.

$\therefore$ The vectors are $\boxed{\text{linearly independent.}}$ ▨