OPTICAL IMAGES DUE TO LENSES AND MIRRORS*

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ABSTRACT

The properties of real and virtual images formed by lenses and mirrors are reviewed. Key ideas are summarized in tables and rules of thumb. Simple conceptual problems illustrate the utility of the results. The practical significance of virtual objects is illustrated by the construction of a noninverting optical microscope.

When light from an object of height h located at distance p from a mirror or lens of focal length f is reflected or transmitted by the optical element, an image of height h' is formed at distance q from the mirror or lens. Sign conventions are adopted that p is positive for a real object (i.e., an object located on the incident side of the optical element) and is negative for a virtual object (which occurs when a lens or mirror intercepts the light from another optical element prior to the image location); q is positive for a real image (i.e., one that can be displayed on a screen) and is negative for a virtual image (i.e., that can only be seen by looking "into" the lens or mirror, because it forms behind the optical element relative to the location of an observer's eye); and f is positive for a converging lens or concave mirror [i.e., one that focuses incident collimated light to a spot, as illustrated in Fig. 1(c) later in this paper], whereas f is negative for a diverging lens or convex mirror. The (lateral) magnification of the image is defined as $M \equiv h'/h$ and can be shown to equal

$$M = -q/p \tag{1}$$

by noting that that a right triangle with sides equal to h and p on the object

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side is similar to a right triangle whose sides are h' and q on the image side of the optical element.† The sign convention for M is that it is positive for an upright image (relative to the orientation of the object) and negative for an inverted image. If $|M| > 1$ then the image is enlarged, whereas it is reduced if $|M| < 1$. We conclude from Eq. (1) that an image is enlarged if it is located farther away from the lens or mirror than is the object, and conversely an image is reduced in size if it is nearer the optical element than is the object.

It is proven in introductory physics textbooks[1] that the image and object distances are related to the focal length of a mirror or thin lens according to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \Rightarrow \quad q = \frac{pf}{p - f} \tag{2}$$

for paraxial rays. This relation will be referred to as the "image equation," while Eq. (1) is called the "magnification equation." Note that if one knows the values of p and f for some object and optical element, one can use these two equations to quantitatively answer the following four key questions about the resulting image:

1. Is the image real or virtual? Call the answer the "type" of image; it is determined by the sign of q from the image equation.

2. How far away is the image from the lens or mirror? Refer to that as the "location" of the image, given by the magnitude of q.

3. Is the image upright or inverted? That defines the "orientation" of the image and it is specified by the sign of M from the magnification equation (found by substituting into it the value of q from the image equation).

4. How big is the image relative to the object? This "magnification" of the image is determined by the magnitude of M.

The answers or range of values for these four quantities are summarized in Table I for a real object and depend on whether the optical element has a

†One hypoteneuse is a ray connecting the tip of the object to the vertex of the mirror or lens, and the other hypoteneuse is a ray connecting the tip of the image to the vertex.

positive (converging/concave) or negative (diverging/convex) value of the focal length. In addition, it turns out there are three different cases for a positive optic, depending on whether the object is located farther away than twice the focal length (i.e., beyond the center of curvature C in the case of a spherical mirror), between $2f$ and f (i.e., between C and the focal point F for a curved mirror), or nearer than a focal length (i.e., between F and the vertex V for a concave mirror).

Table I. Key properties of the image formed by an optical element for a REAL object.

	Concave Mirror or Converging Lens			*Convex Mirror or Diverging Lens*
	$p > 2f$	$2f > p > f$	$p < f$	
type	real	real	virtual	virtual
location	$2f > q > f$	$q > 2f$	$q < 0$	$0 > q > f$
orientation	inverted	inverted	upright	upright
magnification	reduced	enlarged	enlarged	reduced

The entries in this table can also be obtained by standard ray tracing and it is an excellent exercise to ask students to do so. Excluding the column on the left, number the four columns of answers as 1 to 4 from left to right. If a ray-tracing diagram corresponding to column 1 ($p > 2f$) is prepared, then the entries for column 2 can be immediately obtained from it by reversing the directions of all rays and interchanging the roles of the object and image. Thus it is sufficient to ask students to prepare (or find in a textbook) three careful ray-tracing sketches for spherical mirrors and three for thin lenses, in order to reproduce Table I. Incidentally, notice that for a real object and a diverging lens, the image is always virtual and reduced. Therefore the image is always sandwiched between the object and the lens, regardless of how far the object is from the lens. In contrast, for a real object and a converging lens, the image can never be located between them; the image is on the transmitted side of the lens for a distant object, approaches infinity as the object approaches the focal point, and then

loops around to negative infinity and starts approaching the object on the incident side as the object continues to approach the lens.

