Examples of Functions expressed in terms of Hypergeometric Series

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$$\boxed{F(a,b,c;x) \equiv \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{x^n}{n!}}$$

$F(1,1,1;x) = \dfrac{1}{1-x}$	geometric series
$xF(\frac{1}{2},1,\frac{3}{2};-x^2) = \tan^{-1}(x)$	inverse trignometric functions
$xF(1,1,2;-x) = \ln(1+x)$	logarithms
$\dfrac{\pi}{2} F(\frac{1}{2},\frac{1}{2},1;k^2) = K(k)$ $\dfrac{\pi}{2} F(\frac{1}{2},-\frac{1}{2},1;k^2) = E(k)$	complete elliptic integrals
$F(-l,l+1,1;\frac{1-x}{2}) = P_l(x)$	Legendre polynomials

and in terms of Confluent Hypergeometric Series

$$\boxed{M(a,c;x) \equiv \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} \frac{x^n}{n!}}$$

$\dfrac{2}{\sqrt{\pi}} xM(\frac{1}{2},\frac{3}{2};-x^2) = \text{erf}(x)$	error function
$\dfrac{e^{-ix}}{\Gamma(p+1)}\left(\dfrac{x}{2}\right)^p M(p+\frac{1}{2},2p+1;2ix) = J_p(x)$	Bessel functions
$(-1)^n \dfrac{(2n)!}{n!} M(-n,\frac{1}{2};x^2) = H_{2n}(x)$ $(-1)^n \dfrac{(2n+1)!}{n!} 2xM(-n,\frac{3}{2};x^2) = H_{2n+1}(x)$	Hermite polynomials
$M(-n,1;x) = L_n(x)$	Laguerre polynomials