LETTERS TO THE EDITOR

Letters are selected for their expected interest for our readers. Some letters are sent to reviewers for advice; some are accepted or declined by the editor without review. Letters must be brief and may be edited, subject to the author's approval of significant changes. Although some comments on published articles and notes may be appropriate as letters, most such comments are reviewed according to a special procedure and appear, if accepted, in the Notes and Discussions section. (See the "Statement of Editorial Policy" at http://www.kzoo.edu/ajp/docs/edpolicy.html.) Running controversies among letter writers will not be published.

REPAIRING AN ELEMENTARY EXPLANATION OF RADIATION PRESSURE

In the February 2009 issue,[1] Rothman and Boughn convincingly pointed out a fallacy in a familiar introductory explanation of how light exerts an electromagnetic force on a metal surface. Consider a free electron (of charge $-e$ and mass m) in the metal, as in their Fig. 1, but assume for simplicity that the metal is a two-dimensional plate in the xy plane with area A so that the z-component of the electron's velocity can be ignored. For an electric field $E_x = E_0 \cos(\omega t)$, where ω is the angular frequency of the light, $m d\upsilon_x / dt = -eE_x$ implies that $\upsilon_x = -\upsilon_0 \sin(\omega t)$, where $\upsilon_0 = eE_0 / m\omega$. But the magnetic field is in phase with the electric field, $B_y = B_0 \cos(\omega t)$ so that the time-averaged pressure on a plate of N free electrons is $P = N\langle F_z \rangle / A = Ne\upsilon_0 B_0 \langle \sin(\omega t)\cos(\omega t)\rangle / A = 0$. In other words, modeling a metal as an ideal gas of electrons cannot give rise to radiation pressure. This conclusion makes sense because an electromagnetic plane wave sweeping through a plasma does not drive the ions forward (in contrast to laser wakefield acceleration).

Therefore it is clear that a different model of electrons in a metal needs to be adopted to explain radiation pressure. The key problem above is that the velocity of the electron and the electromagnetic field are 90° out of phase with each other. To fix matters, the phase difference needs to be reduced. Halliday *et al.*[2] pointed out that the nonzero resistivity of the metal implies that the oscillations of the electron are damped so that $F_x = -eE_x - b\upsilon_x$ (as in the Drude model for which $b = m/\tau$ where τ is the relaxation time). Then the oscillations are phase shifted by $\theta = \tan^{-1}(b/m\omega)$ relative to what they would be in the absence of damping so that $\upsilon_x \propto -\sin(\omega t + \theta)$ and thus P is now nonzero.

[1] T. Rothman and S. Boughn, "The Lorentz force and the radiation pressure of light," Am. J. Phys. **77**, 122–127 (2009).

[2] D. Halliday, R. Resnick, and K. S. Krane, *Physics*, 5th ed. (Wiley, New York, 2002), Vol. 2, p. 872.

Carl E. Mungan
Department of Physics,
U.S. Naval Academy,
Annapolis, Maryland 21402

REPLY TO "REPAIRING AN ELEMENTARY EXPLANATION OF RADIATION PRESSURE"

We have received several letters from readers making the same suggestion and acknowledge that we hadn't been previously aware of the Halliday, Resnick, and Krane (HRK) argument.[1] We agree that if one includes a damping term on the electrons, then the resultant radiation pressure is nonzero. (This is, after all what the Abraham–Lorentz model does.)

Some models, however, are more transparent than others. For example, if the driving force on the electron is $eE_0 \cos(\omega t)$, then the electron's maximum displacement is $x_{\max}=eE_0/m\omega^2$. Taking an optical laser with $\omega \sim 10^{15}\ \mathrm{s}^{-1}$ and a power output of $100\ \mathrm{W/cm}^2$ gives $E_0 \sim 10^4\ \mathrm{V/m}$ and $x_{\max} \sim 10^{-13}$ cm, approximately the classical radius of the electron. But the HRK demonstration relies on the Drude model (the one showing that current density is proportional to the electric field), which assumes that the damping arises from collisions between electrons and lattice ions whose spacing is much larger than the size of the electron. Specifically, HRK invoke the drift velocity v_d in their expression for the magnetic force on the electron, $F_B=ev_dB$, but one might question whether the concept of drift velocity is meaningful on length scales much smaller than the lattice spacing. The electrons in the Drude model are usually taken to be subject to a DC field as well as random (thermal) velocities, which are assumed to average to zero; however, a proper treatment takes one beyond freshman physics.

Furthermore, as stated in Mungan's letter,[2] in the HRK argument the pressure on the system depends on the number of electrons N, which is not a constant associated with light. At this elementary level, the model only shows that the ratio of the electromagnetic force on the system ($\sim Ne^2E^2/c$) to the power absorbed by the system ($dU/dt \sim Ne^2E^2$) is $1/c$, but it does not establish the standard expression for the Poynting flux. To derive the Poynting relation, one must solve Maxwell's equations for the B-field (which is also phase shifted) within the conductor and then integrate over the electron distribution. All of which goes to show, once again, that freshman physics is not always for freshmen.

[1] D. Halliday, R. Resnick, and K. S. Krane, *Physics*, 5th ed. (Wiley, New York, 2002), Vol. 2, p. 872.

[2] Carl E. Mungan, "Repairing an elementary explanation of radiation pressure," Am. J. Phys. **77**(11), 965–965 (2009).

Tony Rothman
Princeton University

Stephen Boughn
Haverford College