Here are some examples of the kinds of questions and problems one can easily answer using Table I:

(A) A mirror is installed near the ceiling of a video store to provide security against theft. What kind of mirror should be used and what can one say about the properties of the resulting image?

To be useful, the image needs to be upright and to show a large portion of the store. The latter requirement implies that the image be reduced in size. Studying Table I, we see that only column 4 gives an upright, reduced image. Consequently the mirror has to be convex, and the image is virtual and located less than one focal length behind the mirror.

(B) How is a magnifying glass constructed and used?

We want an enlarged, upright image, which corresponds to column 3 in Table I. Consequently one uses a converging lens with the object placed just inside the focal point, to give a virtual object near infinity (allowing the observer's eye to be as relaxed as possible), similar to what is illustrated in Fig. 1(c), so that the magnification is large and positive. (A slightly larger angular magnification is possible if the image is instead located at the observer's near point.[1])

(C) What kind of lens is used in a slide projector and what can one say about the object?[1]

We wish to project a large image onto a screen. A real, enlarged image is only obtained for column 2 of Table I. The slide therefore needs to be inserted into the projector upside-down (since the image is inverted) and the lens must be positioned between one and two focal lengths away from the slide by moving the barrel in and out to focus the image. Similarly, by interchanging the object and image and reversing all optical rays, we see that the image on the film or CCD of a camera is real, reduced, and inverted, in agreement with column 1 of Table I.

(D) How can a lens be used to form a life-size image of an object?

Examining Table I, we see that the only way to "straddle" the boundary between a reduced and enlarged image for a fixed kind of lens is to use a converging lens and locate the object exactly $2f$ away from it. In that case, the image equation implies that q will also be $2f$, and the magnification equation then gives $M = -1$. The image is necessarily real and inverted. This interesting situation is sometimes called "one-to-one imaging" and is illustrated in Fig. 1(a).

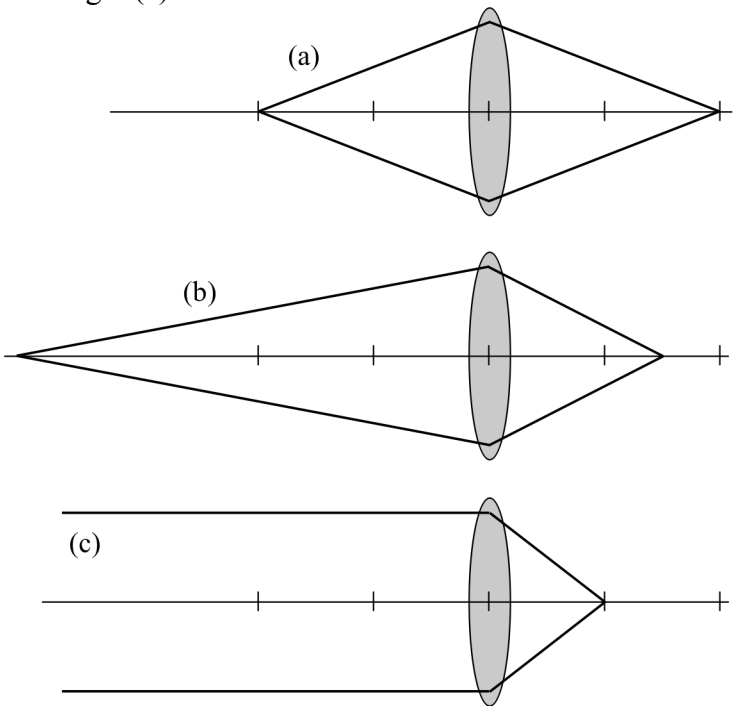

Fig. 1. Sketches of the rays leaving a small, real object and refracting through the periphery of a converging lens to form a real image. In accordance with the principle of optical reversibility, either the object is on the left and the image is on the right of the lens, or vice-versa. The principal axis is marked off in intervals of a focal length of the lens. If we fold the transmitted rays over to the incident side, these diagrams could alternatively represent a concave mirror. (a) One-to-one imaging when p and q are both equal to $2f$. (b) If we increase p then q decreases, and vice-versa. For example, if one moves an overhead projector closer to the screen, then one has to raise the lens away from the transparency to refocus the image.[2] (c) If the object is infinitely far away, then the image is at the focal point, and vice-versa. Notice that if we are to have a real object and a real image, then neither can be closer than one focal length to the lens.

