

Show ALL work. All work must be done on these test sheets. There is no way for you to receive credit for work that is not explicitly written down. I also recommend showing work for the multiple choice questions as partial credit may be possible on a few of them.

Credit for problems will be awarded based upon "our standard problem solving approach" which emphasizes starting from fundamentals and following standard problem solving procedures. Problem solving style is important here.

Moments of Inertia of Rolling Objects about their center-of-mass and symmetry axis:

$$I_{\text{solid ball}} = 2/5 MR^2 \quad I_{\text{disk}} = 1/2 MR^2 \quad I_{\text{loop}} = MR^2$$

Moments of Inertia of Objects spun about an edge:

$$I_{\text{rectangle about an axis along edge}} = 1/3 ML^2$$

$$\begin{aligned} 10 \text{ mph} &= 44.7 \text{ m/s}, 1 \text{ inch} = 2.54 \text{ cm}, \sin 30^\circ = 0.5, \sin 45^\circ = 0.707, \sin 60^\circ = 0.866 \\ G &= 6.67e-11 \text{ Nm}^2/\text{kg}^2, M_{\text{Earth}} = 5.96e+24 \text{ kg}, M_{\text{Moon}} = 7.35e+22 \text{ kg}, M_{\text{Sun}} = 2.0e+30 \text{ kg} \\ \text{Radius of Moon} &= 1740 \text{ km}, \text{Radius of Sun} = 6.96e+5 \text{ km}, \text{Radius of Earth} = 6370 \text{ km} \\ 1 \text{ eV} &= 1.602e-19 \text{ J}, 1 \text{ hp} = 746 \text{ W}, c = 3.00e+8 \text{ m/s} \end{aligned}$$

True/False

$$\text{Work: } \int \vec{F} \cdot d\vec{s}$$

T 1. A force which is always perpendicular to the velocity of a particle does no work on the particle.

F 2. A kiloWatt-hour is a unit of power. Watt is power. Watt  $\times$  time is energy

T 3. Internal forces do not affect the motion of the center-of-mass of the system.

T 4. The momentum of a system can be conserved even if the mechanical energy is not.

F 5. In a perfectly inelastic collision all of the kinetic energy of the particles is lost. *Conservation of momentum persists*

T 6. All parts of a rotating wheel have the same angular velocity.

T 7. All parts of a rotating wheel have the same angular acceleration.

T 8. The moment of inertia of an object depends upon the location of the rotation axis.

F 9. The moment of inertia of a body depends on the angular velocity of the body.

F 10. If the net torque on a body is zero, the angular momentum must be zero.

T tells us how the angular momentum is changing, not what its value is at a split second.

### Multiple Choice

MC1. A particle moves in uniform circular motion. The work done on it by the centripetal force is

- A) zero
- B) positive
- C) negative

MC2. The potential energy of a particle is given by  $U(x) = 3x - 5x^2$ . The x-component of the force  $\vec{F}$  acting on the particle obeys

$$\begin{aligned} F_x &= -\frac{dU}{dx} \\ &= -[3 - 10x] \\ &= -3 + 10x \end{aligned}$$

$$= -3 + 10x$$

MC3. Which of the following statements about the gravitational and elastic potential energies is true?

- A) An arbitrary constant can be added to each without altering the force.
- B) An arbitrary constant can be added to neither
- C) An arbitrary constant can be added to the elastic energy but not the gravitational.
- D) An arbitrary constant can be added to the gravitational energy but not the elastic.
- E) All of the above.

MC4. A ball is dropped from a height  $h$  and hits the ground with speed  $v$ . To have the ball hit the ground at a speed of  $2v$  it should be dropped from a height

- A)  $h/2$
- B)  $h$
- C)  $2h$
- D)  $3h$
- E)  $4h$

$$\begin{array}{c} 2v \\ \downarrow \\ \overbrace{\hspace{1cm}}^{2h} \\ \text{will have 4 times} \\ \text{the kinetic energy of} \\ \text{the other picture} \end{array}$$

MC5. Two blocks of different masses are projected up a frictionless inclined plane with identical speeds. Which block rises to a greater height?

