SP352 HW #7

"The Dirac Delta: Properties and Representations" handout (with problems on pages 30 to 38)

<u>due Tuesday 26 March by 0900:</u>

(a) DD #28

(b) DD #15

(c) DD #16—Start by graphing $|x|$ and $\frac{d}{dx}|x|$. Use your graph to figure out how to express $\frac{d}{dx}|x|$ in terms of the Heaviside step function.

(d) Integrate by parts twice to evaluate $\int_{-\infty}^{\infty} f(x)\frac{d^2}{dx^2}\left[\delta(x-x_0)\right]dx$. Make sure you clearly explain why the *uv* term in each of the two integrations by parts is zero!

(e) DD #5—Hint: What is the area under each distinct portion of the graph of $\frac{d}{dx}\left[{}^2D_n(x-x_0)\right]$? Be sure you shift the triangle representation from being centered at the origin to being centered at x_0.

(f) DD #12—That is, you need to show two things. First, integrate the Yukawa representation over the volume of a sphere of small radius ϵ enclosing the origin and show that the result equals 1 in the limit that $n \to \infty$. Second show that the Yukawa representation is equal to zero for any nonzero value of r in the limit that $n \to \infty$. L'Hôpital's rule might be helpful.

(g) Let $J_{1/3}(z)$ be the Bessel function for $m = 1/3$. What is the independent and what is the dependent variable? With that in mind, write down Bessel's differential equation that this function satisfies. (Do NOT write primes for derivatives; write them out as explicit derivatives or you will get very confused below!) Next let

$$y(x) = \sqrt{x}\, J_{1/3}\left(\tfrac{2}{3}\sqrt{\lambda}\, x^{3/2}\right)$$

where λ is a constant. Compute d^2y/dx^2 and CAREFULLY write the result in terms of z and $J_{1/3}(z)$; i.e., change independent and dependent variables. [Hint: Compute the first derivative and use the chain rule $d/dx = (dz/dx)d/dz$ to rewrite the derivative of $J_{1/3}(z)$. What is z? Then repeat for the second derivative.] Now use Bessel's differential equation that you wrote down above to prove that $d^2y/dx^2 = -\lambda xy$. [Hint: Rewrite $-\lambda xy$ in terms of z and $J_{1/3}(z)$.] <u>Comment</u>: This problem shows that the differential equation $y'' + \lambda xy = 0$ has the Bessel solution given above.