A sign of the times

I applaud the effort and the tools used by Kate Hayes and Michael C. Wittmann[1] to understand "Max's" train of thought, but I would just suggest that Max and the other five students are applying what most first-year texts espouse when they introduce vector addition in one dimension. For many first-year texts it is standard procedure, in a one-dimensional problem, to state the magnitude (hence positive) of a quantity and use +/- signs to indicate direction. This is the same reasoning Max has applied. I completely understand his reasoning at line ¾: "It assumes *a* is positive." Max is stating that his variable *a* is the *magnitude* of the vector (hence positive) and the direction is given with a negative sign. In the article the authors seemed confused as to why Max treats variables as constants: "Perhaps the problem lies in his asserting that letters are like constants (which have a value that is only positive)," but a common approach to 1-D kinematics problems is to do exactly that. And from a *vector equation* point of view, this reasoning is correct. Unfortunately for Max, at the level of second year, he has to be more careful. Since we might expect a solution to Eq. (2), his downfall is in translating this equation to the calculus. Since Max used Fig. 1 to guide his signs in this scalar equation, when *a* is translated into calculus he must write *a* using absolute value signs: $|dv/dt|$. Only in this way can he insure that his application of the negative sign is consistent. If this is done, then by applying the rules of the absolute value, we note that since dv/dt is negative, $|dv/dt|$ must be replaced by: $-dv/dt$ and Newton's law á la Max becomes $-m(-dv/dt) = mg - cv$ or $mdv/dt = mg - cv$. (Here we see perhaps the origin of the inner minus and outer minus.) However, the use of the absolute value sign in this way aligns the physics and the math. I would suggest that Max's error was mathematical: simply not applying (not knowing to apply?) the absolute value operator.

As a final thought, I would add that perhaps the physical misconception of Max in applying Newton's second law, and in particular how to remember to apply it as Eq. (1b), is in the use of the "equals" sign. It was pointed out to me by a mathematician that the "equals" sign in fact has many different meanings: equality (strictly logic); "follows from," as seen on many equal signs in one line of an equation; "is given by," in the use of mathematical function. Note this fact is backed up by the use of different symbols in computer programming languages. So I would suggest that Newton's second law is an application of the last, but with a slightly more physical bent: acceleration is a *result* of forces. Pointing out this subtle detail may help students realize that the acceleration *a* is the function $a(t)$, and is calculated using the forces.

1. Kate Hayes and Michael C. Wittmann, "The role of sign in students' modeling of scalar equations," *Phys. Teach.* **48**, 246–249 (April 2010).

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Sign issues for one-dimensional vectors

In a recent article,[1] Hayes and Wittmann consider a ball thrown vertically downward (defined as the $+y$ direction) with an initial speed greater than its terminal speed. Assuming a drag force for air resistance that is linear[2] in the ball's speed, they write Newton's second law in vector form as

$$m\mathbf{a} = m\mathbf{g} - c\mathbf{v}, \tag{1}$$

but subsequently fault students for writing a scalar version of this equation as

$$-ma = mg - cv\,. \tag{2}$$

In my opinion, it is Hayes and Wittmann who have it wrong, not the students. Equation (2) is perfectly correct for the conventional interpretation in which a vector quantity written in italics (or without an arrow on top of the symbol, in some textbooks) means the *magnitude* of the quantity. All standard textbooks with which I am familiar follow this convention and it could certainly explain why the students understand *a* to be positive! To solve Eq. (2), one must next

note that

$$a = -\frac{dv}{dt} \tag{3}$$

because the particle is *slowing down*. (Both a and v are magnitudes and thus positive, but the speed is decreasing with time.[3]) Substitution of Eq. (3) into (2) immediately gives Eq. (1c) in Hayes and Wittmann, and the solution then proceeds just as they describe and expect.