- A) the lighter one because it has less inertia
- B) the heavier one because it has more kinetic energy.
- C) They reach the same height
- D) More information is needed

$$\frac{1}{2}mv^2 = mgh$$

mass cancels

- MC6. A 1.5 kg mass rests atop a spring with spring constant 500 N/m. The spring is compressed a distance 25 cm and released. The ball will ascend to a height
- A) 0.5 m  
B) 0.6 m  
C) 0.8 m  
**D) 1.0 m**  
E) 10.0 m

$$\frac{1}{2} k x^2 = m g h$$

$$\frac{1}{2} (500)(0.25)^2 = 15(9.8) h$$

$$h = 1.0 \text{ m}$$

- MC7. An object is projected vertically upward from a planet of radius R with a velocity equal to one half the escape velocity. The maximum altitude attained by the object is
- A) 0.3 R  
B) 0.5 R  
**C) 1.3 R**  
D) 1.5 R  
E) 2.0 R

$$\frac{1}{2} m v_e^2 + G \frac{m M}{R} = 0 + G \frac{m m}{r}$$

$$v_e = \sqrt{2 G \frac{M}{R}}$$

Now project upward with half that velocity

$$\frac{1}{2} m \left[ \frac{1}{2} (2 G \frac{M}{R}) \right] + G \frac{m m}{R} = 0 + -G \frac{m M}{r}$$

$$\frac{1}{8} 2 G \frac{m M}{R} - G \frac{m m}{R} = -G \frac{m M}{r}$$

$$\frac{1}{4} \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$$

$$-\frac{3}{4} \frac{1}{2} = -\frac{1}{2}$$

$$r = \frac{4}{3} R$$

- MC8. A person ascends a flight of stairs in 20 s. The person weighs 500 N and the vertical height of the stairs is 9 m. The person's power output is

- A) 150 W  
**B) 225 W**  
C) 325 W  
D) 450 W  
E) 600 W

- MC9. Automobile air bags are effective because they:
- A) increase the impulse time  
B) decrease the impulse time  
C) decrease the momentum change  $\Delta p$   
D) decrease the time that the impulse acts  
E) none of the above

- MC10. A small boat collides (actually bounces off) a large ship. During the collision the boat experiences the larger average collision force
- A) the ship experiences the larger average collision force  
B) the boat experiences a larger change of momentum than the ship  
C) they both experience the same average collision force  
D) more information is needed

This is just the  
"Action-Reaction"  
Newton's 3rd Law.

- MC11. When fired, a rifle will recoil backward because the:  
total energy is conserved  
momentum of rifle and bullet is conserved  
momentum of bullet is greater than the energy of the rifle  
energy of the bullet is greater than the energy of the rifle after the firing  
none of the above

- MC12. When a certain rubber band is stretched a distance x, it exerts a restoring force of magnitude  $F = a + b x^2$ . The work done by an outside agent in extending the rubber band from  $x = 0$  to  $x = L$  is:

$$\begin{aligned} W_{\text{ext}} &= \int \vec{F} \cdot d\vec{x} = \int_0^L (a + b x^2) dx \\ &= \left[ ax + \frac{1}{3} b x^3 \right]_0^L \\ &= \frac{1}{3} b L^3 + \frac{1}{3} b L^3 \\ &= \frac{2}{3} b L^3 \end{aligned}$$

- MC13. Shown below are masses  $m_1$  and  $m_2$  with initial momenta  $\vec{p}_1$  and  $\vec{p}_2$  before the collision as well as the final momentum  $\vec{p}'_1$  of mass  $m_1$  after the collision. The vector which could best represent the final momentum  $\vec{p}'_2$  of  $m_2$  after the collision is

- A) **B**  
B) C  
C) D  
D) E

$$\begin{aligned} \vec{p}_1 &= m_1 v_1 \hat{i} \\ \vec{p}_2 &= m_2 v_2 \hat{j} \\ \vec{p}'_1 &= m_1 v'_1 \hat{i} \\ \vec{p}'_2 &= ? \end{aligned}$$

- MC14. A bomb hanging from a string explodes into pieces of different shapes and sizes. After the explosion:
- A) the vector momentum of each piece is identical  
B) the total momentum is increased  
C) the momentum of all the pieces, exhaust, and smoke adds up to zero  
D) not enough information to comment  
E) none of these

1. A car traveling at a certain speed undergoes a head-on collision with an identical vehicle traveling at the same speed, and both come to rest in 10 ms. Now suppose instead, that the car crashes into a brick wall and again comes to rest in 10 ms. From the perspective of the original driver, which would be less dangerous?

A. the two-car crash

B. the car-wall crash

C. both are identical

D. not enough information

E. none of these

$$\text{Impulse} = \Delta P = F_{\text{ave}} \Delta t$$

Both situations, as described,  
have the same average  
force.

