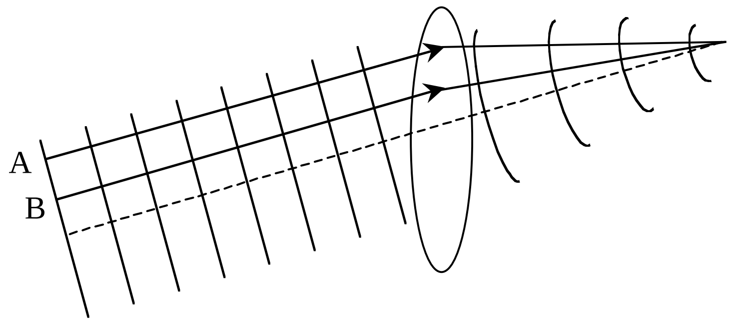
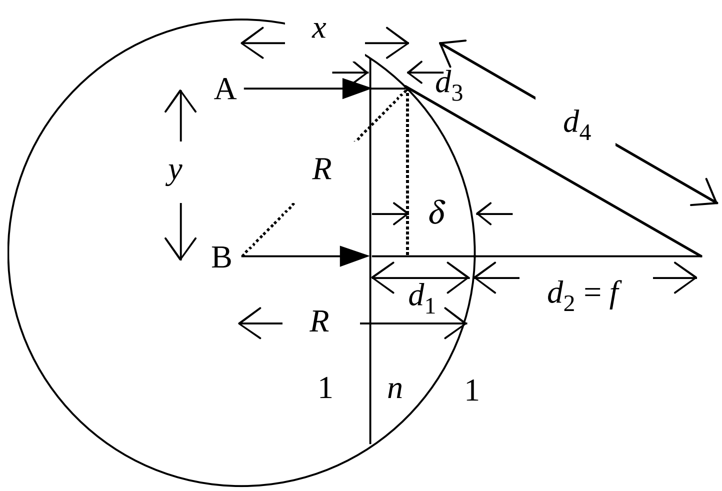
The Lensmaker Formula—C.E. Mungan, Fall 1998

Two parallel beams of light which originate on a common plane wavefront travel the same optical pathlength (OPL) when focused to a point by a lens.



That is, OPL(A) = OPL(B) in the above diagram, since the rays are everywhere orthogonal to the wavefronts and both rays thus undergo the same number of optical cycles. The above diagram is identical to Fig. 5 on p. 949 of Halliday, Resnick, and Krane. As shown in Hecht Secs. 5.2.2–5.2.3, we can use this to derive the Lensmaker formula. For example, consider the simpler case of a wavefront perpendicularly striking a plano-convex thin lens. In the following diagram, *R* is the radius of curvature of the lens and we follow Hecht's convention for measuring *f* from the surface of a thin lens. The indices of refraction of the air and glass are as indicated.



Now,

$$x^2 + y^2 = R^2 \Rightarrow x = \sqrt{R^2 - y^2} \cong R - y^2/2R \qquad (1)$$

since $y \ll R$ in the paraxial approximation. Also,

$$d_3 = x - (R - d_1) \cong d_1 - y^2/2R \qquad (2)$$

using Eq. (1), so that

$$\delta = d_1 - d_3 \cong y^2/2R. \qquad (3)$$

Therefore,

$$d_4^2 = y^2 + (d_2 + \delta)^2 \cong y^2 + d_2^2(1 + 2\delta/d_2) \text{ since } \delta \ll f \text{ for a thin lens}$$

$$\cong y^2 + d_2^2 + d_2 y^2/R \text{ using Eq. (3)}$$

$$\Rightarrow d_4 \cong d_2\left[1+\frac{y^2}{2d_2^2}\left(1+\frac{d_2}{R}\right)\right] \tag{4}$$

using the paraxial approximation once more. Combining Eqs. (2) and (4), we have

$$nd_3 + d_4 \cong nd_1 + d_2 + \frac{y^2}{2d_2R}\left(R + d_2 - nd_2\right).$$

But equality of the optical pathlengths for rays A and B implies that $nd_3 + d_4 = nd_1 + d_2$. Thus the quantity in parentheses must be zero; substitute $d_2 = f$ into it and rearrange to obtain

$$\frac{1}{f} = (n-1)\frac{1}{R}$$

which is the Lensmaker formula for a plano-convex lens. You can now glue two such lenses together to get a biconvex lens.