SP352 HW #6

"The Laplace Equation—Solution & Applications" handout (with problems on pages 66 to 98)

<u>due Thursday 7 March by 0900:</u>

(a) Prove that the following function satisfies the Laplace equation in 2 rectangular coordinates,

$$f(x,y) = \tan^{-1}\left[\frac{\sin y}{\sinh x}\right]$$

by explicitly doing all the derivatives by hand. <u>Hint</u>: Use the Pythagorean identity for sine and cosine ($\sin^2\phi + \cos^2\phi = 1$ for any ϕ), and the analogous identity for cosh and sinh. <u>Comment</u>: When combined with the fact that the Griffiths form satisfies the boundary conditions for the slot problem, this result proves that this form is the unique solution and thus the sum of the series found in class.

(b) Now write down the complete Griffiths form of the potential $V(x,y)$ for an infinite slot (given below Eq. SL.13 in the SL handout). Use it to compute the electric field just above the bottom edge of the slot in vector form. <u>Hints</u>: What derivative of V do you need to take? (You already computed a similar derivative in problem *b* above.) At what coordinates do you need to evaluate that derivative to get the bottom edge?

(c) SL #16+1—That is, do the Pollock & Stump problem on page 74 of the handout. After you have solved the problem, plot both the potential (use the Mathematica command Plot3D) and the electric field (use the command StreamPlot); set both V_0 and a equal to 1 in these plots.

(d) SL #33—There are accidentally two problems numbered 33 on page 82! I want you to do the second one, which starts "Charge is glued on …." Note the sign error in the equation for σ. The derivative of $V^<$ should have a double negative sign.