Test #1 Review Sheet

Key testable concepts are marked with an asterisk.

Vector Spaces

fields: 10 axioms

vector spaces: 10 axioms (egs. polynomials, Taylor spaces, oscillators, Fourier series)

inner products: 4 properties

bra, ket, dagger operator, matrix representation of vectors, span, basis, dimension

component calculator (3 cases: nonorthogonal, orthogonal, orthonormal bases)

*linearly dependent if determinant of coefficients is zero (eg. if column 1 equals column 2)

*Gram-Schmidt process (egs. ordinary vectors, Legendre polynomials)

Fourier Series

$$f(t) = c_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(n\frac{2\pi}{T}t\right) + b_n \sin\left(n\frac{2\pi}{T}t\right)\right]$$

$$\langle f|g\rangle = \frac{1}{T}\int_{-T/2}^{T/2} f^*(t)g(t)\,dt$$

c_0 equals the average value of f over a period

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(n\frac{2\pi}{T}t\right)dt \quad \text{and likewise for } b_n$$

(prefactor of 2 comes from reciprocal average of square of sine or cosine over a period)

odd/even integrals

*example calculations for antisymmetric or symmetric square, triangle, parabola waves:

- cosine or sine series
- odd harmonics only
- alternating signs
- coefficients proportional to a reciprocal power of the harmonic number
- Gibbs overshoot

*sums of series:

- evaluate FS at special values of t
- Parseval relation
- switching between sums over odd, even, and all integers

Multipole Expansions

$$V = \int k\,dq\,D(\vec{\mathbf{r}},\vec{\mathbf{r}}')$$

$$V_0 = \frac{kq_{\text{net}}}{r} \quad \text{where} \quad q_{\text{net}} = \sum q_\alpha = \int dq$$

$$V_1 = \frac{k\,\vec{\mathbf{p}}\bullet\vec{\mathbf{r}}}{r^3} \quad \text{where} \quad \vec{\mathbf{p}} = \sum q_\alpha \vec{\mathbf{r}}_\alpha = \int dq\,\vec{\mathbf{r}}'$$

$$V_2 = \frac{k}{2r^5}\sum_{i,j=1}^{3} Q_{ij}x_i x_j \quad \text{where} \quad Q_{ij} = \int dq\left(3x'x'_j - r'^2\delta_{ij}\right) \text{ is symmetric \& traceless}$$

(standard Q_{zz} and Q_{yz} quadrupoles made of two $+q$ & two $-q$ charges)

*eg. 1: discrete point charges arranged parallel to one or two coordinate axes

*eg. 2: $\sigma(\theta,\phi)$ on the surface of a sphere of radius R

computation of $\int \sin^n\theta\cos\theta\, d\theta$ and $\int \cos^n\theta\sin\theta\, d\theta$

calculating the electric field in high-symmetry directions and the surfaces of zero potential

centering multipole moments on the origin

expansion in Legendre polynomials