Summary of Formulae for Collision Problems—C.E. Mungan, Fall 2000

We assume that we have an isolated system of two objects and that we are given m_1, m_2, $\boldsymbol{\upsilon}_{1i}$, and $\boldsymbol{\upsilon}_{2i}$ and are trying to solve for $\boldsymbol{\upsilon}_{1f}$ and $\boldsymbol{\upsilon}_{2f}$. (If instead you are given some or all of the information about the final velocities, you can rearrange the following equations appropriately.) In the case of a perfectly inelastic collision, $\boldsymbol{\upsilon}_f \equiv \boldsymbol{\upsilon}_{1f} = \boldsymbol{\upsilon}_{2f}$.

<u>1D Perfectly Inelastic</u>

Conservation of linear momentum:

$$\upsilon_{fx} = \frac{m_1\upsilon_{1ix} + m_2\upsilon_{2ix}}{m_1 + m_2} \tag{1'}$$

<u>1D Elastic</u>

Conservation of linear momentum:

$$m_1\upsilon_{1ix} + m_2\upsilon_{2ix} = m_1\upsilon_{1fx} + m_2\upsilon_{2fx} \tag{1}$$

Elastic collision equation (only valid before and after the collision):

$$\upsilon_{1ix} + \upsilon_{1fx} = \upsilon_{2ix} + \upsilon_{2fx} \tag{2'}$$

Solve these two linear equations simultaneously to get υ_{1fx} and υ_{2fx}, as given by Serway Eqs. (8.20) and (8.21) with x subscripts added to every velocity. Make sure you know how to do this! During the collision, Eq. (1) still holds, but Eq. (2)' must be replaced by conservation of mechanical energy (where U is the interactional potential energy):

$$\tfrac{1}{2}m_1\upsilon_{1i}^2 + \tfrac{1}{2}m_2\upsilon_{2i}^2 + U_i = \tfrac{1}{2}m_1\upsilon_{1f}^2 + \tfrac{1}{2}m_2\upsilon_{2f}^2 + U_f \tag{2''}$$

<u>1D Imperfectly Inelastic</u>

Conservation of linear momentum:

$$m_1\upsilon_{1ix} + m_2\upsilon_{2ix} = m_1\upsilon_{1fx} + m_2\upsilon_{2fx} \tag{1}$$

In addition, you need to be given directly or implicitly either υ_{1fx} or υ_{2fx}, in order to be able to solve Eq. (1) for the other.

<u>2D Perfectly Inelastic</u>

Conservation of linear momentum:

$$\upsilon_{fx} = \frac{m_1\upsilon_{1ix} + m_2\upsilon_{2ix}}{m_1 + m_2} \quad \text{and} \quad \upsilon_{fy} = \frac{m_1\upsilon_{1iy} + m_2\upsilon_{2iy}}{m_1 + m_2} \tag{1'}$$

2D Elastic

Conservation of linear momentum:

$$m_1\upsilon_{1ix} + m_2\upsilon_{2ix} = m_1\upsilon_{1fx} + m_2\upsilon_{2fx} \quad \text{and} \quad m_1\upsilon_{1iy} + m_2\upsilon_{2iy} = m_1\upsilon_{1fy} + m_2\upsilon_{2fy} \qquad (1)$$

Conservation of mechanical energy:

$$\tfrac{1}{2}m_1\upsilon_{1i}^2 + \tfrac{1}{2}m_2\upsilon_{2i}^2 + U_i = \tfrac{1}{2}m_1\upsilon_{1f}^2 + \tfrac{1}{2}m_2\upsilon_{2f}^2 + U_f \qquad (2)''$$

Before and after the collision, the interactional potential energy is zero so that (2)" simplifies to:

$$\tfrac{1}{2}m_1\upsilon_{1i}^2 + \tfrac{1}{2}m_2\upsilon_{2i}^2 = \tfrac{1}{2}m_1\upsilon_{1f}^2 + \tfrac{1}{2}m_2\upsilon_{2f}^2 \qquad (2)$$

In addition, you need to be given directly or implicitly υ_{1fx}, υ_{2fx}, υ_{1fy}, υ_{2fy}, υ_{1f}, υ_{2f}, θ_{1f}, or θ_{2f}. Using this together with Eqs. (1) and (2) allows you to solve simultaneously for all the others.

2D Imperfectly Inelastic

Conservation of linear momentum:

$$m_1\upsilon_{1ix} + m_2\upsilon_{2ix} = m_1\upsilon_{1fx} + m_2\upsilon_{2fx} \quad \text{and} \quad m_1\upsilon_{1iy} + m_2\upsilon_{2iy} = m_1\upsilon_{1fy} + m_2\upsilon_{2fy} \qquad (1)$$

In addition, you need to be given directly or implicitly two of (υ_{1fx} or υ_{2fx}), (υ_{1fy} or υ_{2fy}), υ_{1f}, υ_{2f}, θ_{1f}, and θ_{2f}.