Motion of an electric dipole in a static electromagnetic field

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Abstract

Expressions for the force and torque on a moving electric dipole in a magnetostatic field are combined with those due to an electrostatic field. By sending oriented molecular dipoles into a region of crossed magnetic and electric fields, the molecules are selectively rotated based on the direction and magnitude of their velocities. In principle this field configuration could be used to create a molecular isolator that only lets molecules through in one direction.

Keywords: Electric dipole moment, electromagnetic force and torque, molecular orientation.

Resumen

Combinamos expresiones para la fuerza y la torca en un dipolo eléctrico en movimiento en un campo magnetostático con las ocasionadas por un campo electrostático. Enviando dipolos moleculares orientados en la región de cruce de los campos magnético y eléctrico, las moléculas son rotadas selectivamente en la dirección y magnitud de sus velocidades. En principio esta configuración de campo podría ser usada para crear un aislador molecular que sólo permite moléculas en una dirección.

Palabras clave: Momento dipolar eléctrico, fuerza electromagnética y torca, orientación molecular.

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I. INTRODUCTION

A classical electric dipole $\vec{\mathbf{p}} \equiv q\vec{\mathbf{L}}$ consists of a positive charge distribution $+q$ whose centroid is displaced by $\vec{\mathbf{L}}$ relative to the centroid of a negative charge distribution $-q$. (Quantum effects such as Stark mixing can induce dipole moments in molecules which alter this simple picture [1].) For example, in a gaseous NaCl molecule, the much greater electronegativity of the chlorine atom as compared to that of the sodium atom causes Cl to steal an electron away from Na, resulting in an ionic bond between Cl^- and Na^+. One can thereby estimate [2] the magnitude of its dipole moment to be the elementary charge e multiplied by the bondlength L, giving $p \approx 4\times10^{-29}\ \mathrm{C\cdot m} = 12\ \mathrm{D}$. Introductory physics textbooks show that the torque on an electric dipole $\vec{\mathbf{p}}$ in an electric field $\vec{\mathbf{E}}$ is $\vec{\boldsymbol{\tau}}_{pE} = \vec{\mathbf{p}}\times\vec{\mathbf{E}}$. In addition, the motion of point charges in electric and magnetic fields is discussed. Similar ideas are used in the present article to discuss the motion of an electric dipole in static magnetic and electric fields.

II. FORCES AND TORQUES

Label the point midway between the centers of positive and negative charge of the dipole $\vec{\mathbf{p}}$ as O. (This point may or may not coincide with the center of mass of the object.) Decompose the motion of the dipole at any instant into a translation of point O with linear velocity $\vec{\boldsymbol{\upsilon}}$ and a rotation of the dipole about point O with angular velocity $\vec{\boldsymbol{\omega}}$. Now suppose the dipole enters a region of uniform magnetic field $\vec{\mathbf{B}}$. The magnetic forces on the two charge centers will be

$$\vec{\mathbf{F}}_+ = q(\vec{\boldsymbol{\upsilon}} + \vec{\boldsymbol{\omega}}\times\vec{\mathbf{L}}/2)\times\vec{\mathbf{B}} \quad \text{and} \quad \vec{\mathbf{F}}_- = -q(\vec{\boldsymbol{\upsilon}} - \vec{\boldsymbol{\omega}}\times\vec{\mathbf{L}}/2)\times\vec{\mathbf{B}}. \tag{1}$$

Consequently the net force on the dipole becomes the triple vector product

$$\vec{\mathbf{F}}_{pB} = (\vec{\boldsymbol{\omega}}\times\vec{\mathbf{p}})\times\vec{\mathbf{B}}, \tag{2}$$

while the torque about O is

$$\vec{\boldsymbol{\tau}}_{pB} = \vec{\mathbf{p}}\times(\vec{\boldsymbol{\upsilon}}\times\vec{\mathbf{B}}). \tag{3}$$

In the frame of reference of point O, the magnetic field is relativistically transformed into an electric field $\vec{\mathbf{E}}' = \vec{\upsilon} \times \vec{\mathbf{B}}$ and thus $\vec{\tau}_{pB} = \vec{\tau}_{pE'}$ which seeks to rotate $\vec{\mathbf{p}}$ into the direction of $\vec{\mathbf{E}}'$.

