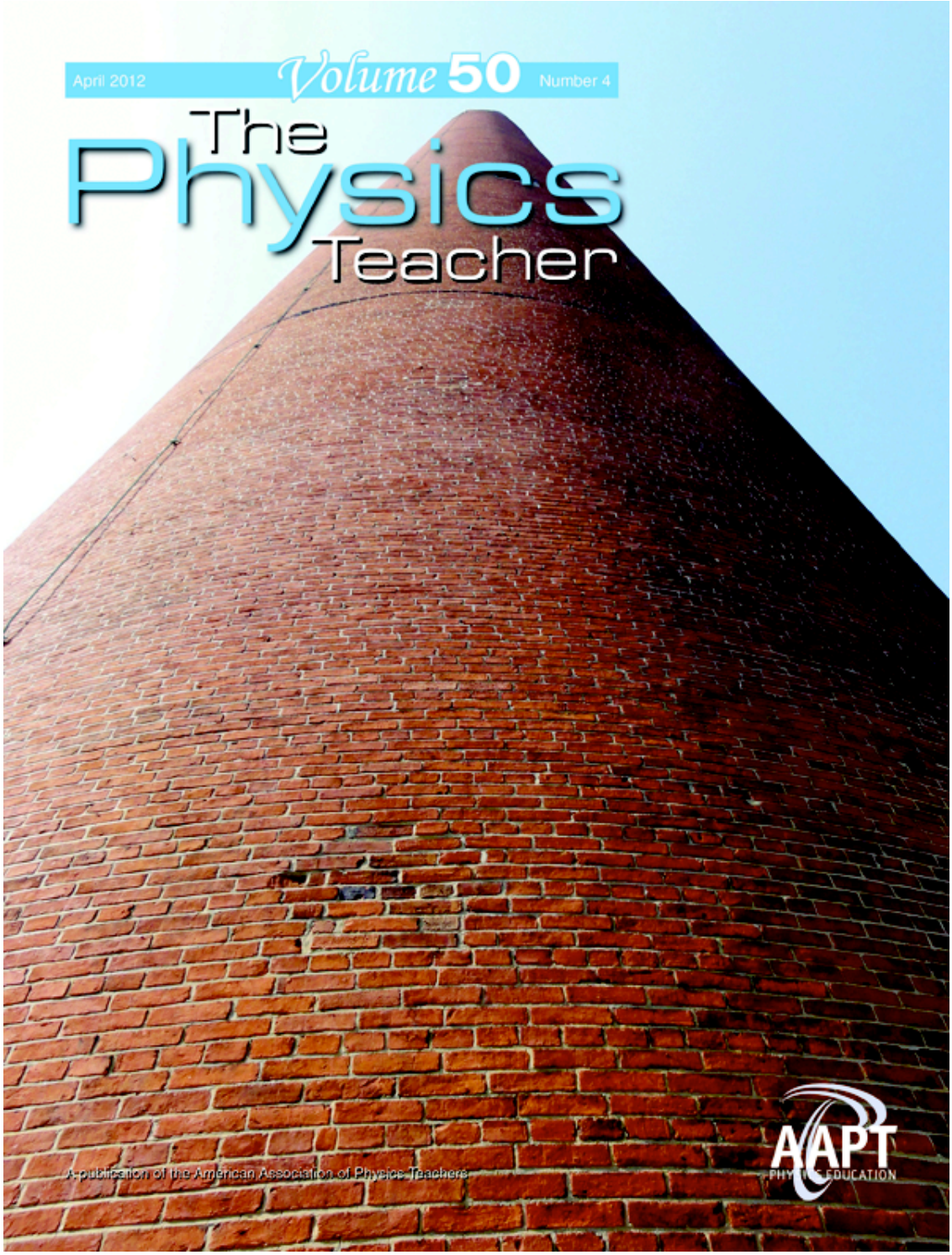

This month's cover shows a view of a shot tower in Baltimore, MD, looking up from its base. The paper beginning on page 218 of this issue discusses the physics behind how lead shot was made in the 18th and 19th centuries. (Photo taken by Carl Mungan.)

The Physics of Shot Towers

Trevor C. Lipscombe, Catholic University of America Press, Washington, D.C.
Carl E. Mungan, U.S. Naval Academy, Annapolis, MD

In the late 18th and throughout the 19th century, lead shot for muskets was prepared by use of a shot tower. Molten lead was poured from the top of a tower and, during its fall, the drops became spherical under the action of surface tension. In this article, we ask and answer the question: *How does the size of the lead shot depend on the height of the tower?* In the process, we explain the basic technology underlying an important historical invention (the shot tower) and use simple physics (Newtonian mechanics and the thermodynamic laws of cooling) to model its operation.