2. When a bullet is fired from a gun which is free to recoil, the bullet and the gun

receive unequal:

A. angular momenta

B. forces

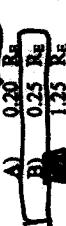
C. impulses

D. kinetic energies

E. linear momenta

5. Which of the following statements about the gravitational and elastic potential energies is true?
- A) An arbitrary constant can be added to each without altering the force.
- B) An arbitrary constant can be added to neither.
- C) An arbitrary constant can be added to the elastic energy but not the gravitational.
- D) An arbitrary constant can be added to the gravitational energy but not the elastic.

6. A rocket of mass  $1.5E6$  kg is launched upward from the surface of the Earth (mass  $M_E = 5.96E24$  kg, radius  $R_E = 6.38E6$  m) with a speed of 5000 m/s. The rocket will reach a maximum altitude above the surface of the Earth of



A)  $0.20 R_E$

B)  $0.25 R_E$

C)  $1.25 R_E$

D) infinity

$$\begin{aligned} \text{Int E} &= F_{\text{ext}} \cdot \epsilon \\ &= -G \frac{m}{r} \cdot \epsilon \\ &= \frac{1}{2} \gamma r^2 + G \frac{m}{r} = 0 + -G \frac{m}{r} \\ &= \frac{1}{2} (5000)^2 - (6.67 \cdot 10^{-11}) \frac{(5.96 \cdot 10^{24})}{(6.38 \cdot 10^6)} = 0 - (6.67 \cdot 10^{-11}) \frac{(5.96 \cdot 10^{24})}{(6.38 \cdot 10^6)} \\ &= -4.98 \times 10^{17} \text{ J} \\ r &= 7.98 \times 10^6 \text{ m} \\ r &= 1.25 R_E \end{aligned}$$

7. If an inelastic collision takes place between two objects and there are no external forces, then:

A) momentum and kinetic energy are conserved.

B) momentum is conserved, but kinetic energy is not.

C) kinetic energy is conserved, but momentum is not.

D) neither momentum nor kinetic energy is conserved.

E) it is not possible to determine if either momentum or kinetic energy are conserved without knowing more of the details of the collision.

7. A person ascends a flight of stairs in 20 s. The person weighs 500 N and the vertical height of the stairs is 9m. The person's power output is
- A)  $150 \text{ W}$
- B)  $225 \text{ W}$
- C)  $325 \text{ W}$
- D)  $450 \text{ W}$

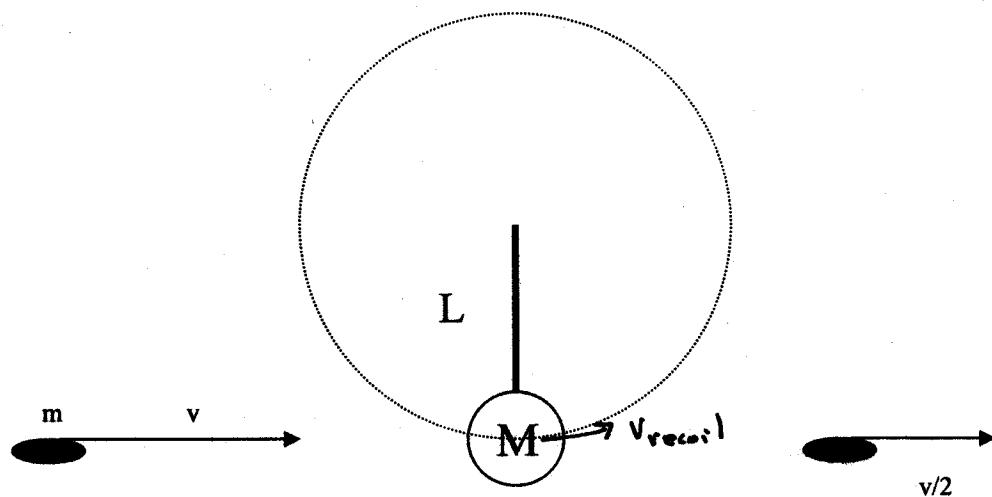
$$\begin{aligned} \text{Power} &= \frac{\text{work}}{\text{time}} = \frac{\text{change in Energy}}{\text{time}} \\ &= \frac{mg h}{t} = \frac{(500)(9)}{20} = 225 \text{ W} \\ F_{y1} &= -\frac{dU}{dy} = -[3x + 18y^2] \end{aligned}$$

8. Automobile airbags are effective because they
- A) increase the impulse time
- B) decrease the impulse
- C) decrease the momentum change
- D) decrease the time the impulse acts

Problem 1. A bullet of mass  $m$  and speed  $v$  is shot into a stationary pendulum bob of mass  $M$ . The pendulum bob is suspended by a stiff rod of length  $L$  and negligible mass.