Suppose the dipole starts out with zero angular velocity but its translational velocity is perpendicular to the applied magnetic field so that the torque in Eq. (3) is maximized. To be specific, choose $\vec{\upsilon}$ to define the +*x* direction and $\vec{\mathbf{B}}$ the +*y* direction. Now the torque is largest if $\vec{\mathbf{p}}$ lies in the *xy* plane. In that case $\vec{\tau}_{pB}$ will also lie in the *xy* plane and will be perpendicular to $\vec{\mathbf{p}}$. As a result, the dipole will begin to librate (rock back and forth) end over end; that is, it will oscillate (indefinitely in the absence of drag) like a pendulum with the apex of its circular arc in the +*z* direction. Associated with these rotational oscillations will be a periodically varying force $\vec{\mathbf{F}}_{pB}$, alternately decelerating and accelerating the translations of the dipole because $\vec{\mathbf{F}}_{pB}$ is parallel to $\vec{\upsilon}$. In turn that force affects the torque by varying $\vec{\upsilon}$ in Eq. (3), although the feedback will be weak if the mass of the dipole and/or its moment of inertia about O is large.

Incidentally, note that we can use a vector identity to rewrite Eq. (3) as

$$\vec{\tau}_{pB} = (\vec{\mathbf{p}} \times \vec{\upsilon}) \times \vec{\mathbf{B}} + (\vec{\mathbf{B}} \times \vec{\mathbf{p}}) \times \vec{\upsilon} \,. \qquad (4)$$

If the dipole moment is initially parallel to the magnetic field, then even after it begins to tumble, $\vec{\mathbf{p}}$ will always lie in the *yz* plane. Therefore $\vec{\mathbf{B}} \times \vec{\mathbf{p}}$ will be parallel to the *x* axis, and the last term in Eq. (4) will be zero. We can then interpret $\vec{\mathbf{p}} \times \vec{\upsilon}$ as a magnetic dipole moment $\vec{\boldsymbol{\mu}}$ in the –*z* direction. Charge +*q* initially located at $y = L/2$ and traveling in the +*x* direction is equivalent to a current *I* circulating clockwise as seen looking down along the *z* axis. Likewise charge –*q* initially located at $y = -L/2$ and traveling in the +*x* direction corresponds to the same clockwise current *I*. We thus have a current loop, corresponding to a magnetic dipole. Equation (4) can now be interpreted as $\vec{\tau}_{pB} = \vec{\tau}_{\mu B}$ where $\vec{\tau}_{\mu B} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ is the torque on a magnetic dipole.

Next suppose that the region also contains a uniform electric field $\vec{\mathbf{E}}$. The electric forces on the two charges constitute a couple, so that $\vec{\mathbf{F}}_{pE}$ is zero and $\vec{\tau}_{pE} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$. As a result, the overall electromagnetic force and torque on the electric dipole are

$$\vec{\mathbf{F}}_p = (\vec{\boldsymbol{\omega}} \times \vec{\mathbf{p}}) \times \vec{\mathbf{B}} \qquad (5)$$

and

$$\vec{\tau}_p = \vec{\mathbf{p}} \times (\vec{\mathbf{E}} + \vec{\upsilon} \times \vec{\mathbf{B}}) \,, \qquad (6)$$

respectively. Equation (6) can be interpreted as the cross-product of $\vec{\mathbf{L}}$ with the Lorentz force. Now suppose we choose to cross the electric field with both the magnetic field and translational velocity, $\vec{\mathbf{E}} = \vec{\mathbf{B}} \times \vec{\upsilon}$ (so that it is equal to $-\vec{\mathbf{E}}'$ and points in the –*z* direction). In that case, $\vec{\tau}_p$ is zero and the dipole will no longer begin to librate. This situation is the rotational analog of a "translational velocity selector" whereby a point charge entering a region of crossed electric and magnetic fields experiences zero net force.