Introduction

In the heart of Baltimore, MD, stands the Phoenix shot tower (Fig. 1). Built in 1828, it measures 71 m in height. Another well-known shot tower in southern Virginia was completed in 1807 and has a 46-m total drop, half above and half below the ground level on top of a cliff. There are similar towers around the world, built in the same era, such as the Clifton Hill shot tower in Melbourne, Australia (constructed in 1882 and 80 m tall); the Chester shot tower in Chester, England (the oldest extant tower, built in 1799 and 41 m in height); and the Drochtersen shot tower, 42 m in height, in Lower Saxony, Germany. The purpose of these towers stems from a patent by an English plumber, William Watts, in 1782. Watts realized that if molten lead were poured from the top of a tower in fine streams, the liquid would break up into drops that would solidify during the fall, and the resulting pellets could be caught in a vat of water. This would serve as a much more economical method by which to produce lead shot for use in muskets than the previous technique of pouring lead into individual molds.

Today the shot towers are of interest historically—the Phoenix shot tower is a National Historic Landmark. But it is interesting to think of them as a way to introduce physics modeling to a high school or introductory college physics class. How does the size of the lead shot depend on the height of the tower? What size shot could the 80-m Clifton Hill shot tower produce that the 41-m Chester shot tower could not? The answers require combining knowledge of the mechanics of falling objects with models of thermal transfer and internal energy.

The shot tower is reminiscent of a popular laboratory experiment, the shot tube. Lead shot is contained in a tube made of a thermally insulated material. When the tube is inverted, the shot falls a distance L. Invert the tube 100 times and the shot falls a distance $100L$. The mechanical work done is primarily converted into thermal energy of the lead. So, by measuring the temperature difference of the shot between the beginning and end of the experiment, a student can infer the heat capacity of lead. In another experiment, wax is heated until it melts, the wax is removed from the heat source, and the temperature of the wax is subsequently recorded as a function of time. Initially there is a plateau in the temperature because of the energy needed for the wax to solidify, corresponding to the latent heat of fusion. These experiments point to the changes in the internal energy of a liquid that solidifies and cools, as needed to understand the physics of a shot tower.

Theoretical model

To produce size number 6 shot in one particular tower used in the early 20th century, five tons of molten lead per hour were poured into a copper pan having 2400 holes in it.[1] The resulting thin cylindrical jets almost immediately pinched off into columns of drops as a result of the Plateau-Rayleigh instability, analogous to the way in which a narrow laminar stream of water from a kitchen faucet breaks up as it falls toward the sink bottom. Up to 10^4 spherical pellets could be made this way *each second*!

Suppose a molten lead drop of mass m falls from the top of a shot tower of height H. To produce a pellet, the lead sphere must solidify during its fall. The amount of energy that must be removed for this purpose is mL, where $L = 24.7$ kJ/kg is the latent heat of fusion of lead.[2] At the bottom of the shot tower is a vat of water that cushions the impact and cools the shot to

Fig. 1. Looking up the interior shaft of the Phoenix shot tower in Baltimore from the location of the catching vat of water.

 DOI: 10.1119/1.3694072

room temperature. To avoid producing copious amounts of steam, the temperature of the solid lead ball as it reaches the water must be less than the boiling point of water. Thus, after solidifying, the ball must lose additional thermal energy during its flight of at least $mc\Delta T$, where $c = 128\ \mathrm{J{\cdot}kg^{-1}{\cdot}K^{-1}}$ is the specific heat of lead and $\Delta T = 227$ K is the temperature difference between the melting point of lead ($T_i = 600$ K) and the boiling point of water ($T_f = 373$ K).[2]

If a pellet takes a time Δt to fall, then the average rate at which it loses thermal energy ΔQ is

$$\frac{\Delta Q}{\Delta t} = m\frac{L + c\Delta T}{\Delta t}. \tag{1}$$

The instantaneous rate at which heat is dissipated (i.e, the power loss) to the environment by convection (forced by the motion of the shot) is

$$P_{\text{convection}} = hA(T - T_a). \tag{2}$$

Here h is the heat transfer coefficient and $A = 4\pi R^2$ is the surface area of a sphere of radius R, while the term in parentheses is the temperature difference between the lead sphere (T, which decreases from T_i to T_f) and the surrounding air ($T_a = 293$ K, which is assumed to be constant). The heat transfer coefficient is estimated in the appendix to be

$$h = \frac{0.185k}{R}\left(\frac{2\upsilon R}{\eta}\right)^{0.6}, \tag{3}$$

where k is the thermal conductivity of air, η is the kinematic viscosity of air, and υ is the speed of the sphere relative to the air. The rate of heat loss is thus

$$P_{\text{convection}} = 0.185k\left(\frac{2\upsilon}{\eta}\right)^{0.6} 4\pi R^{1.6}\left(T - T_a\right). \tag{4}$$

