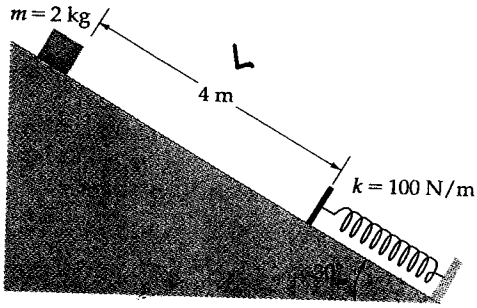
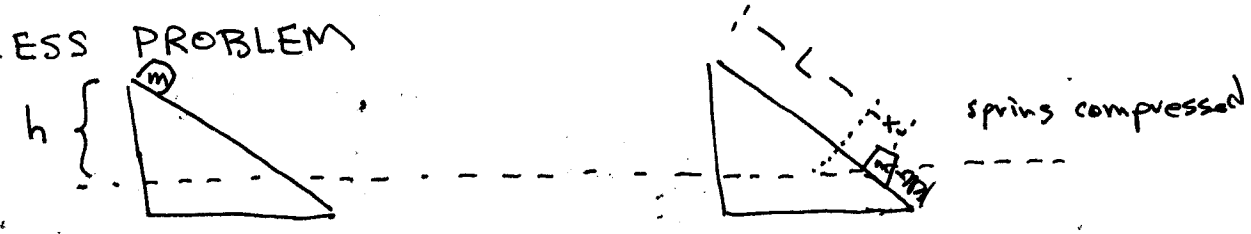
Figure 6-41 Problem 59.



59. A 2-kg block is released 4 m from a massless spring with a force constant $k = 100$ N/m that is fixed along a frictionless plane inclined at 30° as shown in Figure 6-41. (*a*) Find the maximum compression of the spring. (*b*) If the plane is not frictionless and the coefficient of kinetic friction between it and the block is 0.2, find the maximum compression. (*c*) For the rough incline, how far up the incline will the block travel after leaving the spring?

FRICTIONLESS PROBLEM



h is the height above the final location

$h = L \sin 30^\circ$ ← doesn't include spring compression

$= (L + x) \sin 30^\circ$ includes spring compression

Init Total Energy = Final Total Energy

$mgh = \frac{1}{2} k x^2$

$mg(L+x)\sin 30^\circ = \frac{1}{2} k x^2$

$2(9.8)(4+x)\,0.5 = \frac{1}{2} 100 x^2$

$39.2 + 9.8x = 50x^2$

$0 = 50x^2 - 9.8x - 39.2$

$x = \frac{9.8 \pm \sqrt{(9.8)^2 - (4)50(-39.2)}}{2(50)} = \frac{9.8 \pm 89.1}{100}$

$= 0.989$ m

WITH FRICTION

Same basic problem except include the frictional loss as we slide down the plane. The frictional loss will be $f(L+x) \rightarrow \mu N (L+x) \rightarrow \mu mg\cos\theta\,(L+x)$

Init Total Energy − Frictional Losses = Final Tot Energy

$mgh - \mu mg\cos\theta\,(L+x) = \frac{1}{2} k x^2$

$mg(L+x)\sin 30^\circ - \mu mg\cos\theta\,(L+x) = \frac{1}{2} k x^2$

$mg(L+x)[\sin 30^\circ - \mu \cos 30^\circ] = \frac{1}{2} k x^2$

$2(9.8)(4+x)[\,0.327\,] = \frac{1}{2} 100 x^2$ ($\mu = 0.2$)

$0 = 50x^2 - 6.41x - 25.64$

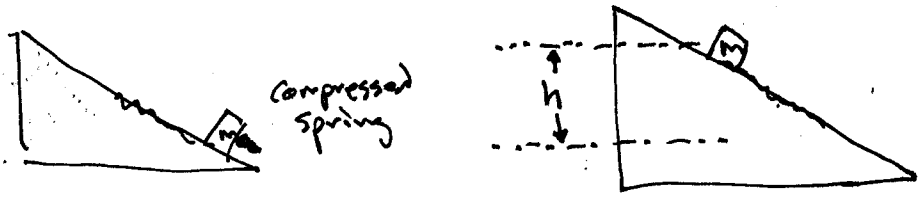
(over)

$$0 = 50x^2 - 6.41x - 25.64$$

$$x = \frac{6.41 \pm \sqrt{(6.41)^2 - (4)50(-25.64)}}{100}$$

$$= \frac{6.41 \pm 71.69}{100} = 0.783 \text{ meters}$$

SHOOTING IT BACK UP THE PLANE



Init Tot. Energy $-$ (frictional loss) $=$ Final Tot. Energy

$$\frac{1}{2}kx^2 - \mu mg\cos\theta(L+x) = mgh$$

$$\frac{1}{2}(100)(0.783)^2 - 0.2(2)(9.8)\cos 30^\circ (L+x) = \frac{mg}{2(9.8)}(L+x)\sin 30^\circ$$

$$30.65 - 3.39(L+x) = 9.8(L+x)$$

$$30.65 = 13.12(L+x)$$

$$2.32 = L+x \quad \text{from previous}$$

$$2.32 = L + 0.783$$

$$L = 1.54 \text{ from natural length of spring.}$$