III. MOLECULAR APPLICATION

As an application, one can imagine a device analogous to a Faraday optical isolator, as sketched in Fig. 1. Consider long, cigar-shaped molecules with one end negatively charged and the other end positively charged so that there is an electric dipole moment directed along their length (such as linear HCN trimers with p = 11 D [3, 4]). Suppose that a beam of them is incident on a horizontal molecular polarizer P1 that only transmits molecules oriented parallel to the *y* direction. (It might be possible to construct such a polarizer by milling nano-sized slits through an impermeable membrane [5].) The molecules enter a region with a magnetic field in the +*y* direction and an electric field in the –*z* direction of appropriate magnitudes. The molecules therefore experience no torque and pass through a second horizontal molecular polarizer P2 and leave the field region. On the other hand, if we reverse the direction of $\vec{\upsilon}$ and send molecules backward through P2, then there will be a torque on them (specifically in the –*x* direction if $\vec{\mathbf{p}}$ is initially in the +*y* direction). By suitable choice of the spacing between the two polarizers, we can arrange for the dipoles to be rotated by exactly 90˚ when they reach P1 and therefore be rejected by it.

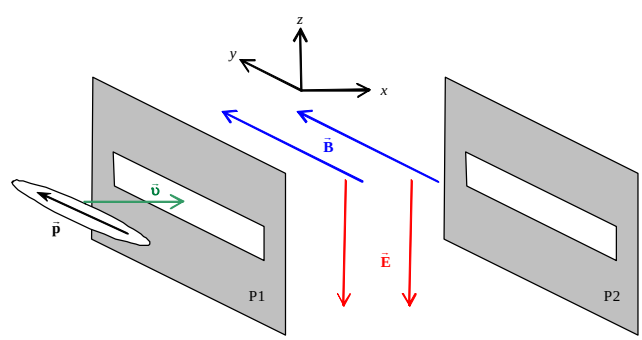

FIGURE 1. Sketch of a molecular isolator. Two horizontal molecular polarizers P1 and P2 sandwich a region of crossed magnetic and electric fields directed along the +*y* and –*z* axes, respectively. A long molecule with an electric dipole moment oriented along its axis is shown entering the device with a translational velocity in the +*x* direction.

Note that the resulting collision of the molecules with P1 should be designed to be inelastic, with the rejected molecules falling vertically into some collection chamber

below the field region.[1] If the molecules instead reflected elastically off P1, their rotational inertia would cause them to continue to rotate as they traveled back toward P2. But since they experience no torque on that return trip, they would rotate by more than 90˚ and overshoot the acceptance slit in P2 (by an angle of $180^{o}/\sqrt{2}-90^{o}\approx 37^{o}$). If the molecules reflected elastically off P2 also, they would then be returned to P1 once again and this time be transmitted by it. In the absence of losses, it is impossible to get a net flow of molecules from the entrance to the exit side of the isolator. Otherwise one would have a Maxwell's demon which, when connected to two chambers of molecules, would maintain a steady-state pressure imbalance between them. The optical analog would be a nonabsorbing valve that permits the flow of radiation in only one direction, creating a permanent temperature imbalance between two connected chambers, in violation of the second law of thermodynamics [11].

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[1]Other practical considerations are that the molecules need to be: low density to avoid collisions [6], rotationally cooled to below 1 K [7, 8] to minimize thermal reorientations, and traveling at high speed (above 1 km/s [9]) to keep the magnetic field strength reasonable (say 0.5 T [10]).