In general, both υ and T vary during the fall of a pellet, so it is a bit involved to find the exact time average[3] of Eq. (4), but roughly we can replace υ by $\bar{\upsilon} = \upsilon_T/2$ according to the appendix and T by $\bar{T} = (T_i + T_f)/2 = 487$ K, so that

$$\frac{\Delta Q}{\Delta t} \approx 0.185k\left(\frac{\upsilon_T}{\eta}\right)^{0.6} 4\pi R^{1.6}\left(\bar{T} - T_a\right). \tag{5}$$

The right-hand sides of Eqs. (1) and (5) are now equated, where the fall time is $\Delta t \approx H/\bar{\upsilon}$ and υ_T is given by Eq. (10), to obtain

$$R = \alpha H^{5/8}, \tag{6a}$$

where

$$\alpha = \left[\frac{0.37k(\bar{T} - T_a)}{L + c\Delta T}\right]^{5/8}\left(\frac{9}{\rho^2\eta}\right)^{3/8}\left(\frac{\rho_a}{16g}\right)^{1/8} = 1.2\times10^{-4}\ \mathrm{m}^{3/8} \tag{6b}$$

upon substituting the numerical values.

Equation (6) predicts that shot with a radius of up to 1.2 mm can be produced from a 40-m tower, in good agreement with the historical fact that number 6 shot was fabricated. A tower twice as high could produce shot that is 1.9 mm in radius, or up to size number 2.

Conclusion

By analyzing the thermal energy lost by a molten lead sphere as it solidifies and cools while falling from a shot tower, a sublinear relationship between the maximum radius of the pellets produced and the height of the tower has been derived that predicts historically plausible values. A similar relationship would be expected to hold for the formation of freezing rain falling from thunderclouds.

The original shot towers were subsequently improved by adding fans that blow upward as the spheres drop. This updraft increases the fall time as a result of the drag force. (For example, if the air speed were υ_T then the drops could actually float in the air.) In addition, the flow of fresh air minimizes the heating of the interior of the tower, which otherwise increases T_a and reduces the convectional cooling. Both effects enabled larger diameter shot to be produced from a given starting height.

Appendix:

Mechanisms of heat loss from a falling lead drop

The Nusselt number (N) is defined to be the dimensionless ratio of heat losses by convection and conduction given by

$$N = \frac{P_{\text{convection}}}{P_{\text{conduction}}} = \frac{hD}{k}, \tag{7}$$

where D is a characteristic length of the cooling object and k is the thermal conductivity of the fluid in which the object is immersed (air in the present situation). For a sphere, the characteristic length is its diameter, $D = 2R$. Experimental measurements for a large range of Reynolds numbers r indicate that the Nusselt number can be expressed in terms of the air flow around the sphere by the power law[4]

$$N = 0.37r^{0.6} = 0.37\left(\frac{2\upsilon R}{\eta}\right)^{0.6}, \tag{8}$$

where η is the kinematic viscosity of air, υ is the speed of the sphere (relative to the air), and R is the radius of the sphere. We can estimate the average Nusselt number $\bar{N}$ by replacing υ by its average value,

$$\bar{\upsilon} = \frac{\upsilon_i + \upsilon_f}{2} = \frac{\upsilon_f}{2}, \tag{9}$$

where $\upsilon_i = 0$ because the drops start nearly from rest. If a pellet were in free fall, its final speed υ_f would be $\sqrt{2gH}$ which is about 30 m/s for an object falling from the shortest shot towers of 40 m height. This value is larger than the terminal speed, which for quadratic drag[5] on a sphere (for which the drag coefficient is 0.5) falling through room-temperature air of density[2] $\rho_a = 1.29\ \mathrm{kg/m^3}$ is given by

$$\upsilon_T = \sqrt{\frac{2mg}{0.5\rho_a\pi R^2}} = \sqrt{\frac{16Rg\rho}{3\rho_a}}, \tag{10}$$

since $m = 4\pi R^3\rho/3$, where lead has a density[2] of $\rho = 11.3$ g/cm^3. Number 6 shot has a weight[1] of 223 pellets per ounce and thus this density implies that $R = 1.39$ mm. Substituting the numbers into Eq. (10) leads to $\upsilon_f = \upsilon_T = 25$ m/s. Finally, the kinematic viscosity η of air at room temperature is its dynamic viscosity[2] $\mu = 0.018$ mPa·s divided by ρ_a, which works out to be 1.4×10^{-5} m^2/s. Inserting these values of $\bar{\upsilon}$, R, and η into Eq. (8) gives $\bar{N} = 40$, implying that convection dominates over thermal conduction.