The bullet passes completely through a pendulum bob and emerges with a speed  $v/2$ . As a result of the recoil, the pendulum bob swings in a complete circle.

What is the minimum value of  $v$  such that the pendulum bob will barely swing through a complete circle?



Do the collision first

Init Momentum = Final Momentum

$$mv_0 = m\frac{v_0}{2} + Mv_{\text{recoil}}$$

=

The bob swinging vertical is a conservation of energy problem.

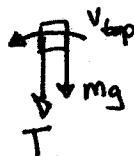
Energy at bottom = Energy at top

$$\frac{1}{2}Mv_{\text{recoil}}^2 = Mg(2L) + \frac{1}{2}Mv_{\text{top}}^2$$

Is the bob moving at the top?

- Well, I want to make sure it goes around.

- If it were moving really fast at the top, then the fbd at the top would be



$$M a_{rad} = \sum F_{rad}$$

$$m \frac{v_{top}^2}{R} = T + mg$$

as the bob goes across the top slower, the tension will also decrease.  
The smallest tension  $\rightarrow 0^+$

$$m \frac{v_{top}^2}{R} = 0^+ + mg$$

$$v_{top}^2 = gR = gL$$

then back-substitute

$$\begin{aligned} \frac{1}{2} M v_{recoll}^2 &= MgZL + \frac{1}{2} M v_{top}^2 \\ &= MgZL + \frac{1}{2} M g L \\ &= \frac{5}{2} MgL \\ v_{recoll}^2 &= 5gL \end{aligned}$$

$$mv_0 = m \frac{v_0}{2} + M v_{recoll}$$

$$\frac{1}{2} mv_0 = M v_{recoll}$$

$$v_0 = 2 \frac{M}{m} v_{recoll}$$

$$v_0^2 = \left[ 2 \frac{M}{m} \right]^2 v_{recoll}^2$$

$$v_0^2 = \left[ 2 \frac{M}{m} \right]^2 5gL$$

SP211.\*

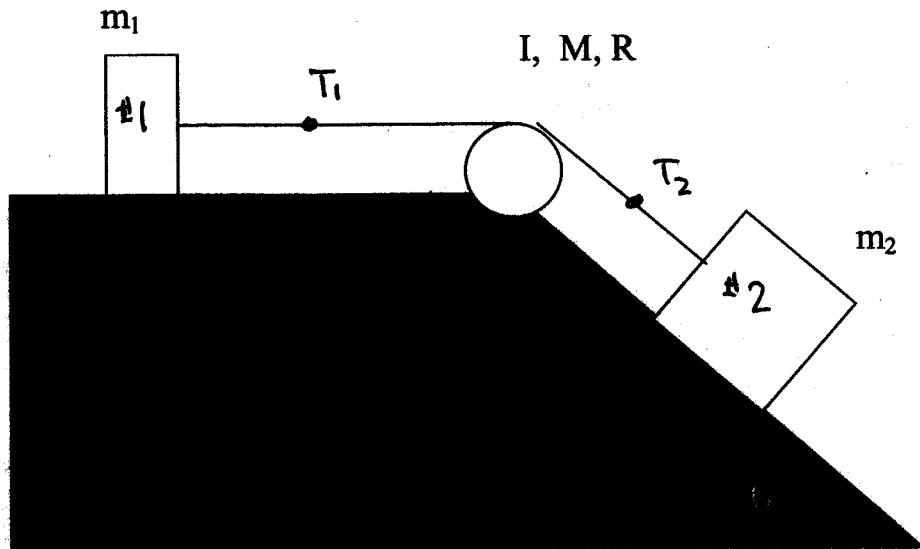
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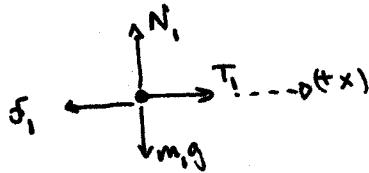
Problem 2. A block of mass  $m_1 = 2 \text{ kg}$  and a block of mass  $m_2 = 6 \text{ kg}$  are connected by a massless string over a pulley in the shape of a disk having radius  $R = 0.25 \text{ meter}$ , mass  $M = 10 \text{ kg}$ . In addition, they are allowed to move on a fixed block-wedge of angle  $\theta = 30^\circ$  as shown in the figure. The coefficient of kinetic friction is 0.36 for both blocks.