(E) A dentist uses a small mirror to form an upright $10\times$ image of a tooth. The mirror is located about 1 cm away from the tooth.[1] What kind of mirror is it? Estimate its radius of curvature. What kind of image is formed and where?

An upright, enlarged image is obtained for column 3 of Table I. Therefore the mirror is concave. Its focal length must be a bit larger than 1 cm since the image is much farther away from the mirror than is the object. The radius of curvature R of the spherical mirror is double its focal length and is thus about 2 cm. The image is virtual and is located about 10 cm behind the mirror. If the dentist moves the mirror farther away from the tooth, notice that the image properties change from column 3 to column 2 of Table I and consequently the image will flip upside-down. Students can check this prediction for themselves using a concave make-up or shaving mirror.

One can similarly work out the image properties that result for virtual objects located at all possible distances away from converging or diverging lenses or mirrors, as summarized in Table II.

Table II. Key properties of the image formed by an optical element for a VIRTUAL object.

	Convex Mirror or Diverging Lens			*Concave Mirror or Converging Lens*
	$p < 2f$	$2f < p < f$	$p > f$	
type	virtual	virtual	real	real
location	$2f < q < f$	$q < 2f$	$q > 0$	$0 < q < f$
orientation	inverted	inverted	upright	upright
magnification	reduced	enlarged	enlarged	reduced

Comparing Tables I and II, we see that there are three sets of differences between them: the positive and negative optical elements are interchanged in the header rows; the real and virtual image types are interchanged; and

all of the inequalities for p and q are reversed. The reversal of the inequalities is necessary because of the opposite signs for p and f for corresponding columns in Tables I and II. If we ignore the signs, then the object and image distances agree for columns having the same number in the two tables. For example, in column 1 of either table, the object is located more than two focal lengths away from the lens; in column 4 of either table, the image is located within one focal length of the mirror; and so forth. On the other hand, the fact that the image types are interchanged between the two tables leads to a memorable rule of thumb:[3] Images are inverted if and only if the image and object are of the same type, i.e., either both real (as in Fig. 1) or both virtual.

As an example of the practical application of Tables I and II, consider constructing a noninverting microscope using two lenses. To obtain maximum overall magnification, we want both lenses to enlarge. The initial object is real; the final image should be virtual (preferably with a large negative image distance so that the observer's eye is as relaxed as possible). The overall image is to be uninverted. Carefully studying the two tables, only two designs satisfy all of these criteria:

(i) Neither lens inverts. This can be done with two converging lenses from column 3 of the real object table, corresponding to two magnifying glasses. However, this scheme is impractical because the virtual image from the first lens has to be within a focal length of the second lens, which means the two lenses have to be very close to one another. One may as well just use a single, more powerful magnifying glass!

(ii) Both lenses invert. This requires a converging lens from column 2 of the real object table and a diverging lens from column 2 of the virtual object table. This design is sensible.

I constructed a microscope according to the latter scheme using standard Pasco™ optics equipment.[4] The object is a light source positioned at 0 cm on an optics bench. The first, converging lens has $f_1 = +20\text{ cm}$. The object has to be a little farther away than one focal length, say $p_1 = +26\text{ cm}$ (i.e., the first lens is positioned at 26 cm). One then calculates $q_1 = +86.7\text{ cm}$ (which can be observed by placing a screen at

112.7 cm on the bench). The magnification is $M_1 = -10/3$ (corresponding to an inverted and enlarged first image). The second, diverging lens has $f_2 = -15$ cm. The first image needs to be a virtual object for the second lens and be located a little farther away from it than its focal length, say $p_2 = -17$ cm. In that case, the separation between the two lenses is $q_1 + p_2 = 69.7$ cm (i.e., the second lens has to be positioned at 95.7 cm on the bench). Now one calculates $q_2 = -127.5$ cm and $M_2 = -15/2$ (i.e., the second image is also inverted and enlarged). Thus the final image is overall uninverted, virtual, and enlarged by $M_1 M_2 = 25\times$. To observe it, simply remove the screen from the bench and look through the second lens toward the light source. You will see an enlarged, upright image of the ground-glass ruling on the source. The field of view is a bit small due to the long distances involved, but the image is reasonably sharp. Such a microscope could be used, for example, to monitor a chemical reaction occurring in a small cuvette from a safe distance.

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References

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4. The components I used to construct the microscope are a +200 mm and a –150 mm mounted lens, a light source, a viewing screen, and a 1.2-m optics bench, all available from <http://www.pasco.com>.