The other mechanism of heat loss is radiation, whose rate is given by the Stefan-Boltzmann law as

$$P_{\text{radiation}} = e\sigma A\left(T^4 - T_a^4\right) \tag{11}$$

(assuming the walls of the shot tower to be at the same temperature as the air), where e is the emissivity of lead and $\sigma = 5.67\times10^{-8}$ W·m^{-2} · K^{-4}. Using Eqs. (2), (7), and (11), we find the ratio of heat losses by convection and radiation is thus

$$\frac{P_{\text{convection}}}{P_{\text{radiation}}} = \frac{Nk}{2e\sigma R\left(T + T_a\right)\left(T^2 + T_a^2\right)}. \tag{12}$$

The smallest possible value this ratio can have is given by using the largest possible numbers in the denominator, namely $e = 1$ and $T = 600$ K. Noting that the thermal conductivity of room-temperature air[2] is $k = 26$ mW·m^{-1}· K^{-1} and using $N = 40$, the ratio is therefore no smaller than about 20, implying that convection also dominates over thermal radiation.

In summary, the dominant mechanism of heat transfer from the lead spheres to the environment is convection. Substituting Eq. (8) into (7) leads to Eq. (3).

References

1. G. V. Sanders, "Gravity molds shot in a modern tower," *Pop. Sci.* **145**, 123–127 (Oct. 1944), available online by searching on the title of the article.
2. P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers,* 6th ed. (W.H. Freeman, New York, 2008).
3. An exact calculation proceeds as follows. First compute the fall distance X required for a molten drop to solidify (during which time the temperature of the lead remains constant at its melting point, T_i) by solving

$$0.185k\left(\frac{2}{\eta}\right)^{0.6} 4\pi R^{1.6}\left(T_i - T_a\right)\int_0^X \upsilon^{-0.4}dy = \frac{4\pi}{3}\rho R^3 L$$

(with the positive y-axis pointing downward). Given a formula for $\upsilon(y)$ —e.g., $(2gy)^{1/2}$ in free fall or υ_T at terminal speed—this integral can be computed to determine $X(R)$. Next find the distance Y over which the solid sphere cools. The convectional cooling power equals the rate of *decrease* of the internal energy U of a pellet, so that

$$P_{\text{convection}} = -\frac{dU}{dt} = -\frac{dy}{dt}\frac{dU}{dy}$$

using the chain rule. Note that dy/dt equals the speed υ of the shot, while $dU = mcdT$. Hence the right-hand side of Eq. (4) can be equated to $-\upsilon mc\, dT/dy$. Again given a formula for $\upsilon(y)$, a separable differential equation is obtained that can be solved for $Y(R)$. Finally, add $X + Y = H$ to relate the height of the tower to the maximum radius R of the shot that can be produced. For example, if one uses $\upsilon = \upsilon_T$ for both parts of the motion, then one obtains Eq. (6) with $\alpha = 1.1\times10^{-4}m^{3/8}$, essentially the same result that was found above by approximating the averages. (Incidentally, in this case $Y/X \approx 2$ independent of R, indicating that it takes about one-third of the height of the tower for the sphere to solidify and the remaining two-thirds for it to cool down.)
4. F. Kreith and M.S. Bohn, *Principles of Heat Transfer,* 6th ed. (Brooks/Cole, Pacific Grove, CA, 2001), p. 437.
5. Using the numbers cited after Eq. (10), the average Reynolds number is $\bar{r} = \upsilon_T R/\eta = 2500$, which is large enough to justify modeling the air resistance as being quadratic rather than linear in the speed of a drop.

Trevor C. Lipscombe, *a physicist by training, is the author of* The Physics of Rugby *(Nottingham Univ. Press, 2009) and coauthor of* Albert Einstein: A Biography *(Greenwood, 2005). His current position engages his other passion in medieval theology, but he still finds free time to indulge his interests in physics.*
Catholic University of America Press, Washington, DC 20064; lipscombe@cua.edu

Carl Mungan *is a frequent contributor to educational physics journals. His current research interests include measurements of upconversion in erbium-doped laser materials and computer simulations of Brillouin scattering in infrared optical fibers.*
U.S. Naval Academy, Department of Physics, Annapolis, MD 21402; mungan@usna.edu