Determine the acceleration of the two blocks.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}10(0.25)^2 = 0.313 \text{ kgm}^2$$



THREE OBJECTS

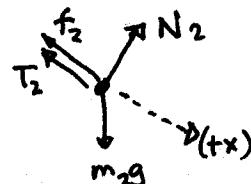


$$\begin{aligned} m_1 a_x &= T_1 - f_1 \\ &= T_1 - \mu N_1 \\ &= T_1 - \mu m_1 g \end{aligned}$$

$$I \alpha = T_2 R - T_1 R$$

$$\text{no slipping } a = R\alpha$$

$$\frac{I}{R^2} \alpha = T_2 - T_1$$



$$\begin{aligned} m_2 a &= m_2 g \sin \theta - T_2 - f_2 \\ &= m_2 g \sin \theta - T_2 - \mu N_2 \\ &= m_2 g \sin \theta - T_2 - \mu m_2 g \cos \theta \end{aligned}$$

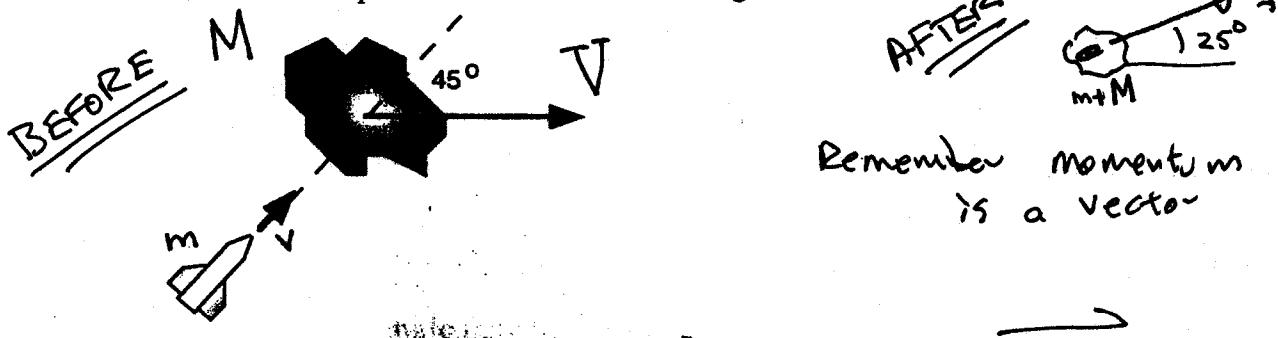
Add all three equations

$$(m_1 + m_2 + \frac{I}{R^2}) a = m_2 g \sin \theta - \mu m_2 g \cos \theta - \mu m_1 g$$

$$a = \frac{m_2 g \sin \theta - \mu m_2 g \cos \theta - \mu m_1 g}{m_1 + m_2 + I/R^2}$$

2. A large rock with a mass of  $2.50 \times 10^6$  kg is observed moving at 2000 m/s through free space in a direction toward the Earth as indicated in the figure below. You have been given the task of deflecting this object by 25 degrees from its current path to ensure that it won't collide with Earth. Your tool is a  $5.00 \times 10^4$  kg space probe that you will crash into the rock in a direction that makes an angle of 45 degrees with respect to the motion of the rock.

What speed is necessary for the probe to accomplish its mission? You can assume that the probe sticks to the rock during the collision.



Remember momentum  
is a vector

$$\frac{mv \cos 45^\circ + MV}{\cancel{mv \cos 45^\circ} + \cancel{MV}} = \frac{\text{Initial Momentum}}{\text{Final Momentum}}$$

$$mv \sin 45^\circ = (m+M) v_f \sin 25^\circ$$

To do the algebra, divide the equations

$$\frac{mv \sin 45^\circ}{mv \cos 45^\circ + MV} = \frac{(m+M) v_f \sin 25^\circ}{(m+M) v_f \cos 25^\circ} = 0.466$$

Then

$$mv \sin 45^\circ = [mv \cos 45^\circ + MV] 0.466$$

$$mv [\sin 45^\circ - 0.466 \cos 45^\circ] = MV 0.466$$

$$(5 \times 10^4) v [\sin 45^\circ - 0.466 \cos 45^\circ] = (2.50 \times 10^6)(2000)(0.466)$$

$$v = 1.23 \times 10^5 \text{ m/s}$$