In principle that resolves all questions of sign in the original article. However, there is more that can be said from a pedagogical point of view. I think all of us physics instructors recognize that the approach I have taken above, while strictly *correct*, is not the most *helpful* way to teach the topic of vectors. A far better approach is to formally invoke components *even when the vectors are one-dimensional*. The y-component of Eq. (1) is

$$ma_y = mg_y - cv_y. \tag{4}$$

Because downward is positive, we have $g_y = +g$. (Presumably Hayes and Wittmann would agree that g is always positive, regardless of choice of axes. I find it curious that they do not treat other vectors in the same way.) Furthermore $a_y \equiv dv_y / dt$, in contrast to Eq. (3). Making these two substitutions, Eq. (4) becomes a more explicit version of Eq. (1c),

$$m\frac{dv_y}{dt} = mg - cv_y, \tag{5}$$

and *components* (unlike *magnitudes*) can be either negative or positive.

As a final note, many textbooks are careful about signs of one-dimensional vectors until they come to the treatment of collisions, where they may get sloppy. For example, Eq. (9.16) in Serway and Jewett[4] really should be written as

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \tag{6}$$

instead of leaving off the x subscripts.

1. K. Hayes and M. C. Wittmann, "The role of sign in students' modeling of scalar equations," *Phys. Teach.* **48,** 246–249 (April 2010).
2. Technically quadratic drag would be a better model for a thrown ball than linear drag, but that's not germane to the sign issues.
3. On a related note, I prefer to always refer to v as speed and never as velocity (which is **v**), in contrast to what Hayes and Wittmann do in their article.
4. R. A. Serway and J. W. Jewett, *Physics for Scientists and Engineers,* 7th ed. (Thomson Brooks/Cole, Belmont CA, 2008).

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Authors' response

We thank the two writers for their comments, which indicate the difficulty of dealing with letters in equations. These letters can represent functions, variables, parameters, and constants—it's no wonder that even experts see things differently.

Paetkau and Mungan assert that Max is correctly interpreting the equation as a magnitude equation. It is notable that this discussion of his thinking is disconnected from his actual words—he does not use the vocabulary that we use to describe what he's doing. The evidence we have is that he is inconsistent in his treatment of letters in equations. He never talks explicitly about a magnitude or a scalar equation, for example.

Both writers say that one can write $F = -ma$ as long as one writes $a = -dv/dt$. We, and all the colleagues we have asked, find these definitions to be deeply problematic. The first violates Newton's second law, the second the definition of acceleration. We could find no textbooks supporting the writers' claims, while our brief review of textbooks by Tipler, Cummings et al., and others, supports our assertion that scalar equations can contain negatively valued variables. Magnitude equations are scalar equations, but scalar equations need not be magnitude equations. More to the point, we and the textbooks we consulted treat one-dimensional equations as equivalent to component equations (though without the subscripts), as Mungan describes.

Our main point was not that Max was incorrect, at times, but that he was inconsistent. In conversations not presented in the paper due to length constraints, Max numerically solved for a negatively valued acceleration when asserting a positive number for velocity, and also asserted a positive value for the acceleration before trying to solve for the velocity. He accepted negative values and asserted positive values not just in the variables but also in the numbers.

Finally, Prof. Mungan asks why we do not treat all vectors like g, which he says is always positive. In simple kinematics questions, g is not treated as a (field) vector; it is a constant and constants are positive in physics (the CRC shows only one constant that is not). This is the most familiar situation for students, at this stage in their career, and we do not expect them to think otherwise. Of course, the sign of g is fraught with complications: some say g is 9.8 m/s^2 and one picks g or $-g$ in an equation, while others say equations just have g, and we can pick the value to be 9.8 or –9.8 m/s^2. We leave this discussion to our readers and their colleagues.

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MY COMMENT: I would never write $F = -ma$ because both F and a are positive *magnitudes*. I agree with Hayes and Wittmann that we need to explain to students the difference between scalars like charge q (which can be positive or negative) and magnitudes like F (or of any other vector) which can only be positive. -Carl